SABRE NOZZLE

Supersonic Air-Breathing Redesigned Engine Nozzle

Critical Design Review

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Air Force Research Lab

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Presentation Outline

- Project Description
- Design Solution
- Critical Project Elements
- Design Requirement Satisfaction
- Risk Analysis
- Verification and Validation
- Project Planning

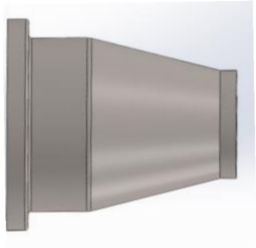
Project Description

Project Description

Model, manufacture, and **verify** an **additive manufactured nozzle** capable of accelerating flow to **supersonic exhaust** produced by a **P90-RXi JetCat** engine maintaining the **T/W ratio** from its stock configuration.

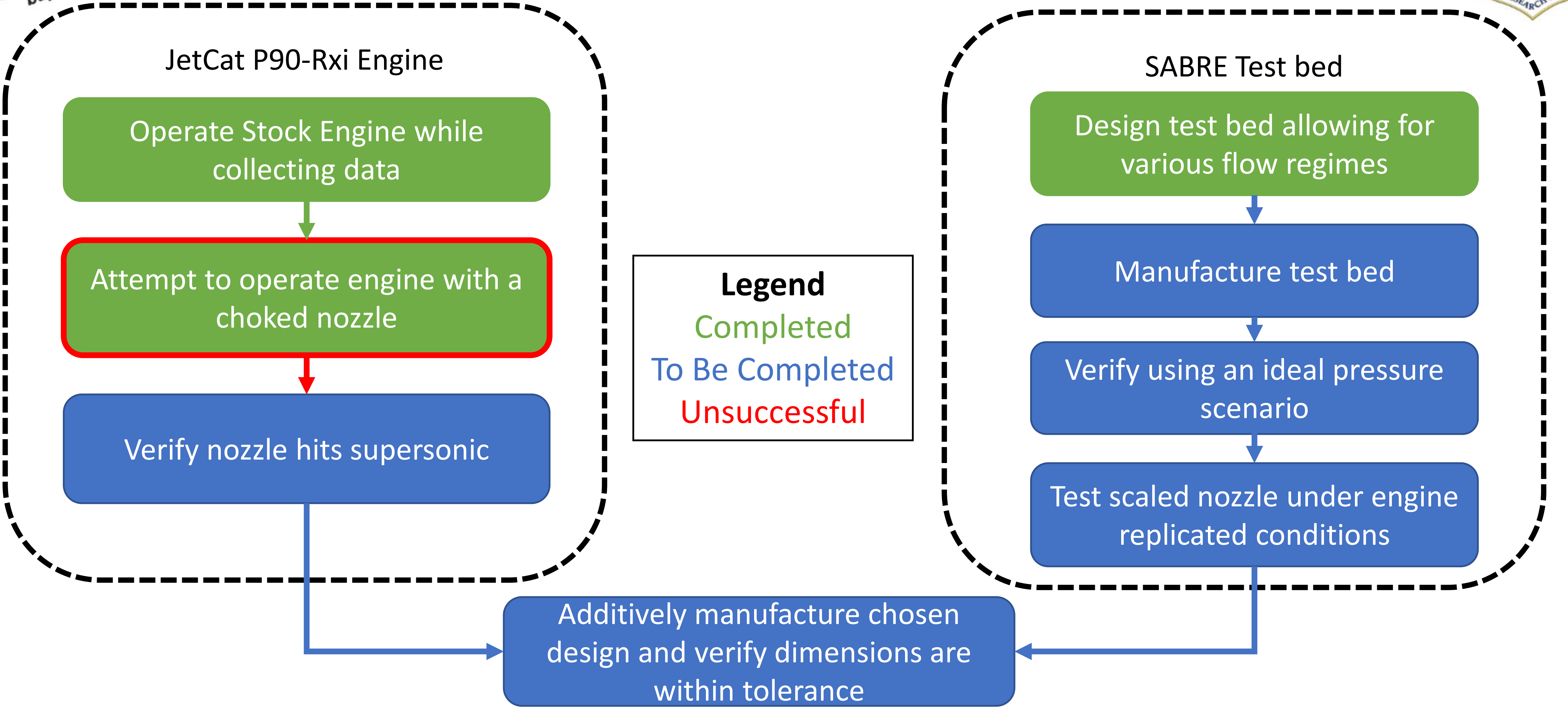
Stock Nozzle

Vs.



Supersonic Nozzle

Project Pathway



JetCat P90-Rxi Engine

Operate Stock Engine while collecting data

Attempt to operate engine with a choked nozzle

Verify nozzle hits supersonic

Legend

Completed

To Be Completed

Unsuccessful

SABRE Test bed

Design test bed allowing for various flow regimes

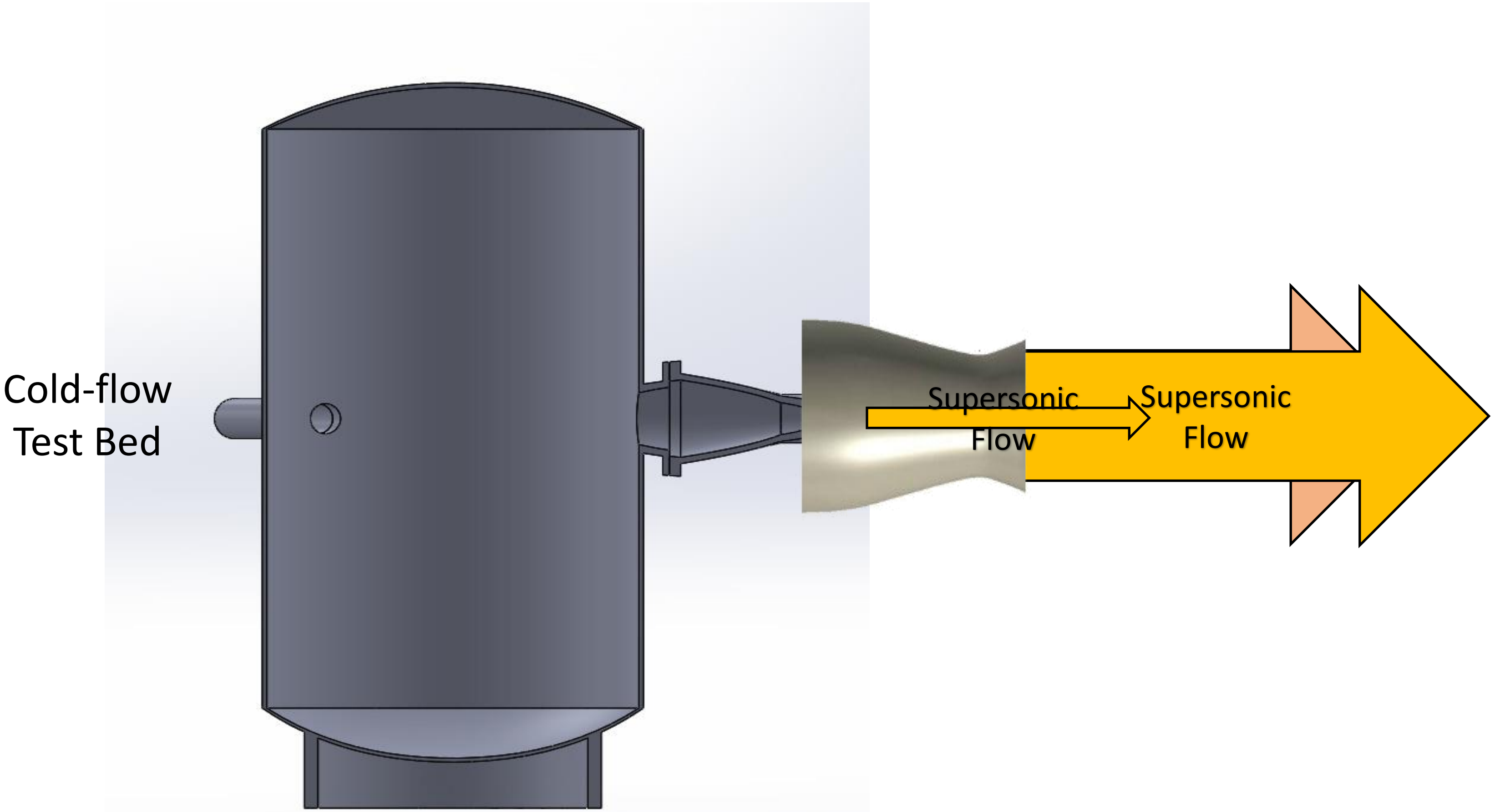
Manufacture test bed

Verify using an ideal pressure scenario

Test scaled nozzle under engine replicated conditions

Additively manufacture chosen design and verify dimensions are within tolerance

Testbed Concept of Operations



1. Integrate Nozzle to JetCat
2. Operate Supersonic Engine
3. Replace Engine with Test Bed Design
4. Scale Down Nozzle
5. Integrate and test supersonic nozzle

Objectives/Requirements

- **FR 1:** The Nozzle accelerates flow from subsonic to supersonic conditions.

- **FR 2:** The Nozzle shall maintain/increase the Thrust-to-Weight Ratio.

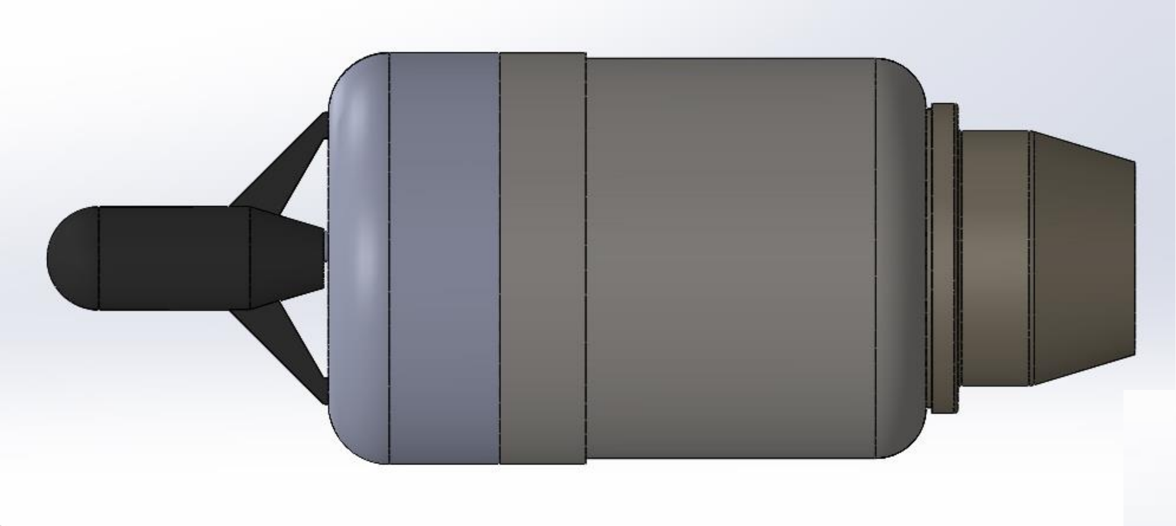
- **FR 3:** The Nozzle shall be designed and manufactured such that it will integrate with the JetCat Engine.

- **FR 4:** The Nozzle shall be able to withstand engine operation for at least 30 seconds.

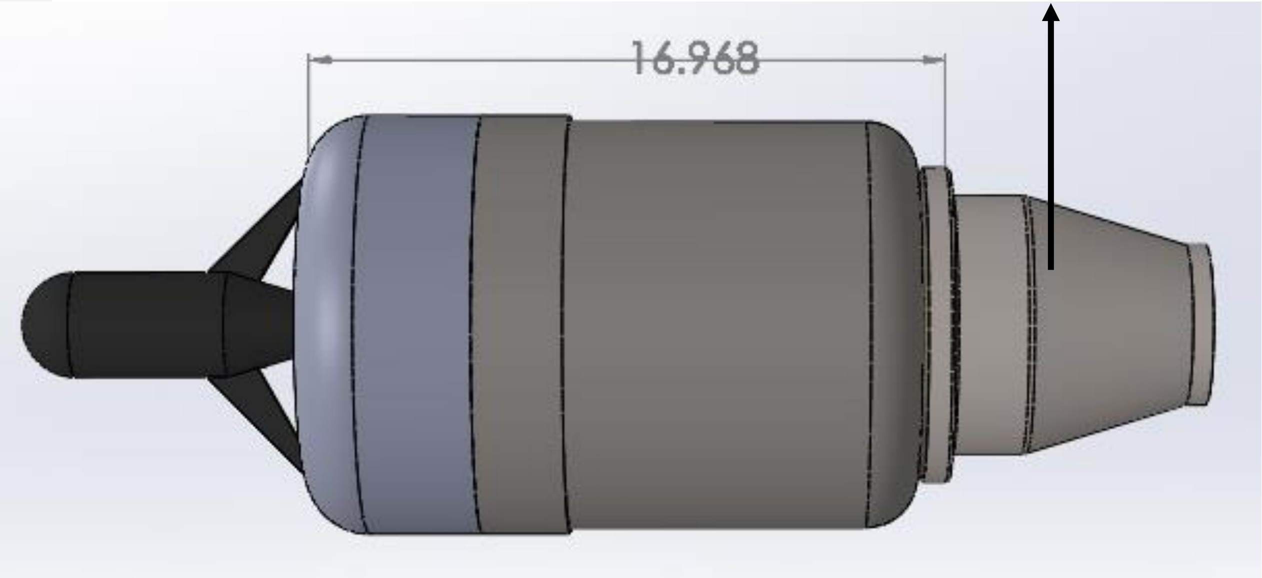
- **FR 5:** The Nozzle's performance shall be verified and validated through the use of an alternate cold-flow test bed.

Design Solution

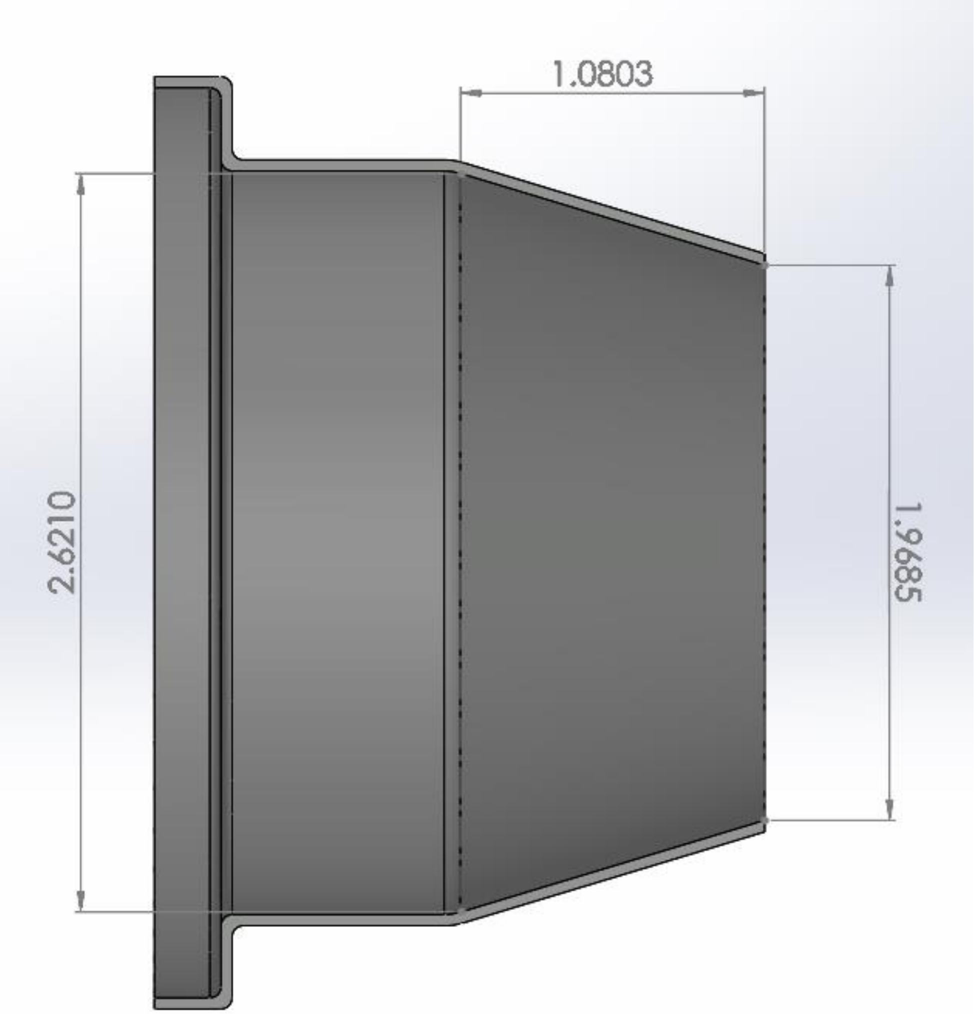
Design Solution- Nozzle



SABRE Nozzle Design unchanged

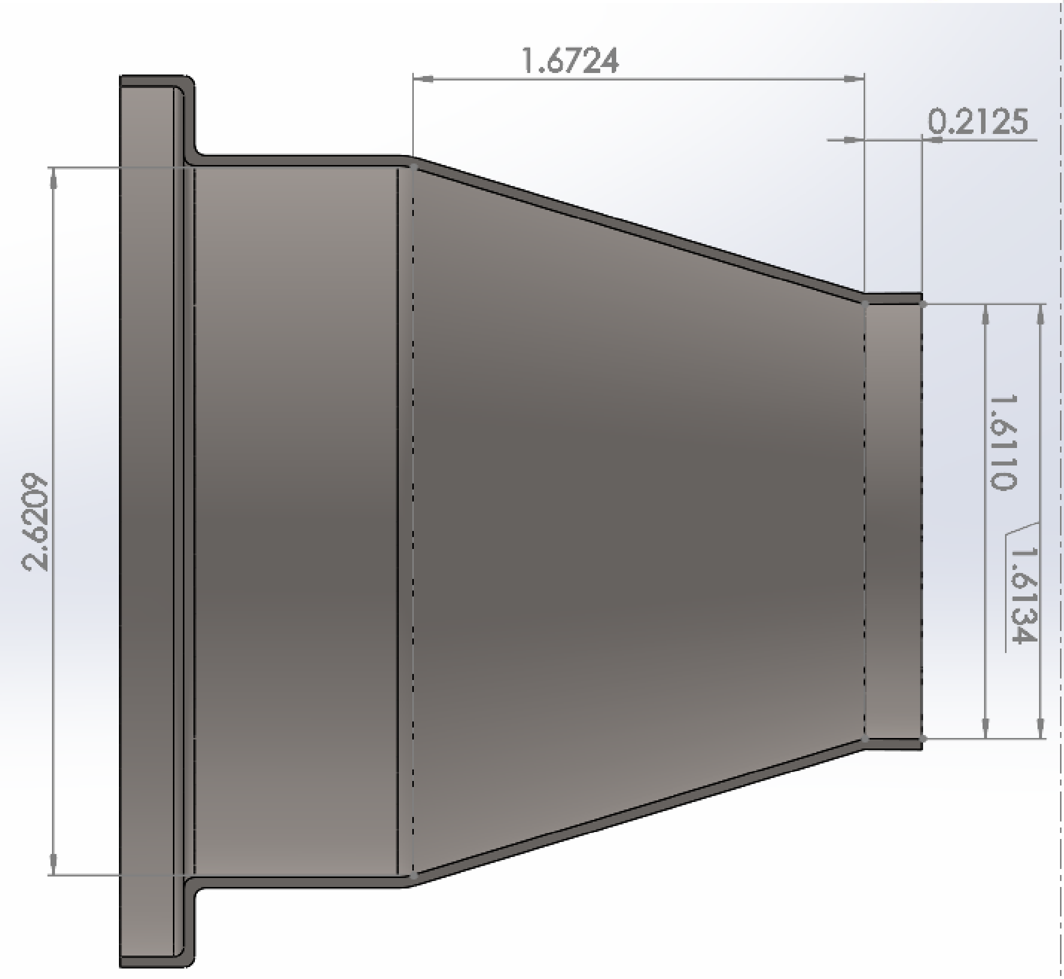


Design Solution – Full-scale Nozzles



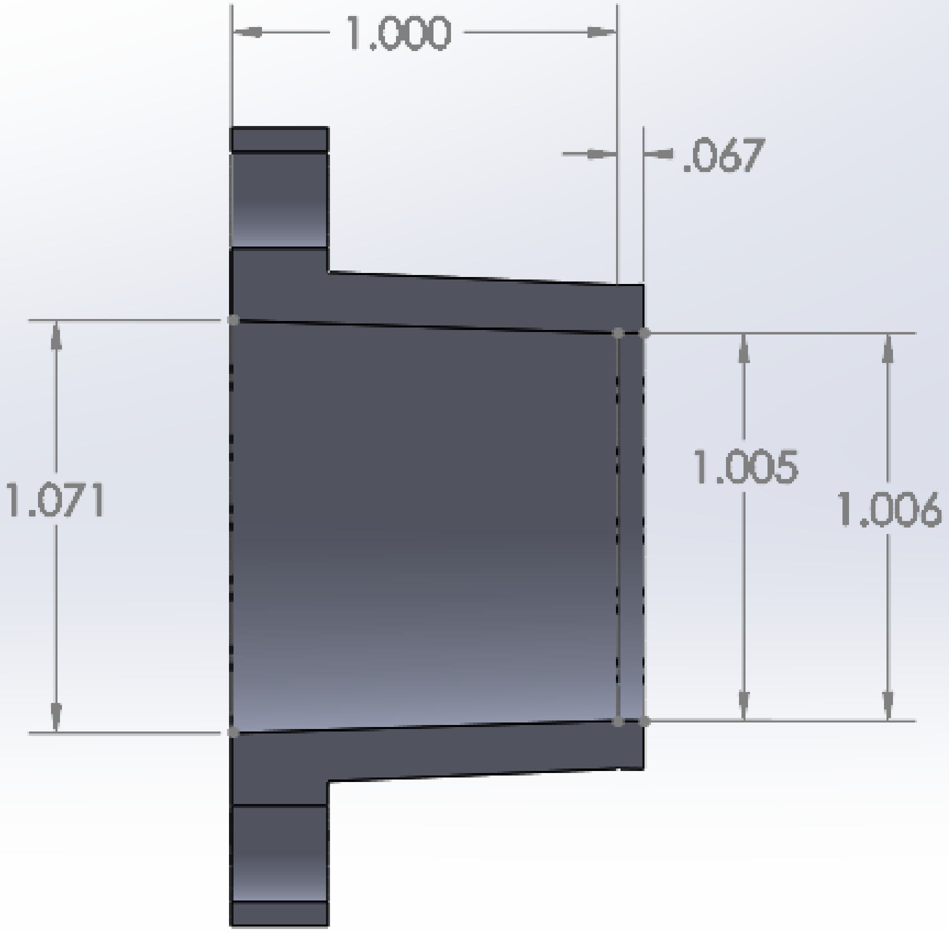
Left:
Original Nozzle
M = 0.65
Steel
Existing

Right:
SABRE Nozzle
M = 1.06
Cobalt Chrome
DMLS



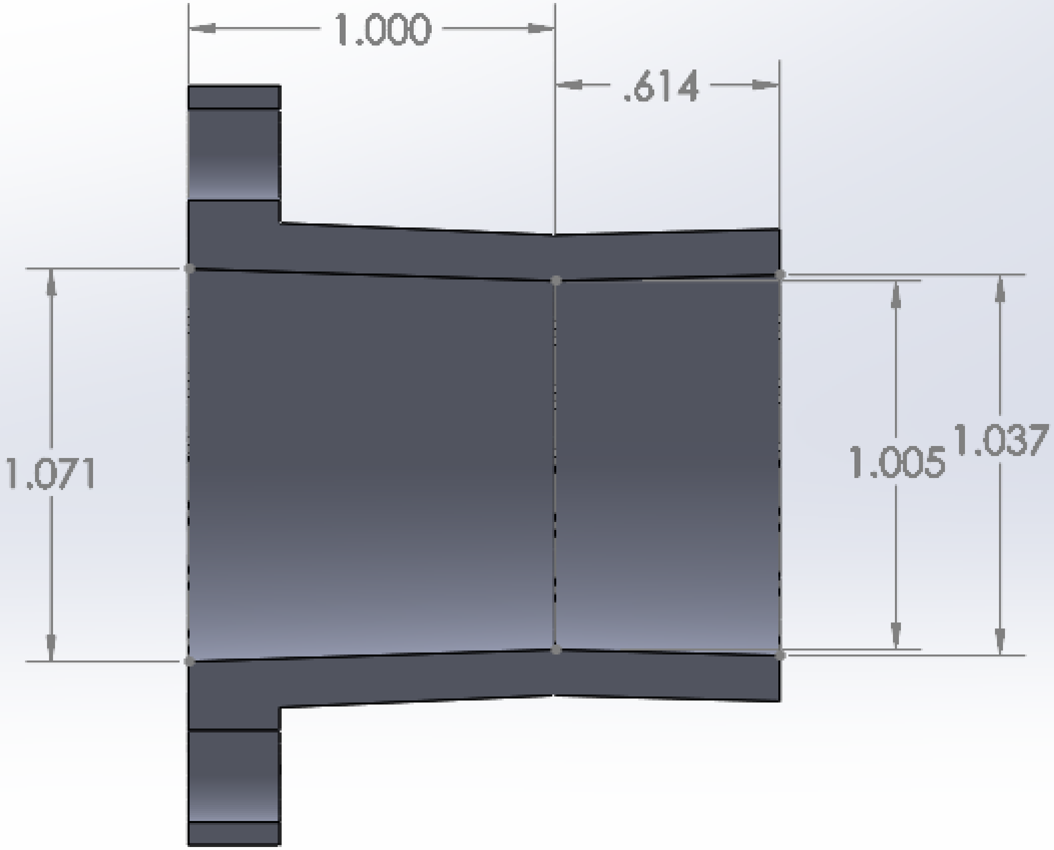
All units in inches

Design Solution - Scaled Nozzles



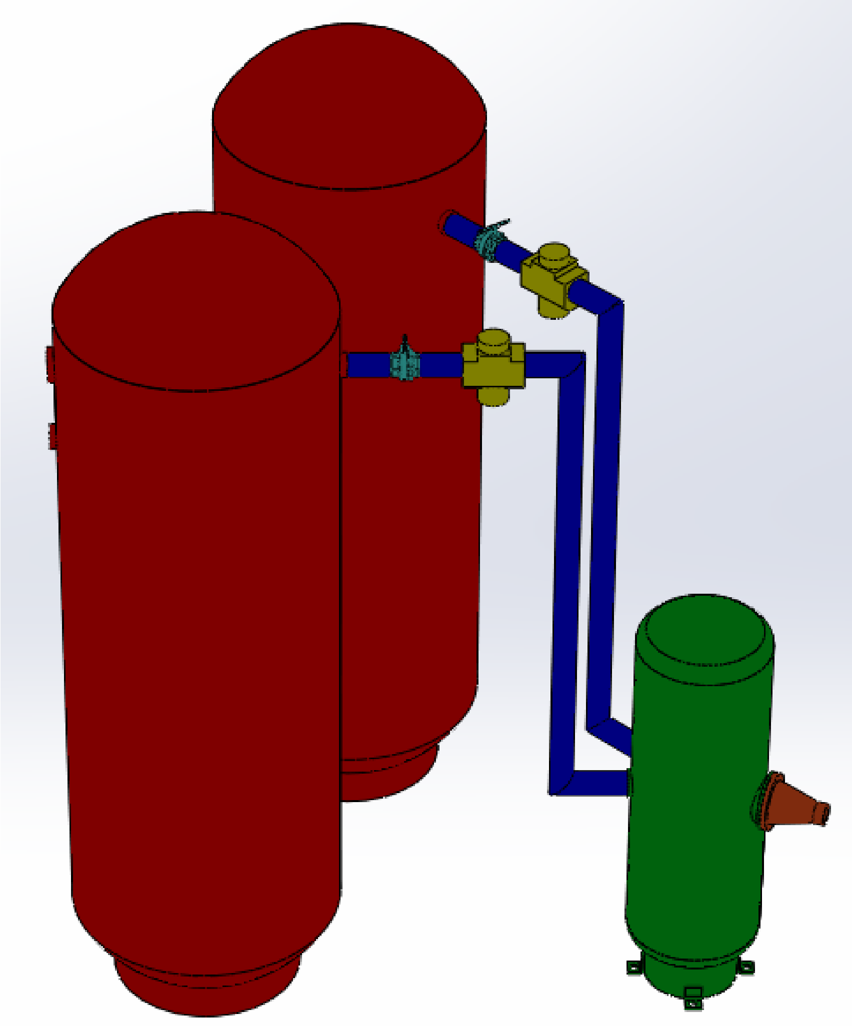
Left:
Scaled SABRE Nozzle
M = 1.03
Clear FLGPCL02
3D Printed

Right:
Validation Nozzle
M = 1.3
Clear FLGPCL02
3D Printed



All units in inches

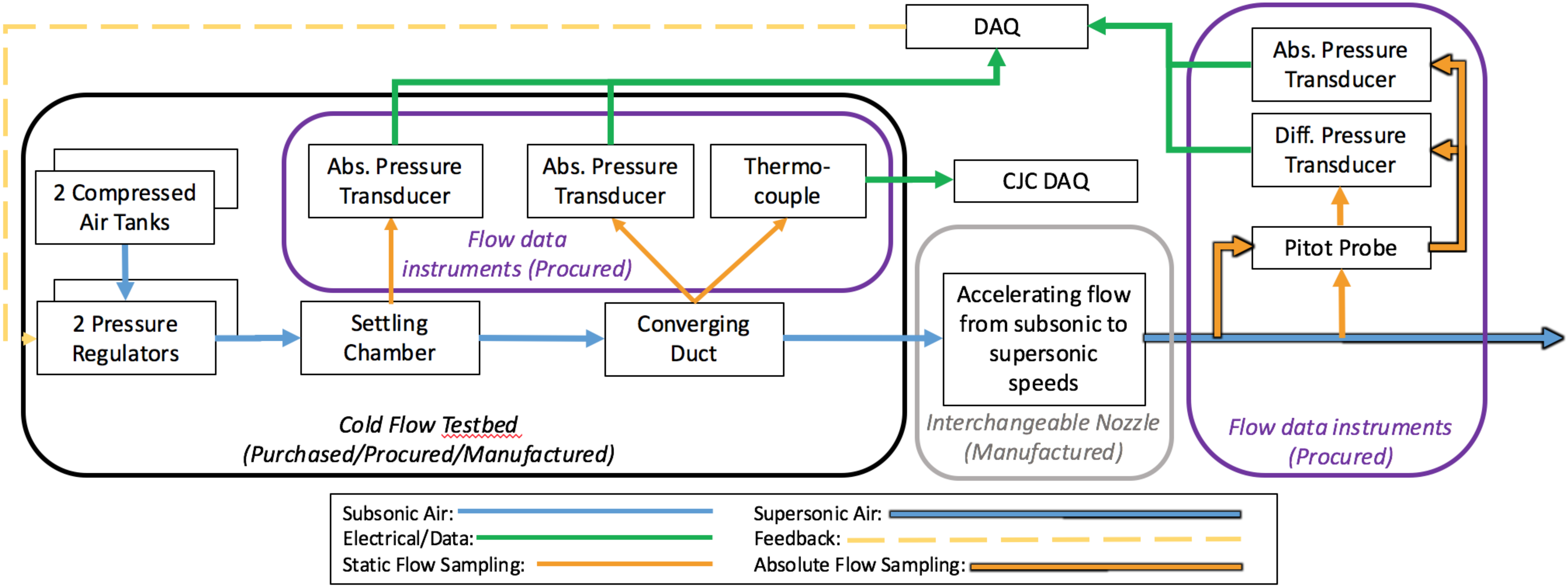
Design Solution - Test Bed



Major Components

- 2x Air supply tanks (175 psi)
- 2x Shut-off valves
- 2x Pressure regulators
- 1x Settling chamber
- 1x Converging nozzle
- 3x Passive pressure release valves
- ASME Compliant
- Filled with external tank @ 3000 psi

Design Solution - Test Bed



Critical Project Elements

CPE 1: Engine Operation

Stock Test Verification

Additive Manufacturing

Nozzle Survivability

Modified Nozzle Verification

CPE 2: Test Bed Operation

Test Bed Verification

Nozzle Survivability

Nozzle Design Validation

Design Requirements & Satisfaction

CPE 1 – Engine Operation

FR 1	The Nozzle shall accelerate the flow from subsonic to supersonic conditions.
DR 1.1	The flow through the nozzle shall be choked such that the nozzle exit flow Mach is greater than 1.

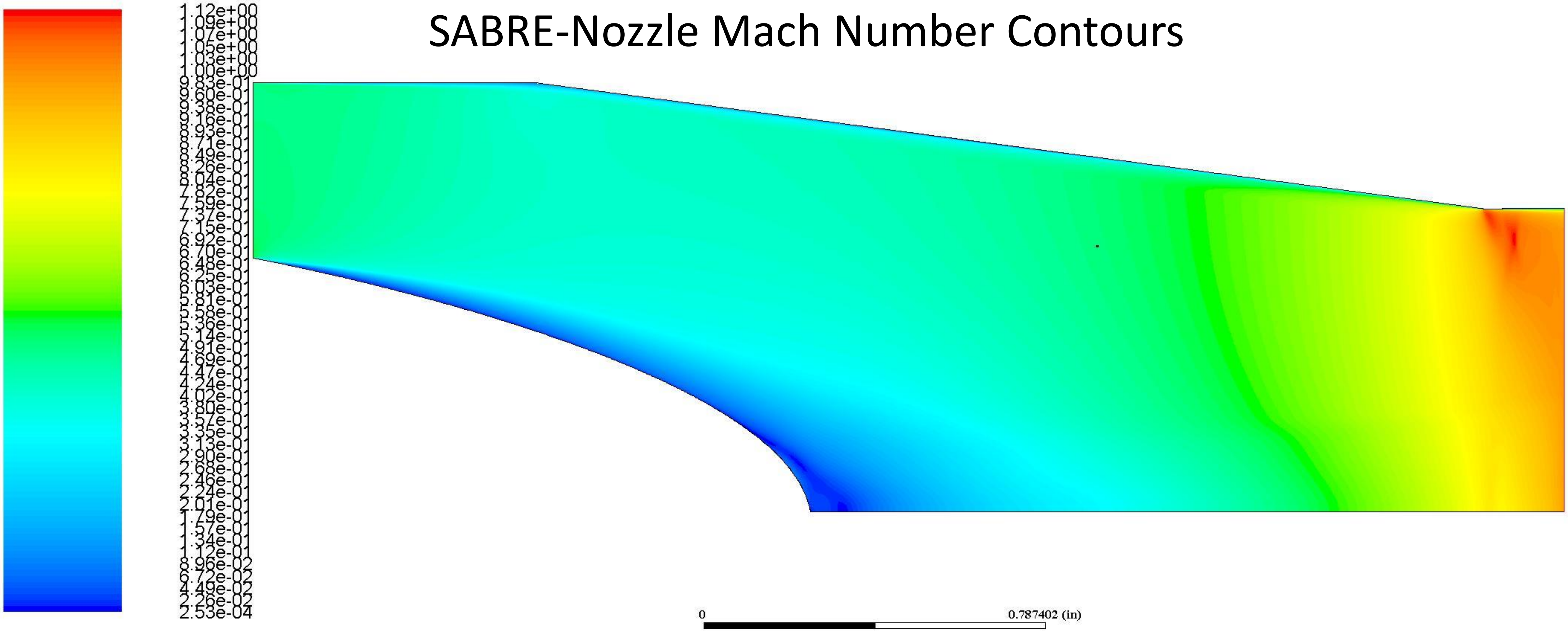
FR 2	The Nozzle shall not decrease the Thrust/Weight ratio.
DR 2.2	The thrust of the engine shall be increased to 120 N.

FR 3	The Nozzle shall be designed and manufactured such that it will integrate with the JetCat Engine.
DR 3.1	The Nozzle shall be manufactured with additive manufacturing.
DR 3.4	Successful integration of the nozzle shall not render the engine inoperable after the nozzle is detached and the engine is returned to its stock configuration.

FR 4	The Nozzle shall be able to withstand engine operation for at least 30 seconds.
DR 4.1	The nozzle shall have a melting point higher than 1100K.
DR 4.4	The thrust of the engine shall not decrease over a 30 second span.
DR 4.5	The nozzle shall survive the pressure and forces of engine operation.

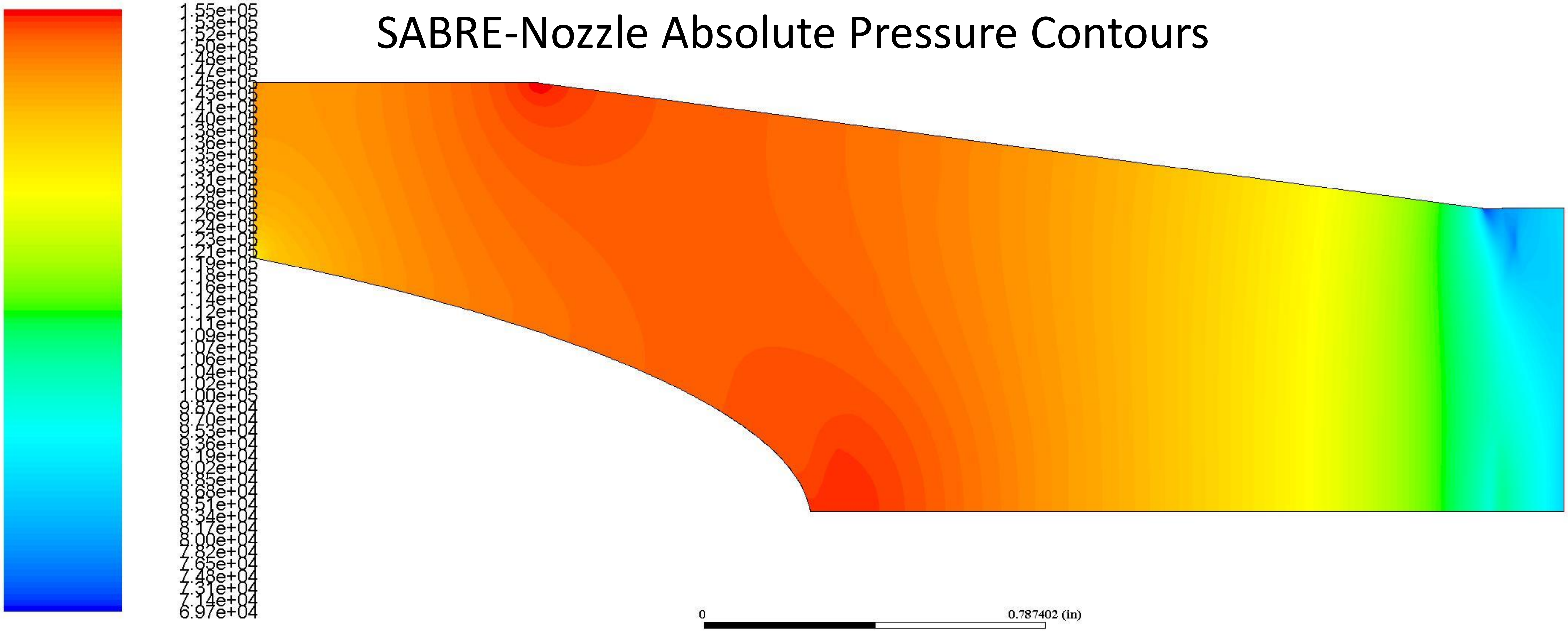
Validation of DR's 1.1

SABRE-Nozzle Mach Number Contours



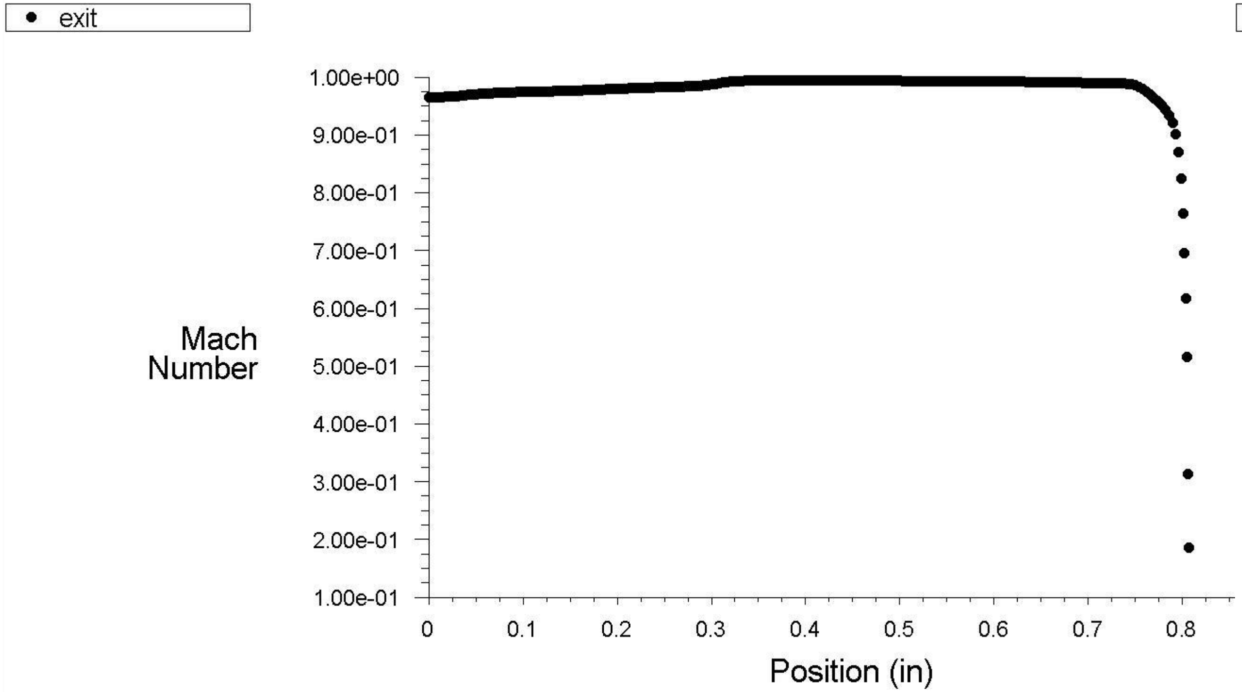
Validation of DR's 1.1

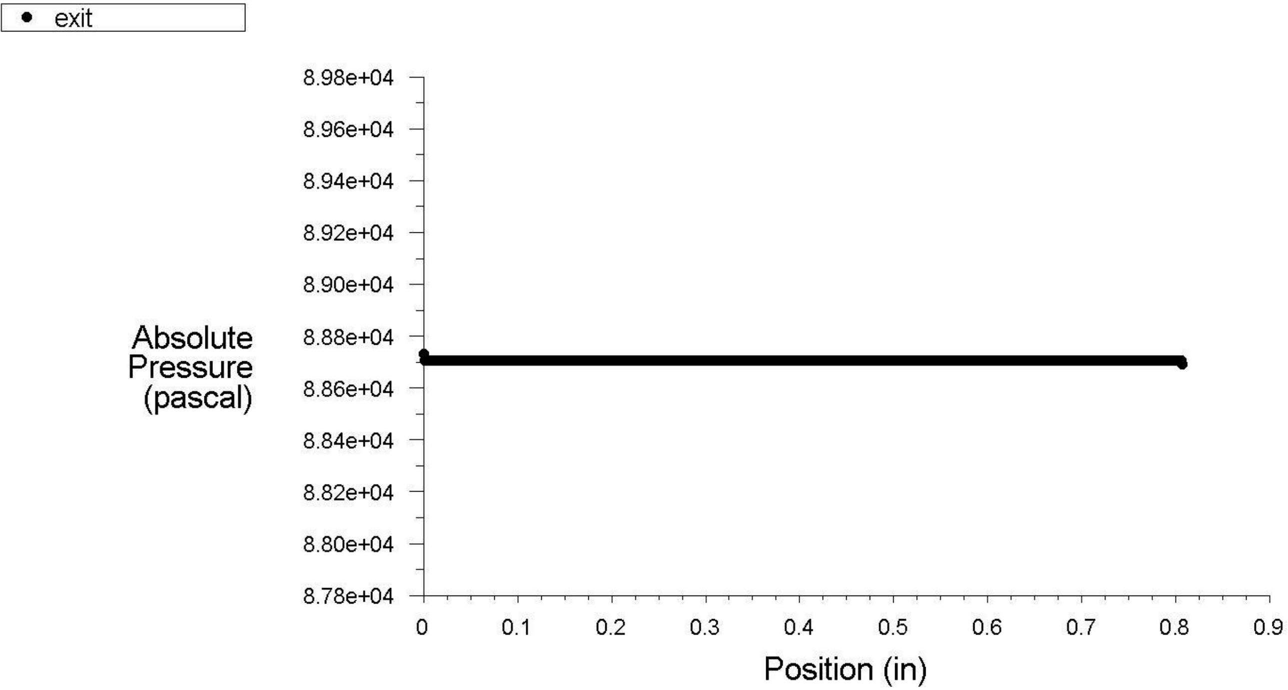
SABRE-Nozzle Absolute Pressure Contours



Validation of DR's 1.1

SABRE-Nozzle Exit Conditions

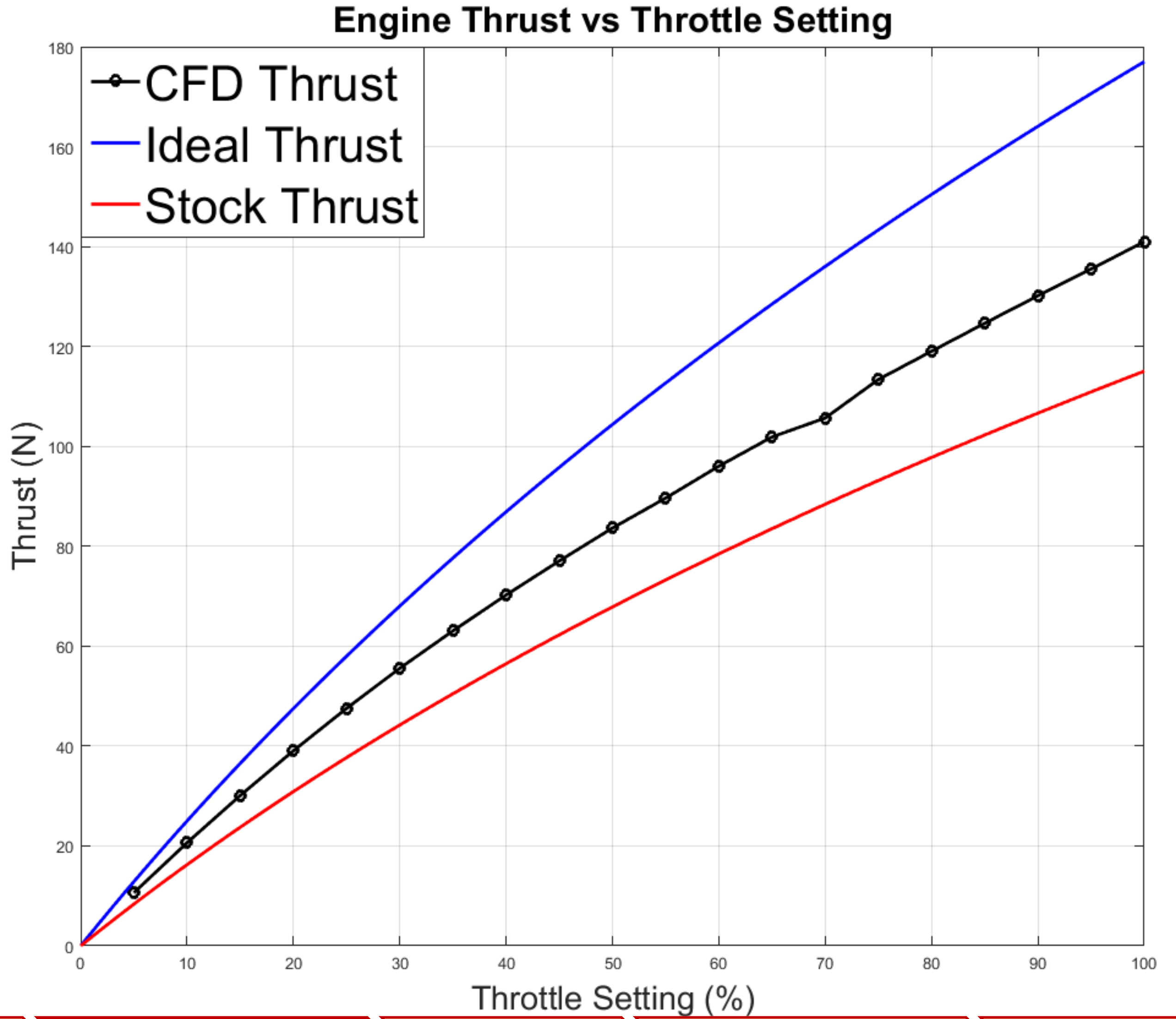




Validation of DR's 2.2

- DR 2.2 Thrust = 120 N
- Maximum Ideal Thrust = 177 N
- Maximum CFD Thrust = 140 N
- Maximum Stock Thrust = 105 N

- Takeaway: DR 1.1 and 2.2 Satisfied by SABRE-Nozzle Design



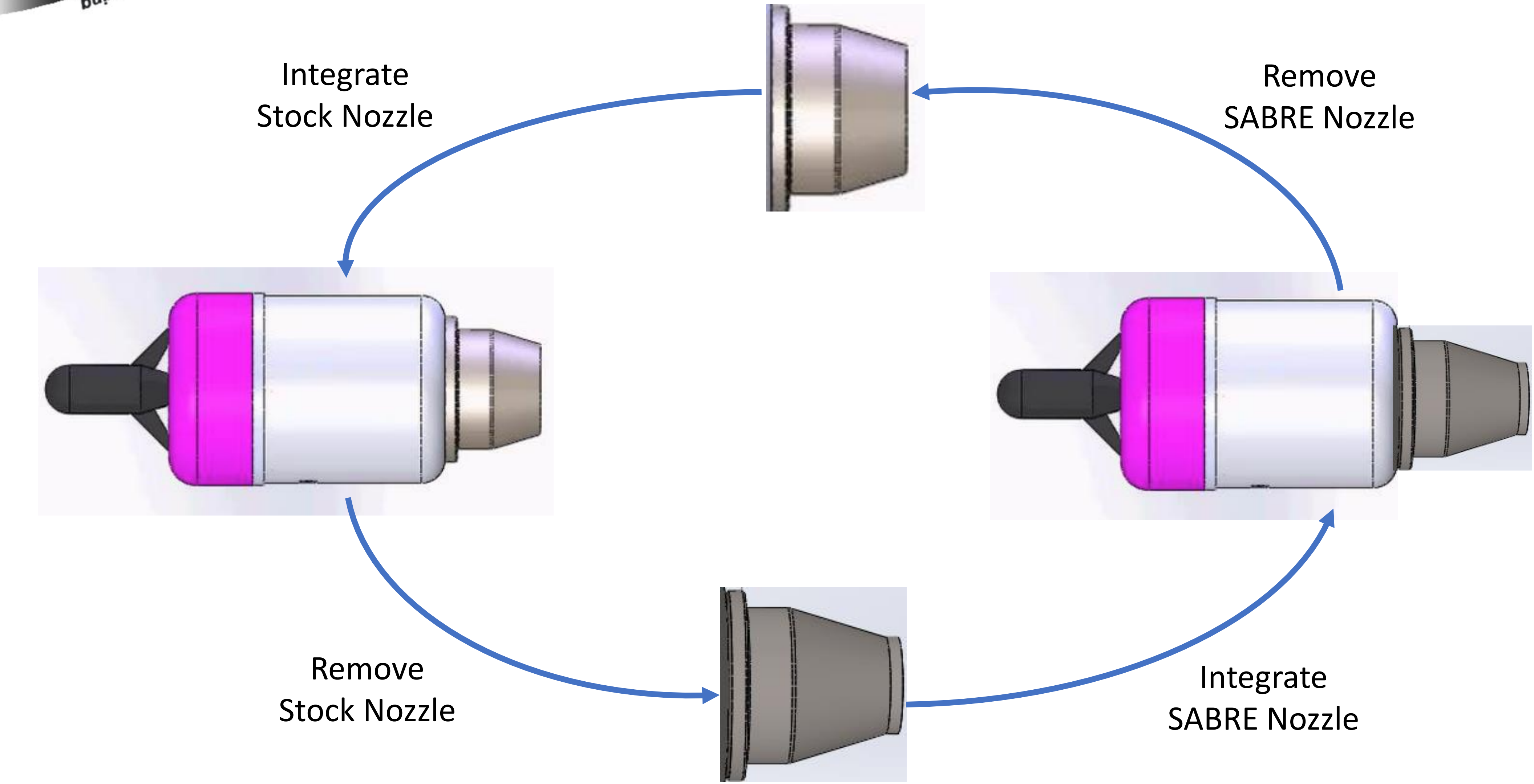
Additive Manufacturing (DR 3.1)

- Direct Metal Laser Sintering (DMLS)
  - Additive Manufacturing Technique
  - Laser binds layers of sinter powdered material together

- Accuracy: 0.0005-0.001"
  - Post Machined: down to 1µm
  - Resources: CU Boulder Machine Shop or Stratysys Company

DMLS	Cobalt Chrome	Inconel 718
Price	$727	$662
Temperature Rating	2100 °F (1423 K)	1200 °F (922 K)
Density	8.5 g/cm^3	8.22 g/cm^3
Mass	137.2 g	132.7 g

Nozzle Interchangeability (DR 3.4)



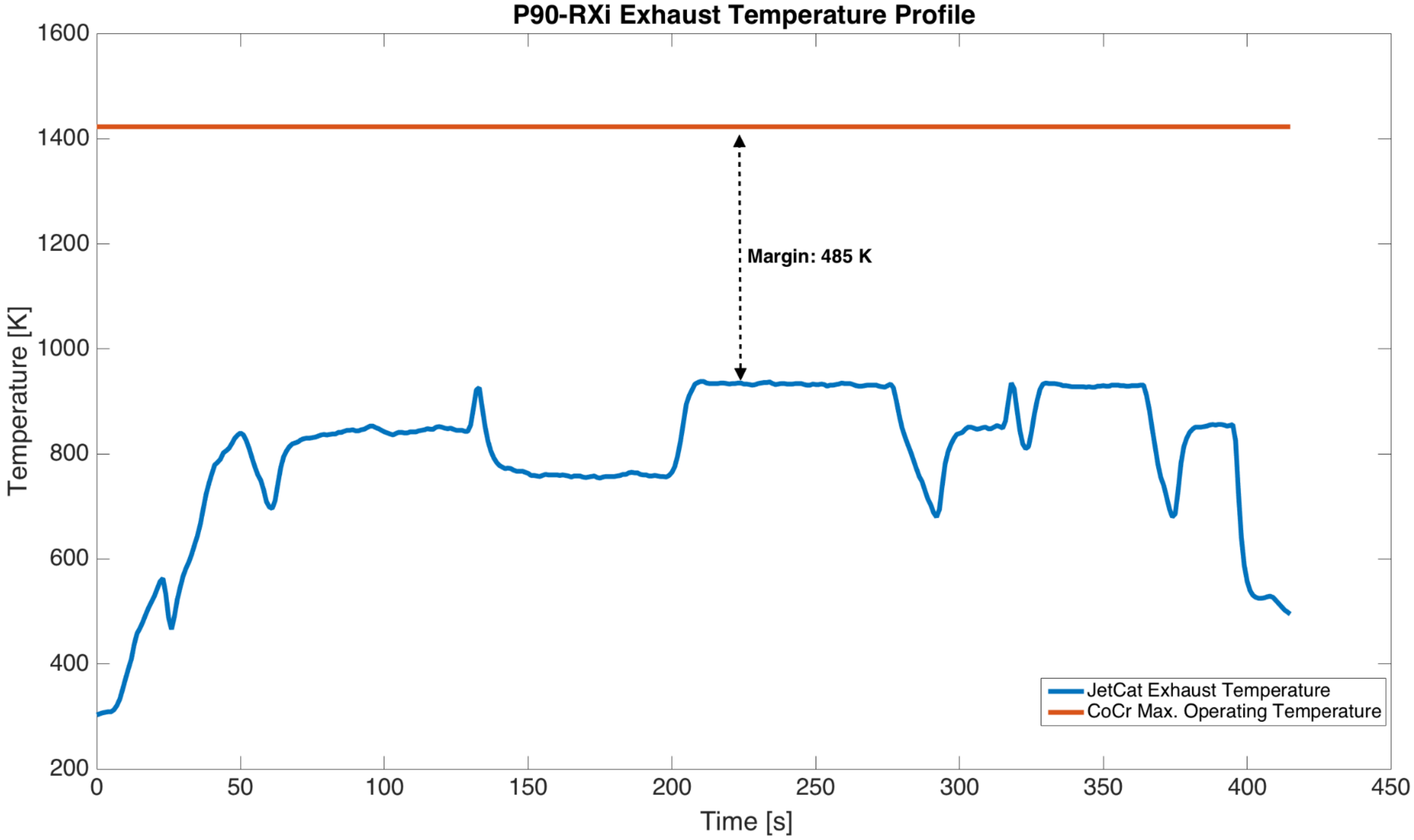
Integrate
Stock Nozzle

Remove
SABRE Nozzle

Remove
Stock Nozzle

Integrate
SABRE Nozzle

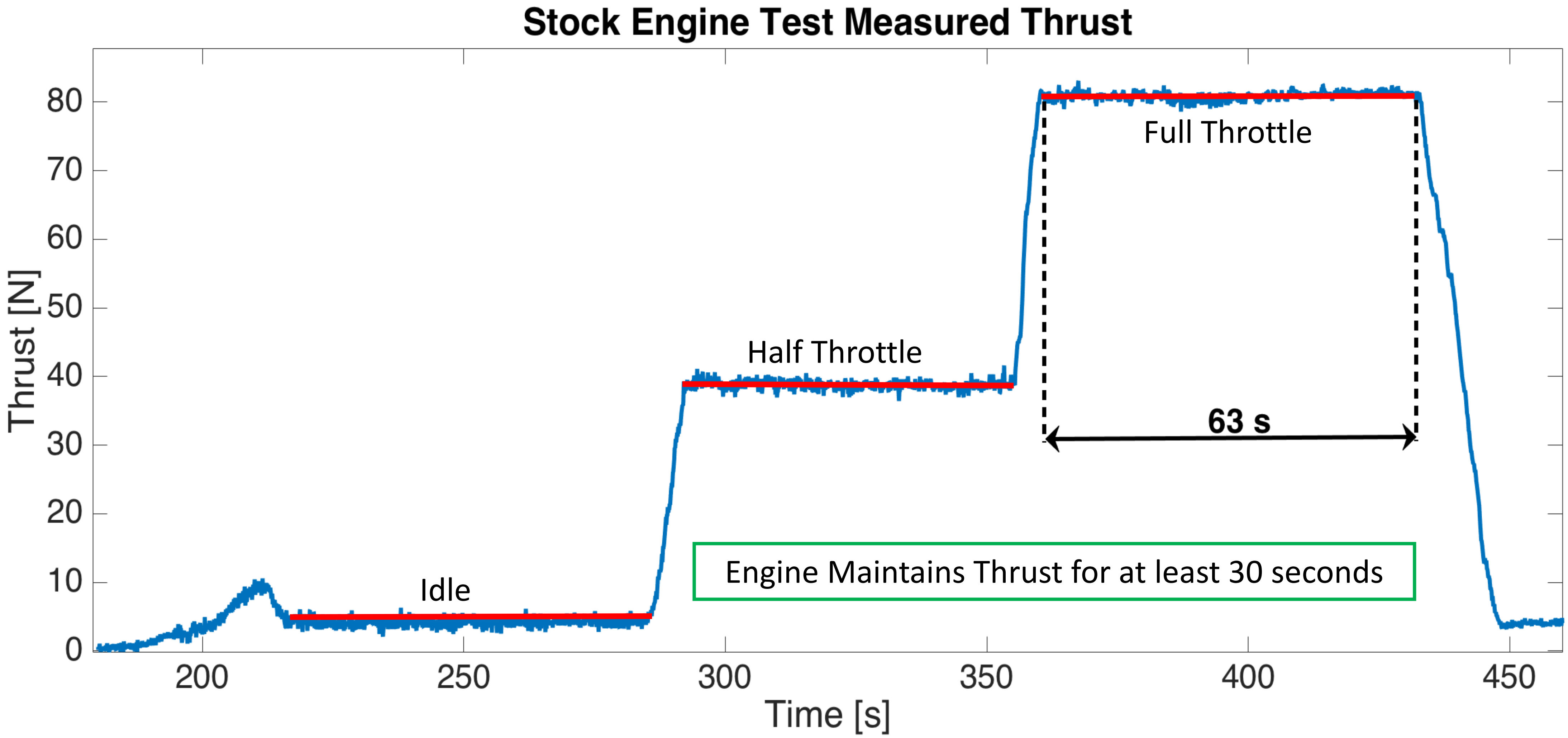
Thermal Survivability (DR 4.1)



- CoCr Max Operating Temp.:
  - 1423 K
- P90-Rxi Max Exhaust Temp.:
  - 938 K
- **Margin:**
  - **485 K**

The exhaust temperature is below the max. operating temperature

Testing Duration (DR 4.4)



Engine Maintains Thrust for at least 30 seconds

Operational Stress (DR 4.5)

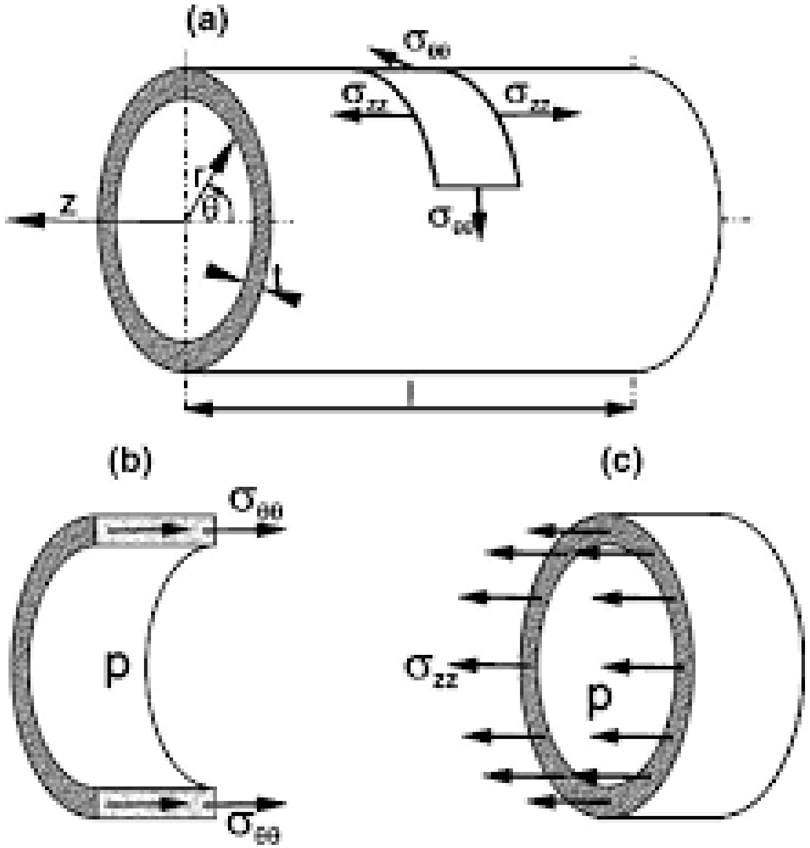
Assuming thin-walled pressure vessel (t/R << 1)

Maximum Gauge Pressure within nozzle: 60kPa
Nozzle Thickness: 1mm

Maximum Thrust Stress: 0.6MPa
Maximum Hoop Stress: 1.5 MPa
CoCrMo Proof Strength: 600 MPa

$$\sigma_{\theta\theta} = \frac{pR}{t}$$

Operational Stresses are well under the Proof Strength



Simulated CEngineOperation CEngine Operation

FR 1	The Nozzle shall accelerate the flow from subsonic to supersonic conditions.
DR 1.1	The flow through the nozzle shall be choked such that the nozzle exit flow Mach is greater than 1.

✓ Ideally, Flow at the exit is supersonic and chokes at throat.

FR 2	The Nozzle shall not decrease the Thrust/Weight ratio.
DR 2.2	The thrust of the engine shall be increased to 120 N.

✓ Thrust of 120 N can be achieved at 80% throttle power.

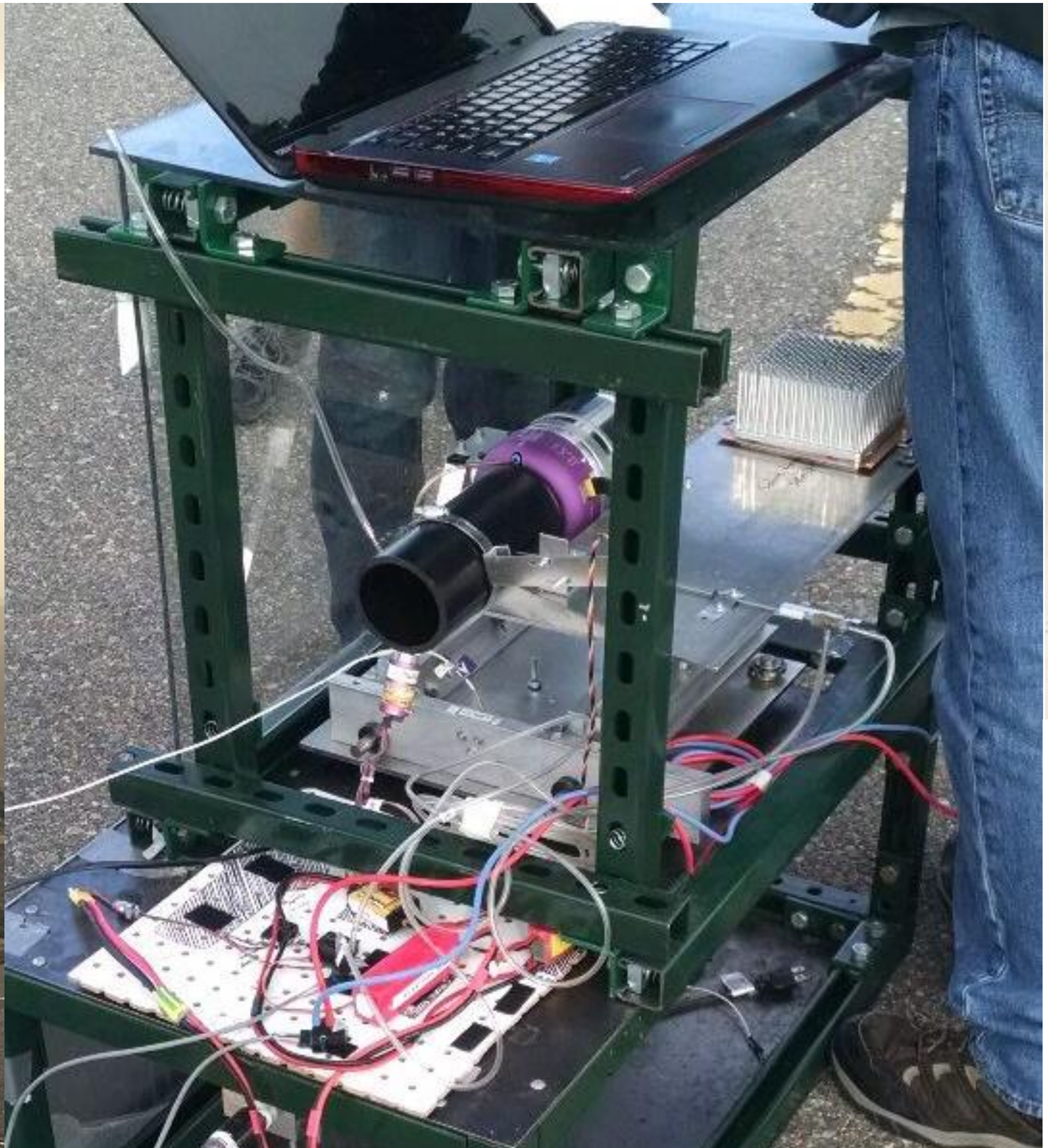
FR 3	The Nozzle shall be designed and manufactured such that it will integrate with the JetCat Engine.
DR 3.1	The Nozzle shall be manufactured with additive manufacturing.
DR 3.4	Successful integration of the nozzle shall not render the engine inoperable after the nozzle is detached and the engine is returned to its stock configuration.

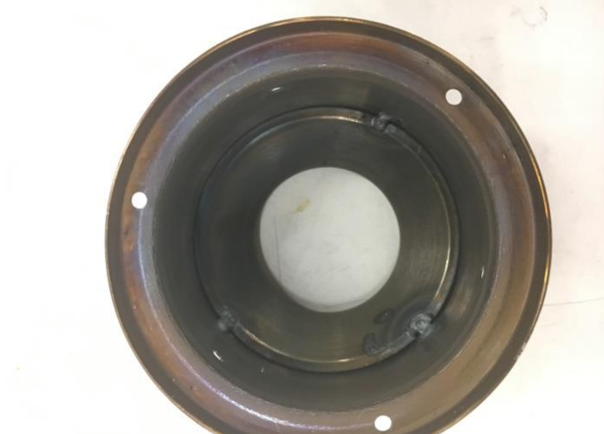
✓ Nozzle additively manufactured by company. Nozzle is interchangeable between stock/SABRE nozzles.

FR 4	The Nozzle shall be able to withstand engine operation for at least 30 seconds.
DR 4.1	The nozzle shall have a melting point higher than 1100K.
DR 4.4	The thrust of the engine shall not decrease over a 30 second span.
DR 4.5	The nozzle shall survive the pressure and forces of engine operation.

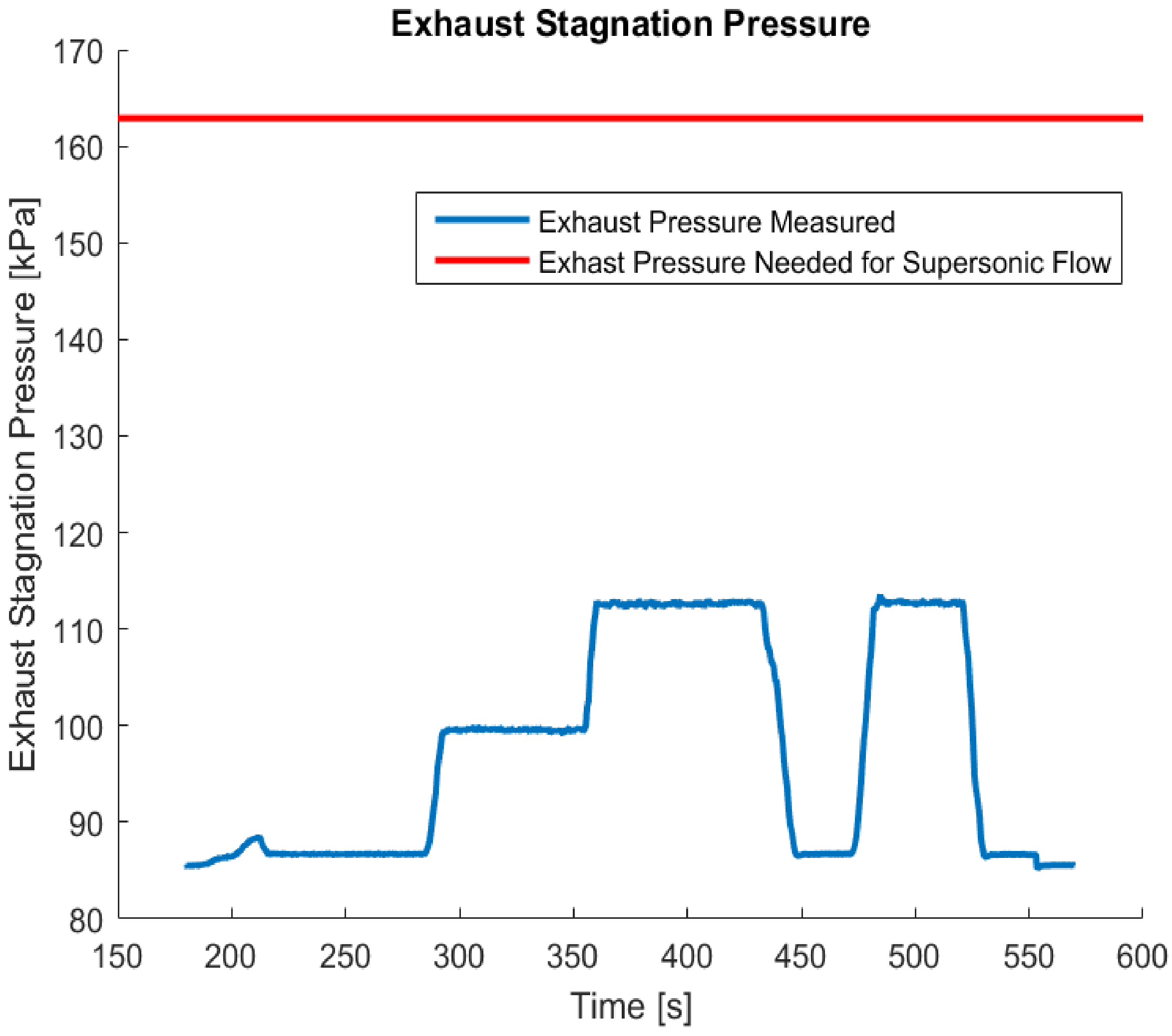
✓ Nozzle has operating temperature of 1423K. Thrust maintained for 1 min. Nozzle can survive engine conditions.

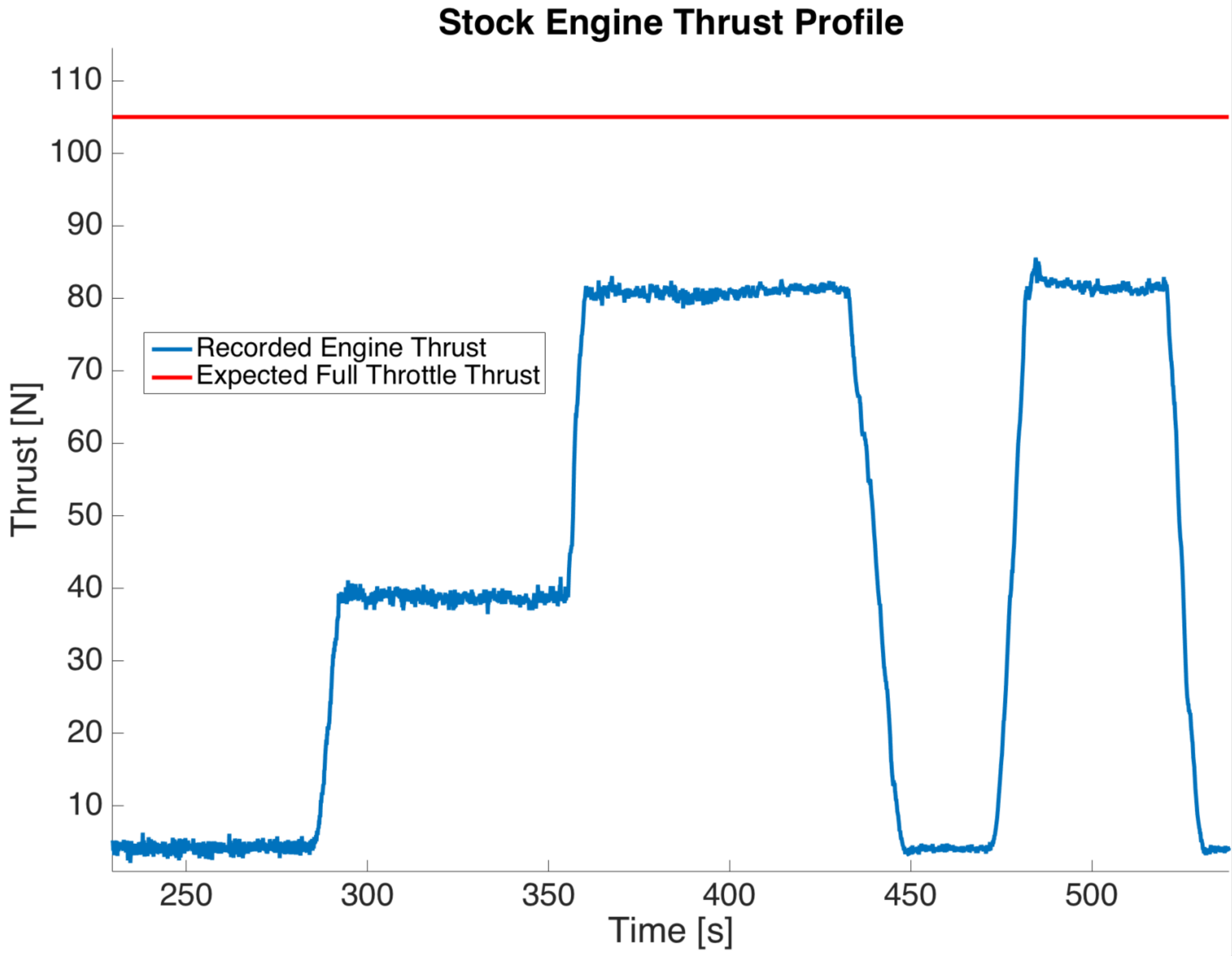
Stock Engine Test





Engine Supersonic Capability (DR 1.1) and Thrust Capability (DR 2.2)





Engine Testing Takeaways

- Engine provides a constant thrust at a given throttle setting
- Pressure ratio is too low to achieve supersonic flow at the exit
- A modified nozzle caused the engine to shut down

FR 1	The Nozzle shall accelerate the flow from subsonic to supersonic conditions.
DR 1.1	The flow through the nozzle shall be choked such that the nozzle exit flow Mach is greater than 1.

✖ **The engine is incapable of providing a pressure ratio at which supersonic flow can exist**

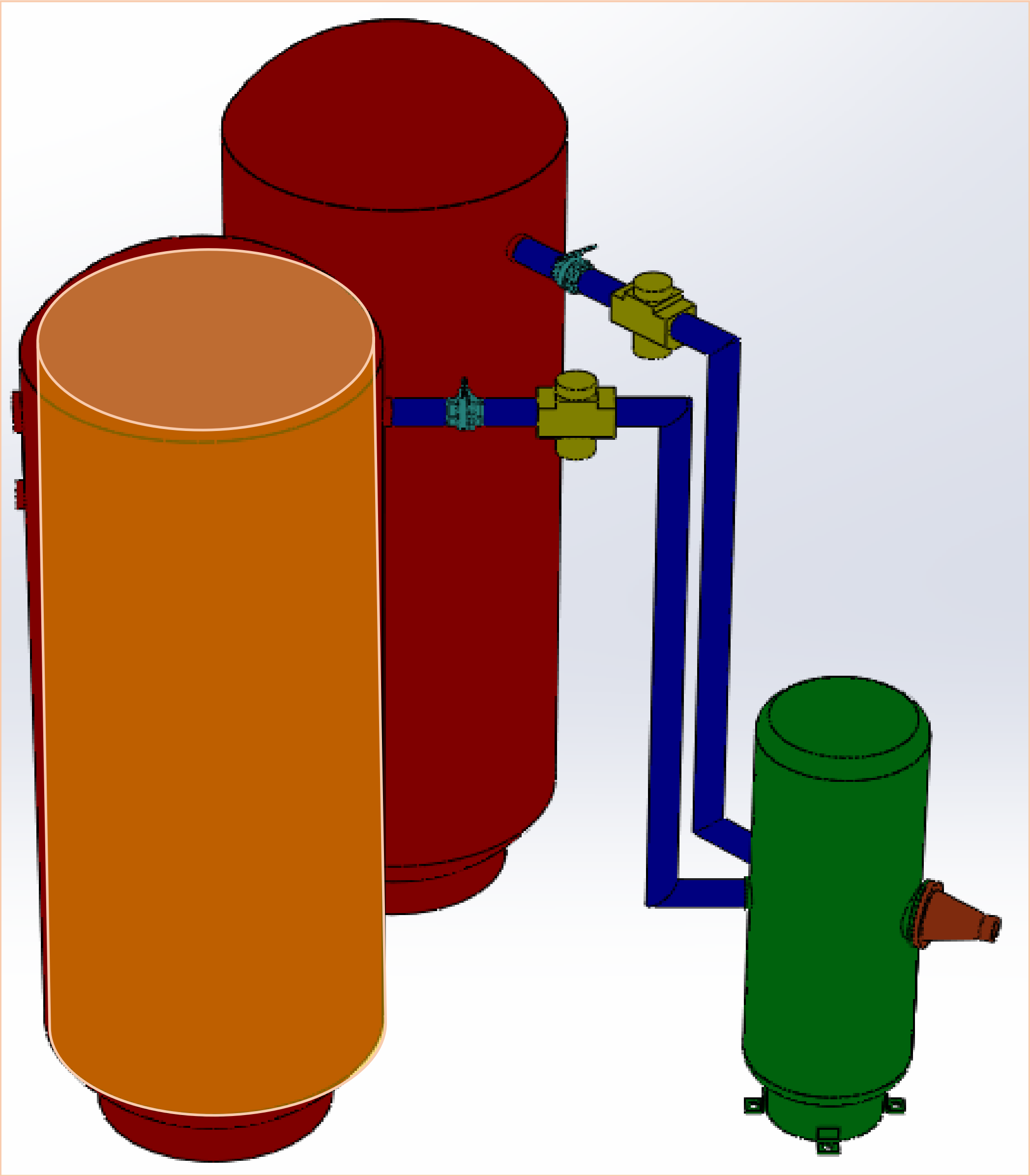
CPE 2– Test Bed Validation

FR 1	The Nozzle shall accelerate the flow from subsonic to supersonic conditions.
DR 1.1	The flow through the nozzle shall be choked such that the nozzle exit flow Mach is greater than 1.

FR 4	The Nozzle shall be able to withstand engine operation for at least 30 seconds.
DR 4.4	The thrust of the engine shall not decrease over a 30 second span.
DR 4.5	The nozzle shall survive the pressure and forces of engine operation.

FR 5	The Nozzle's performance shall be validated/verified through the use of a cold-flow test bed.
DR 5.1	The test bed shall provide the same pressure and mass flow rate as the engine exit within 5% adjusted for temperature

CPE 2 – Test Bed Design



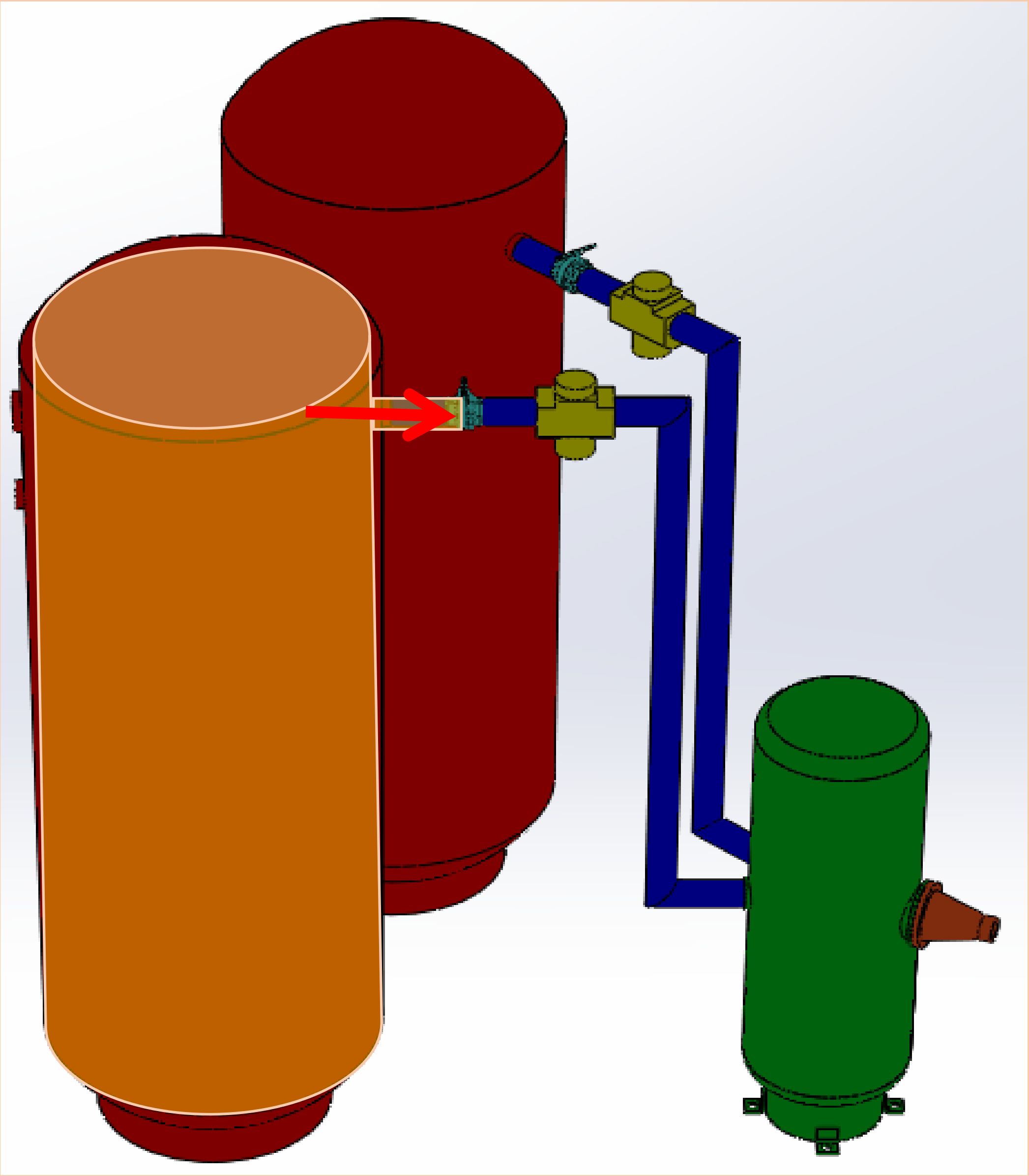
- Objectives:

M = 1.06 Test: $\dot{m} = 0.202 \frac{kg}{s}$ $p_t = 167kPa$

M = 1.3 Test: $\dot{m} = 0.281 \frac{kg}{s}$ $p_t = 233kPa$

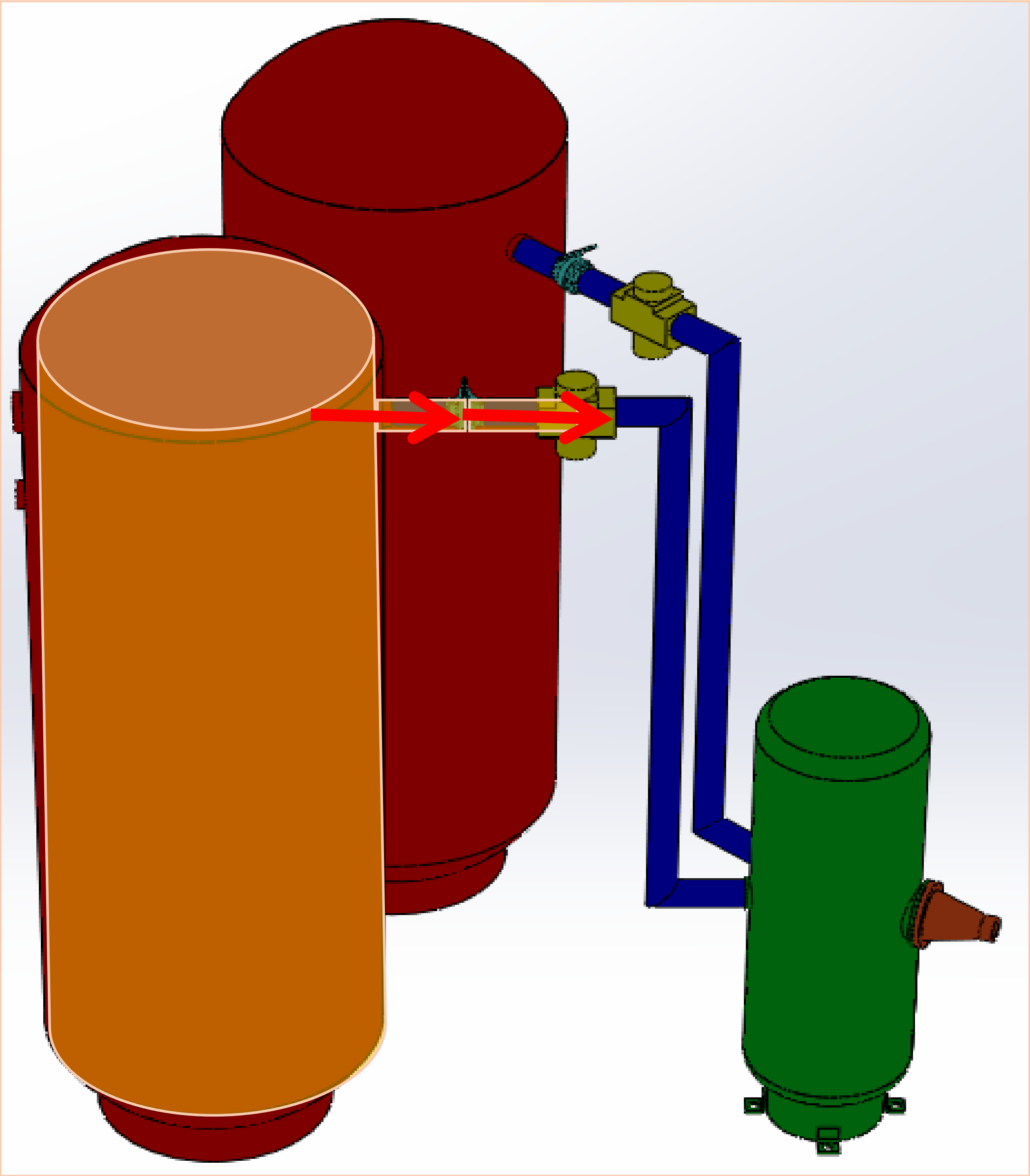
- Pressurized air supply tanks
  - ISC 80 gal. tank @ max of 200 psi
  - Dimensions:
    - Height = 63"
    - Diameter = 20"
  - Provides pressurized air to the ball valves/pressure regulators

CPE 2 – Test Bed Design



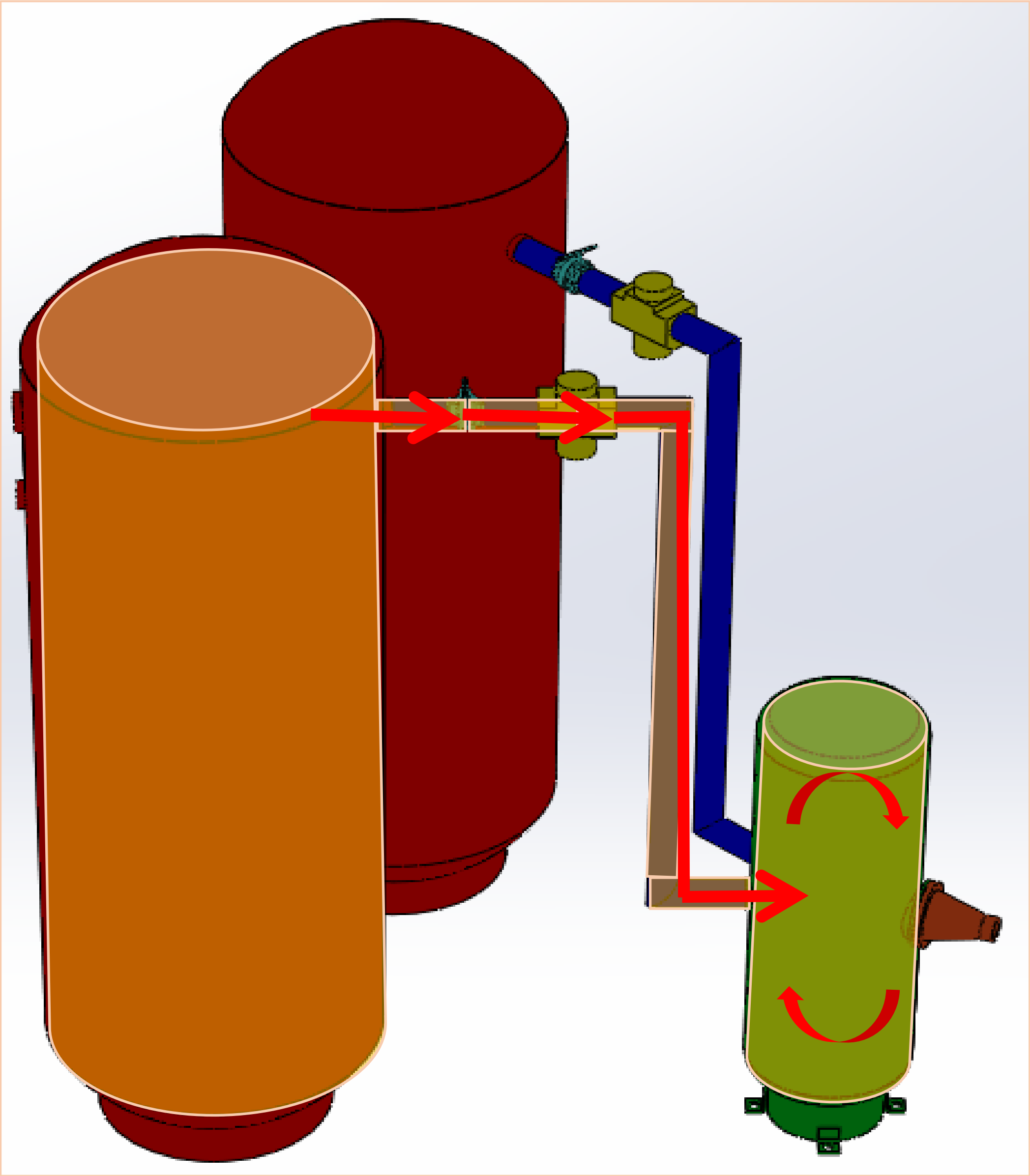
- Ball Valves
  - 1.25" brass shut-off ball valves
  - Dimensions:
    - Height = 2.5"
    - Length = 3.75"
    - Width = 5.5"
  - Max pressure = **600 psi** ✓
  - Allows us to shut-off flow from the tanks to the pressure regulators

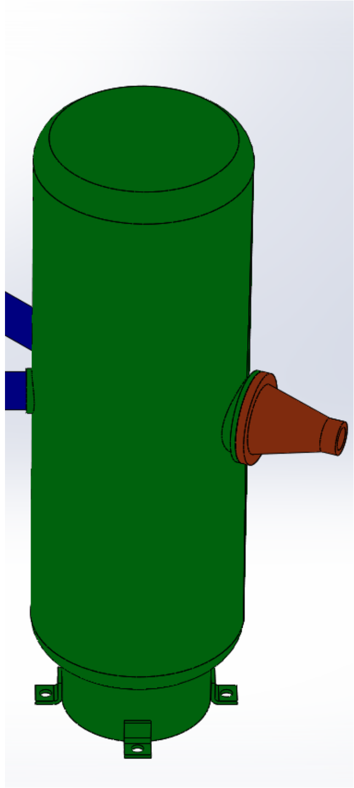
CPE 2 – Test Bed Design



- Pressure Regulator
  - Parker 53R626R
  - Output Pressure Range: **5-160 psig** ✓
  - Max inlet pressure = **300 psig** ✓
  - Max mass flow rate = **0.40 kg/s** ✓
  - Allows us to regulate the static pressure down to the needed 44 and 61 psi
  - 1.25" female NPT threads

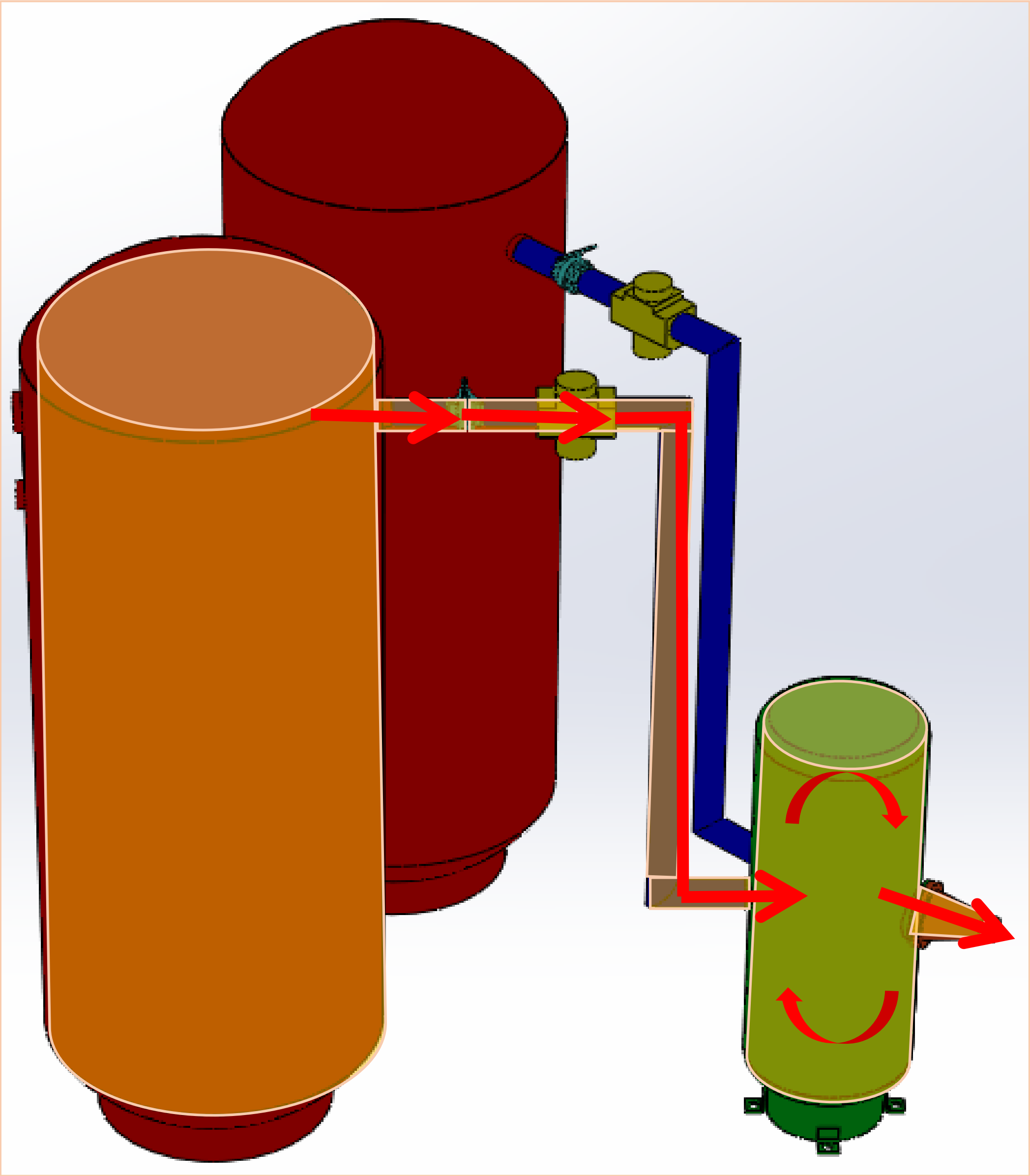
CPE 2 – Test Bed Design

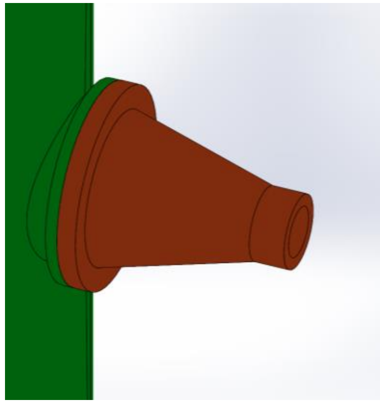




- Settling Chamber
  - Dimensions:
    - Height = 32"
    - Diameter = 10"
  - Max pressure = **100 psi (690 kPa)** ✓
  - Used to mix the incoming flow from the tanks to create approximately stagnant flow before the converging duct

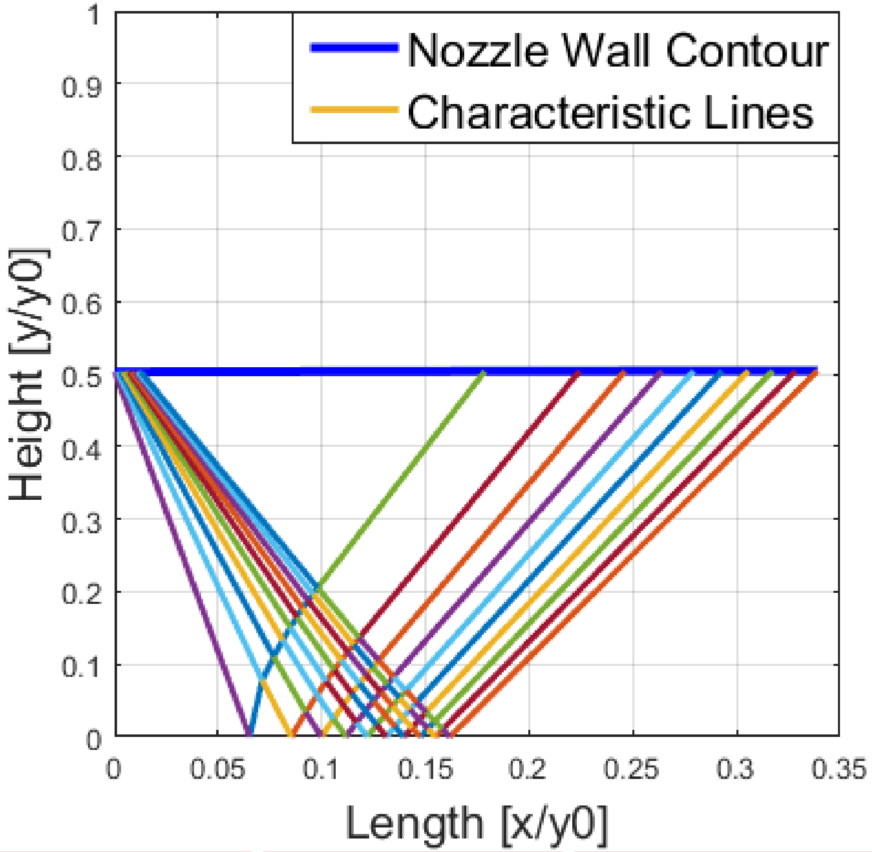
CPE 2 – Test Bed Design

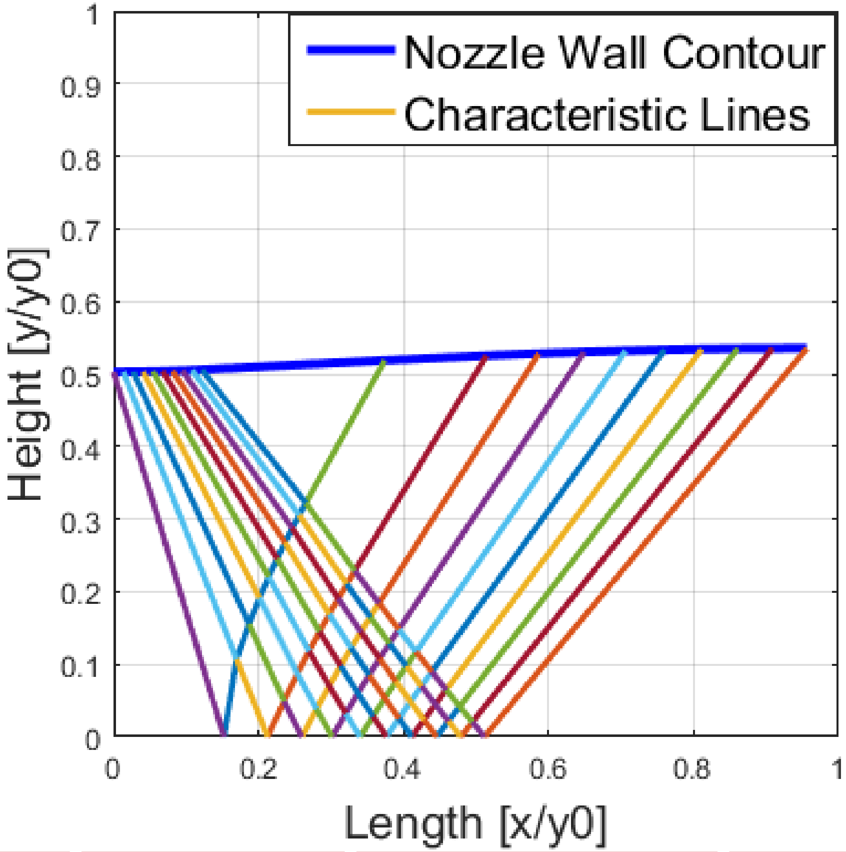




- Diffuser
  - Aluminum construction
  - Dimensions:
    - Inlet Diameter = 2.428"
    - Outlet Diameter = 1.072"
    - Converging Length = 3.601"
    - Straight Pipe Length = 0.750"
  - Accelerates the approximately stagnant flow in the settling chamber to M=0.65

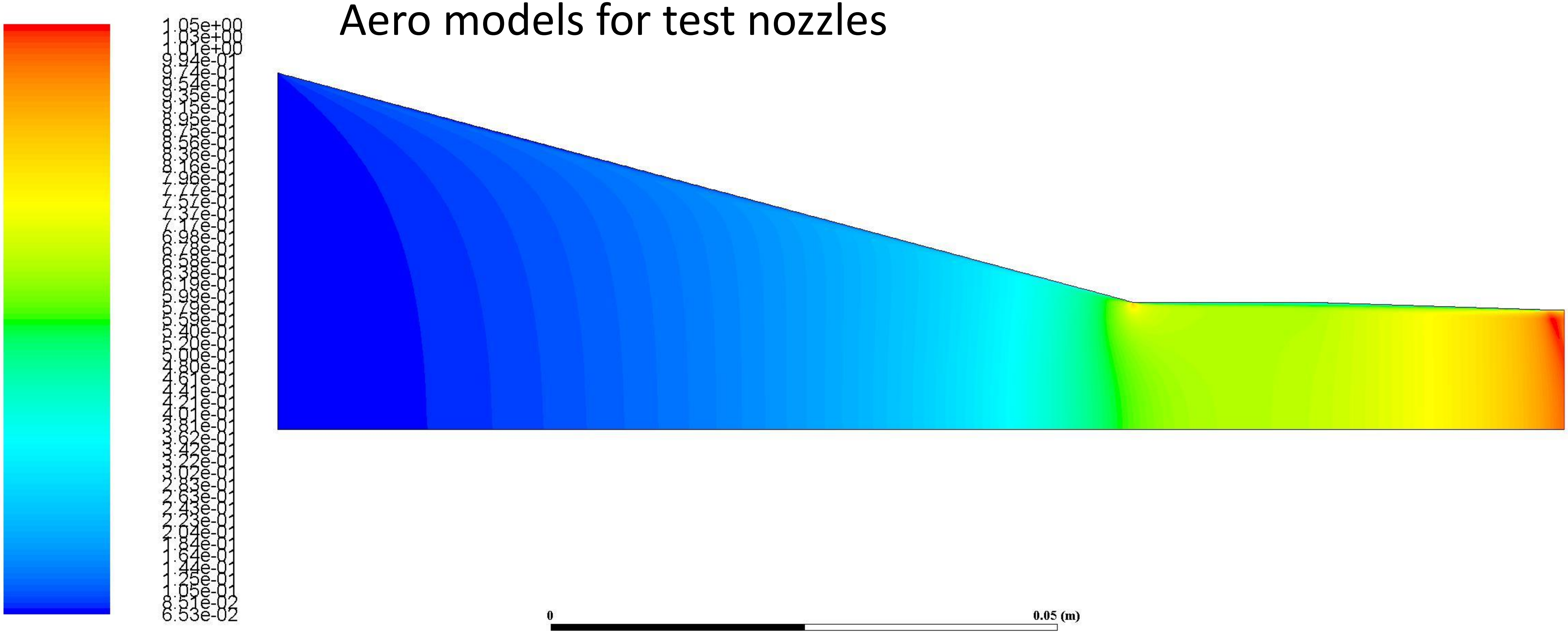
Test Bed Nozzle Wall Design


Mach 1.06 Divergent Section Design


Mach 1.3 Divergent Section Design

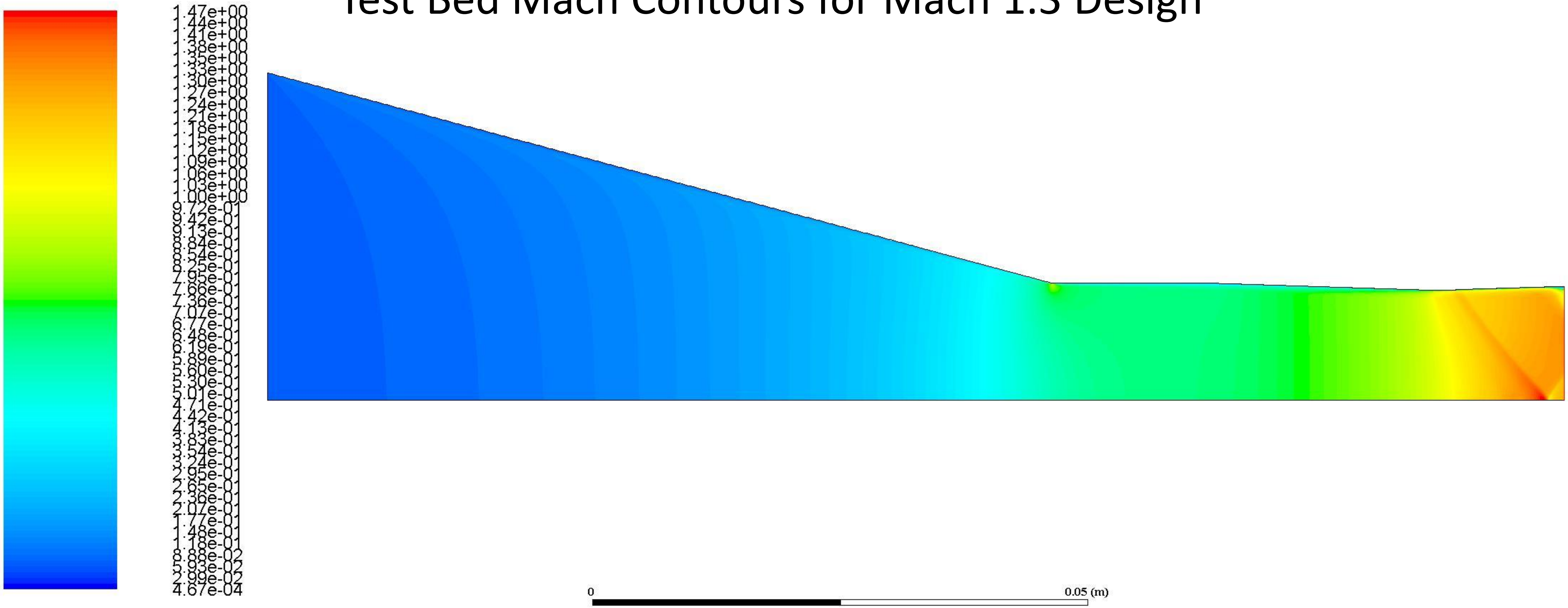
Validation of DR 1.1—Test Bed

Aero models for test nozzles



Validation of DR 1.1—Test Bed

Test Bed Mach Contours for Mach 1.3 Design



Testing Duration (DR 4.4)

- With 2 compressed air tanks
  - 80 gallons each (or 18480 cubic inches)
  - Each at 175 psi

$$P_1V_1 = P_2V_2$$

- Using Boyle's Law:
- With: $P_1 = 175psi$ $V_1 = 18480in^3$ $P_2 = 12.18psi$
- Results in the equivalence of **153.6 cubic feet** of air
- For our M=1.06 test: CFM = 205.5 (each tank)
  - Allows for a **45s** test
- For our M=1.3 test: CFM = 285.8 (each tank)
  - Allows for a **32s** test

We are able to ensure the thrust of the test does not decrease over 30 seconds.

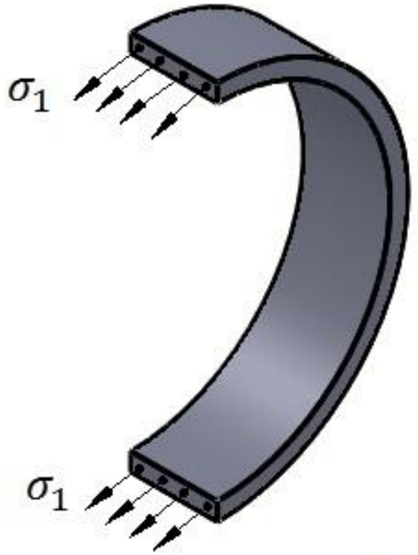
Operational Stress (DR 4.5)

Assuming: Thin pressure vessel

Maximum Gauge Pressure within nozzle: 190kPa
Nozzle Thickness: 0.5mm

Maximum Hoop Stress: 5.17 MPa
Clear FLGPCL02 Proof Strength: 58.5 MPa

Operational Stress is well under the Proof Strength



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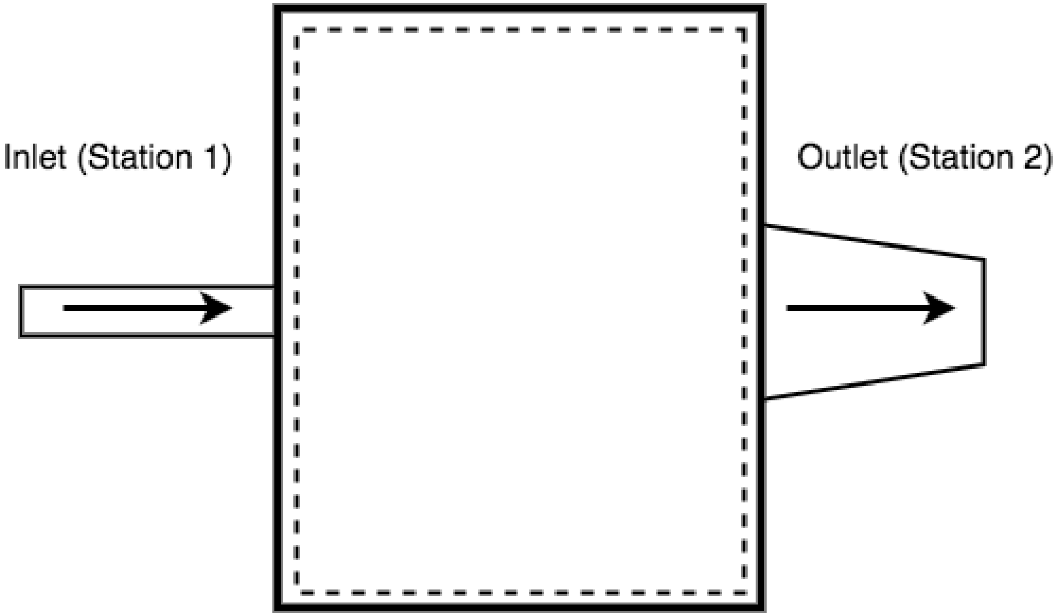
Validation of (DR 5.1)

- Control Volume Analysis
  - Using:
    - Conservation of Mass, Energy, and Momentum
    - Ideal Gas Law
- 4 equations, 4 unknowns
- **IMPORTANCE:**
  - Solve for static pressure out of the regulators



- **RESULTS:**

M = 1.06 Test: $p_1 = 44\text{psi}$ $\dot{m} = 0.202\,\frac{\text{kg}}{\text{s}}$ $M = 0.65$ $p_t = 167\text{kPa}$

M = 1.3 Test: $p_1 = 61\text{psi}$ $\dot{m} = 0.281\,\frac{\text{kg}}{\text{s}}$ $M = 0.65$ $p_t = 233\text{kPa}$

CPE 3 – Test Bed Validation

FR 1	The Nozzle shall accelerate the flow from subsonic to supersonic conditions.
DR 1.1	The flow through the nozzle shall be choked such that the nozzle exit flow Mach is greater than 1.

✓ Flow at the exit is supersonic and chokes at throat.

FR 4	The Nozzle shall be able to withstand engine operation for at least 30 seconds.
DR 4.4	The thrust of the engine shall not decrease over a 30 second span.
DR 4.5	The nozzle shall survive the pressure and forces of engine operation.

✓ Thrust maintained for 30+ sec. Nozzle can survive engine conditions.

FR 5	The Nozzle's performance shall be validated/verified through the use of a cold-flow test bed.
DR 5.1	The test bed shall provide the same pressure and mass flow rate as the engine exit within 5% adjusted for temperature .

✓ Test Bed is capable of achieving the adjusted values for nozzle performance.

Risk Analysis

Risk Assessment

1. Modified Engine

- 1.1 Engine fails to run
- 1.2 Nozzle structurally fails
- 1.3 Nozzle deforms under test conditions
- 1.4 Supersonic flow not achieved
- 1.5 Sensors failure due to test conditions

2. Test Bed

- 2.1 No choke in nozzle due to pressure regulator
- 2.2 Failure of the testing nozzle
- 2.3 Leaks in design connections

3. General

- 3.1 Nozzle Operation damages property/equipment

Risk Assessment Matrix

Near Certainty	🟨	🟨	🟥	🟥 1.4 2.1	🟥
Highly Likely	🟩 2.3	🟨	🟨 1.1 1.3	🟥	🟥
Likely	🟩	🟩 2.2	🟨	🟨	🟥 3.1
Low	🟩	🟩	🟩	🟨 1.2	🟨
Extremely Unlikely	🟩	🟩	🟩	🟩	🟨
	Minimal	**Minor**	**Major**	**Serious**	**Catastrophic**

Severity

1.1 Engine does not run

→ Test stock engine with choked Nozzle

2.1 No choke in nozzle due to pressure regulator

1.2 Nozzle structurally fails

→ Use different pressure regulator

→ CoCrMo Material Selection

2.2 Failure of the testing nozzle

1.3 Nozzle deforms under test conditions

→ Use different material or increase thickness

→ CoCrMo Material Selection

2.3 Leaks in design connections

1.4 Supersonic flow not achieved

→ Use metal sealant, check/tighten connections

→ Revert to Test Bed for Nozzle validation

3.1 Sound wave damages property/equipment

1.5 Sensors fails due to test conditions

→ Isolated location, safety procedures

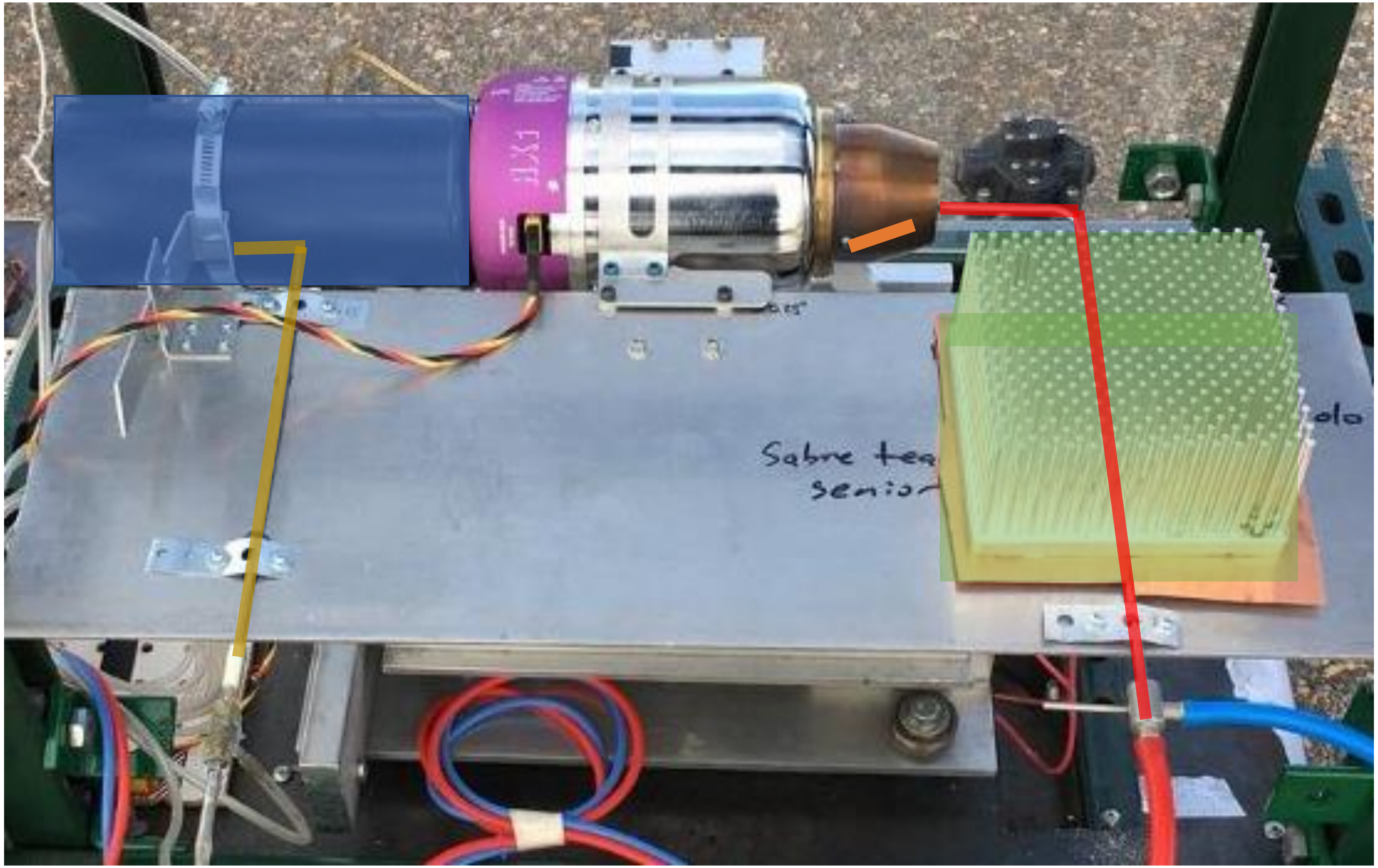
→ Heat sink/insulation

Verification & Validation

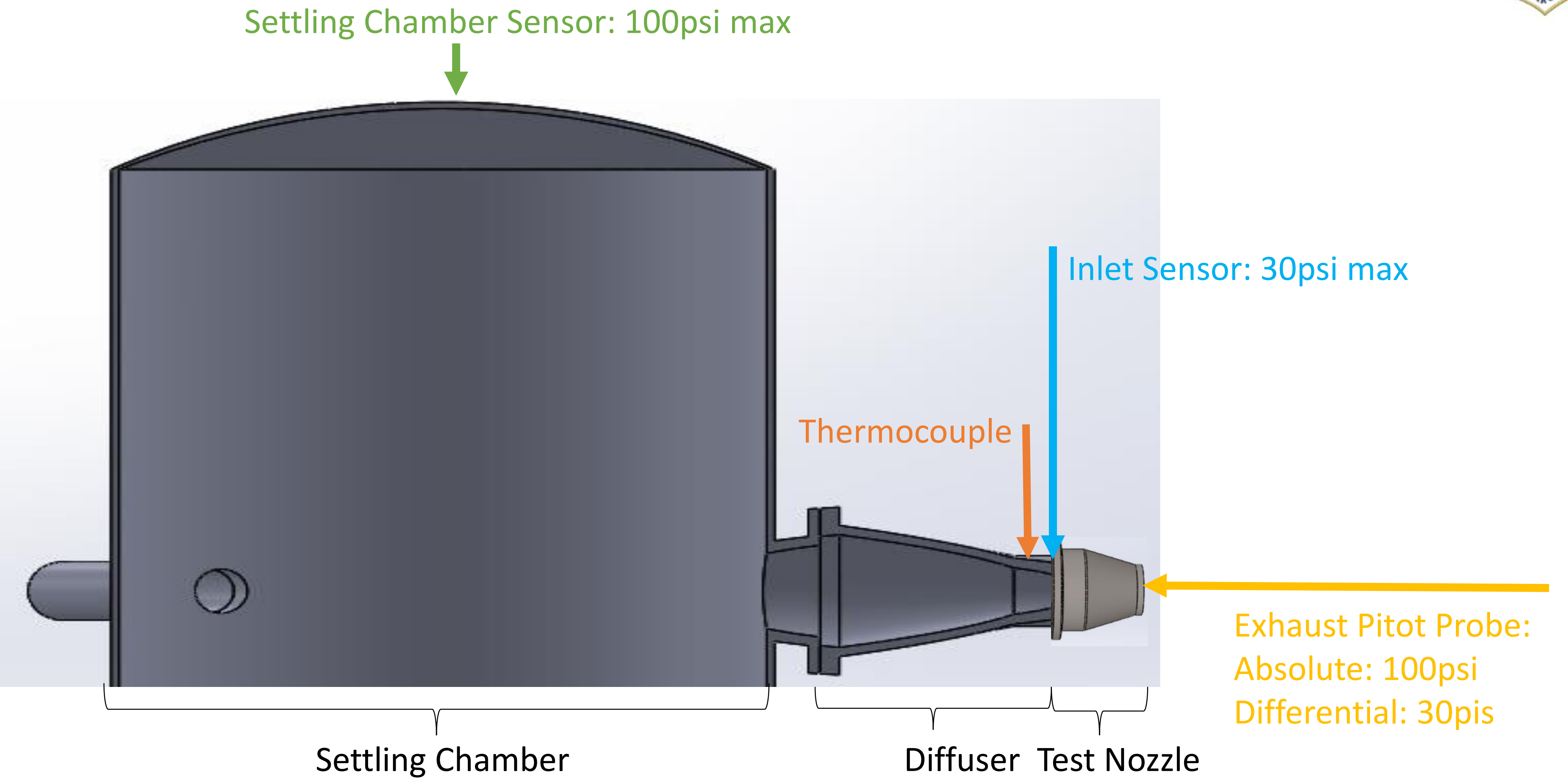
Static Test Stand Sensor Locations

Mass Flow Determination:
Inlet Cowl (removable) and
Inlet Pitot Probe to determine
volumetric flow rate

Exhaust Velocity Determination:
Stock Thermocouple
Exhaust Pitot Probe
Heat Sink



Testbed Sensor Locations



Differential Pressure Transducer

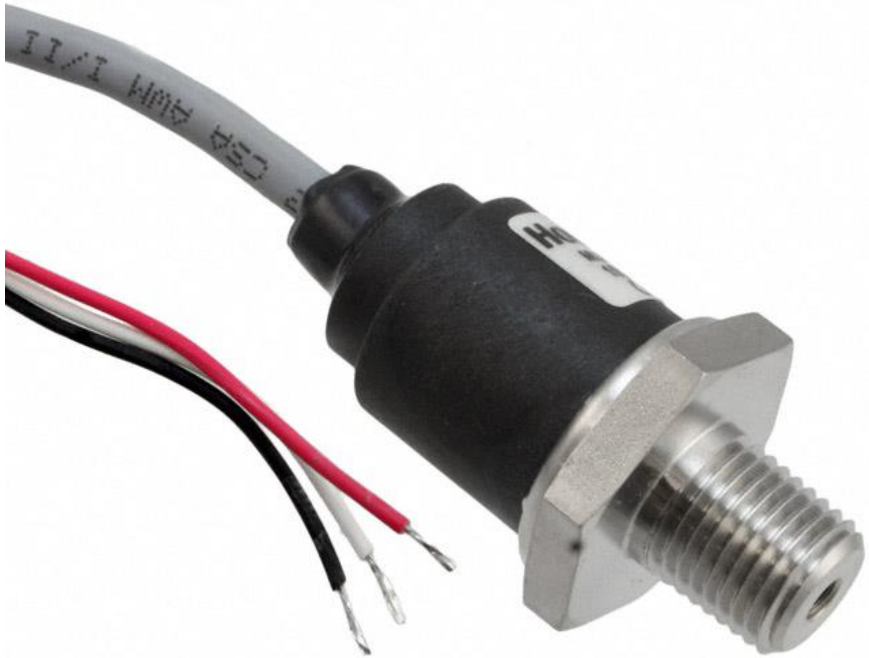
- Sensor Specifications: PX137-015DV and PX137-030DV
- Range: +/- 15 psi and +/- 30 psi
- Resolution: 6 mV/psi and 3 mV/psi
- Operating Temperature Range: 0 – 75°C
- Operating Pressure Range: +/- 45 psi and +/- 90 psi
- Noise: +/- 5 mV

Absolute Pressure Sensor

- Sensor Specifications: PX142-030A5V
- Range: +/- 30 psi
- Resolution: 167 mV/psi
- Operating Temperature Range: -40 - 85°C
- Operating Pressure Range: 60 psi

Absolute Pressure Sensor

- Sensor Specifications: MLH100PGL06A
- Range: +/- 100 psi
- Resolution:35 mV/psi
- Operating Temperature Range: -40 - 125° C
- Operating Pressure Range:200 psi



Data Acquisition

General DAQ

NI-USB-6009

- 4 differential analog input channels
- 14-bit resolution
- 48000 samples per second
- Programmable Digital I/O ports

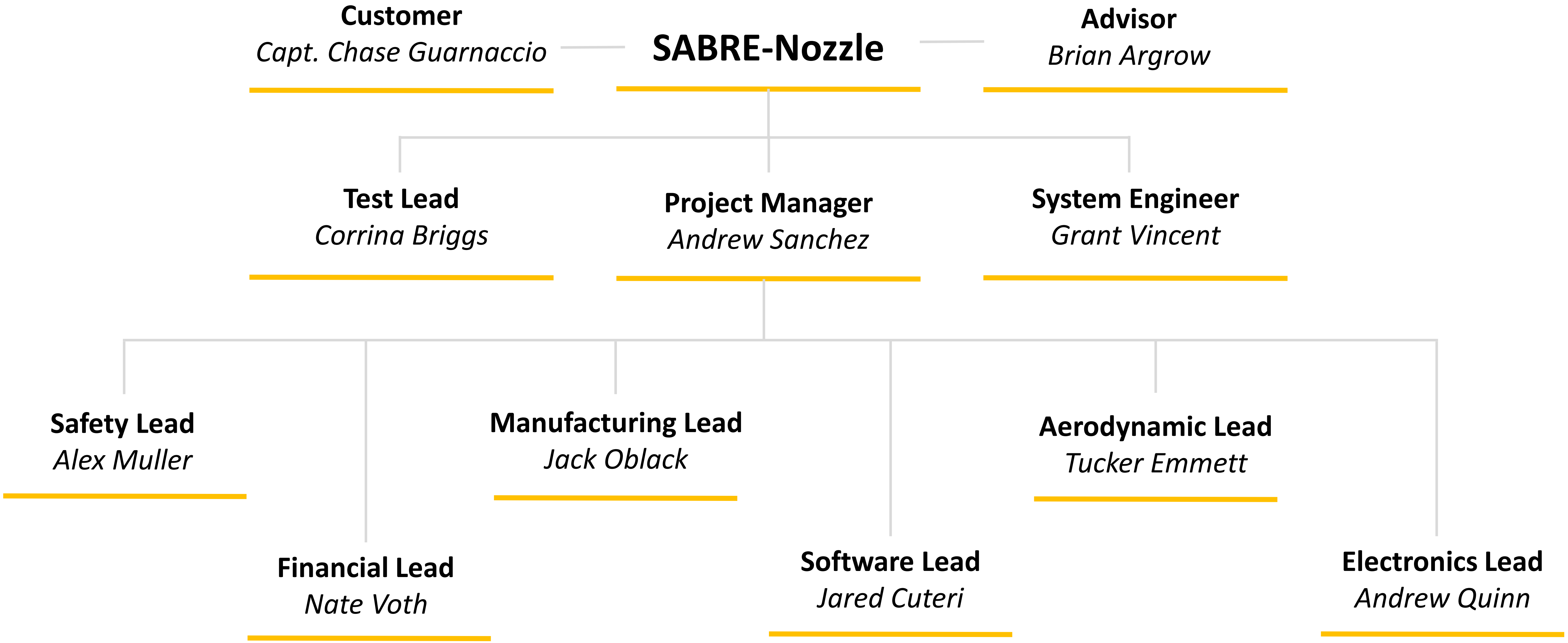
Cold Junction Comparison DAQ

NI-9211

- -40°C - 70°C operating range
- 4 separate thermocouple channels
- 24-bit resolution
- 14 samples per second

Project Planning

Organizational Chart



Customer
Capt. Chase Guarnaccio

SABRE-Nozzle

Advisor
Brian Argrow

Test Lead
Corrina Briggs

Project Manager
Andrew Sanchez

System Engineer
Grant Vincent

Safety Lead
Alex Muller

Financial Lead
Nate Voth

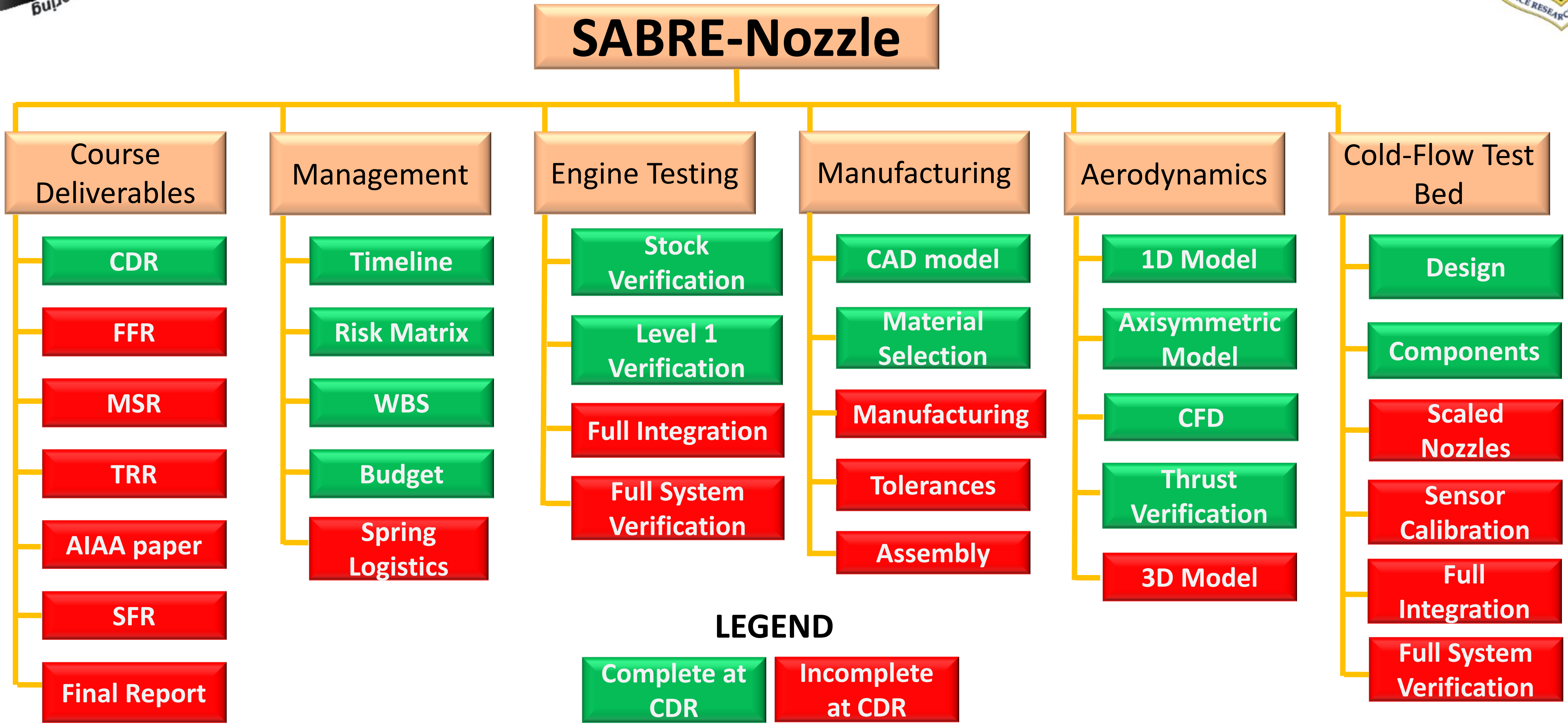
Manufacturing Lead
Jack Oblack

Software Lead
Jared Cuteri

Aerodynamic Lead
Tucker Emmett

Electronics Lead
Andrew Quinn

Work Breakdown Structure



SABRE-Nozzle

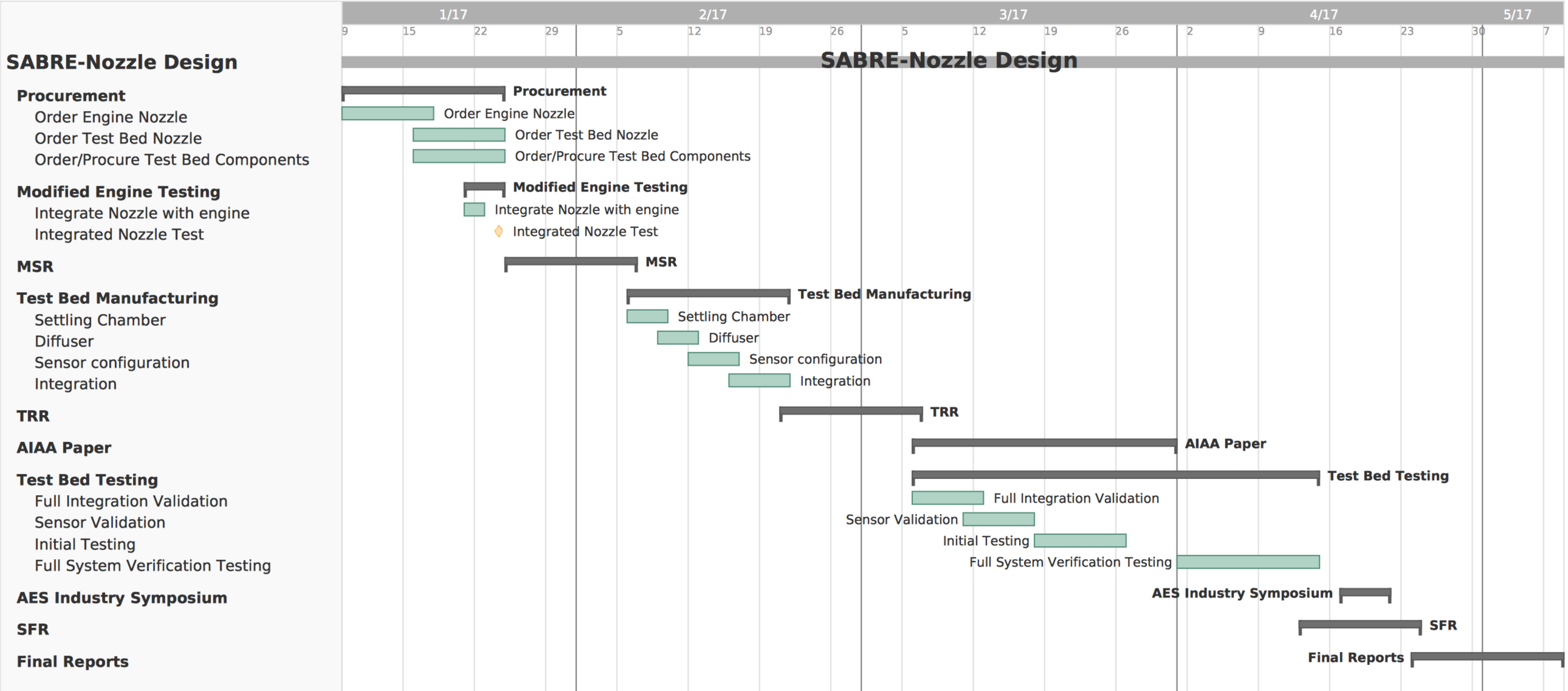
- Course Deliverables
  - 🟩 CDR
  - 🟥 FFR
  - 🟥 MSR
  - 🟥 TRR
  - 🟥 AIAA paper
  - 🟥 SFR
  - 🟥 Final Report
- Management
  - 🟩 Timeline
  - 🟩 Risk Matrix
  - 🟩 WBS
  - 🟩 Budget
  - 🟥 Spring Logistics
- Engine Testing
  - 🟩 Stock Verification
  - 🟩 Level 1 Verification
  - 🟥 Full Integration
  - 🟥 Full System Verification
- Manufacturing
  - 🟩 CAD model
  - 🟩 Material Selection
  - 🟥 Manufacturing
  - 🟥 Tolerances
  - 🟥 Assembly
- Aerodynamics
  - 🟩 1D Model
  - 🟩 Axisymmetric Model
  - 🟩 CFD
  - 🟩 Thrust Verification
  - 🟥 3D Model
- Cold-Flow Test Bed
  - 🟩 Design
  - 🟩 Components
  - 🟥 Scaled Nozzles
  - 🟥 Sensor Calibration
  - 🟥 Full Integration
  - 🟥 Full System Verification

LEGEND

🟩 Complete at CDR

🟥 Incomplete at CDR

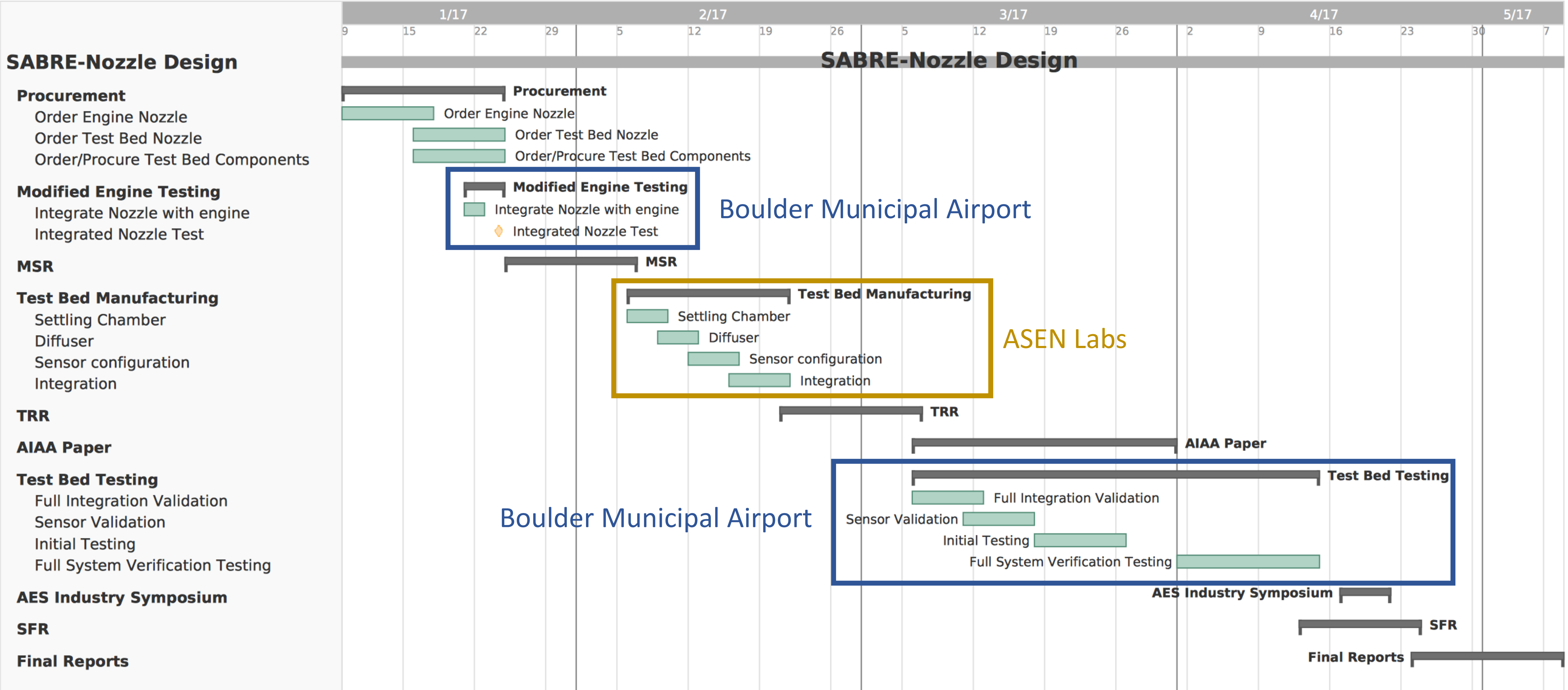
Work Plan



SABRE-Nozzle Design

- **Procurement**
  - Order Engine Nozzle
  - Order Test Bed Nozzle
  - Order/Procure Test Bed Components
- **Modified Engine Testing**
  - Integrate Nozzle with engine
  - Integrated Nozzle Test
- **MSR**
- **Test Bed Manufacturing**
  - Settling Chamber
  - Diffuser
  - Sensor configuration
  - Integration
- **TRR**
- **AIAA Paper**
- **Test Bed Testing**
  - Full Integration Validation
  - Sensor Validation
  - Initial Testing
  - Full System Verification Testing
- **AES Industry Symposium**
- **SFR**
- **Final Reports**

Test Plan



SABRE-Nozzle Design

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Boulder Municipal Airport

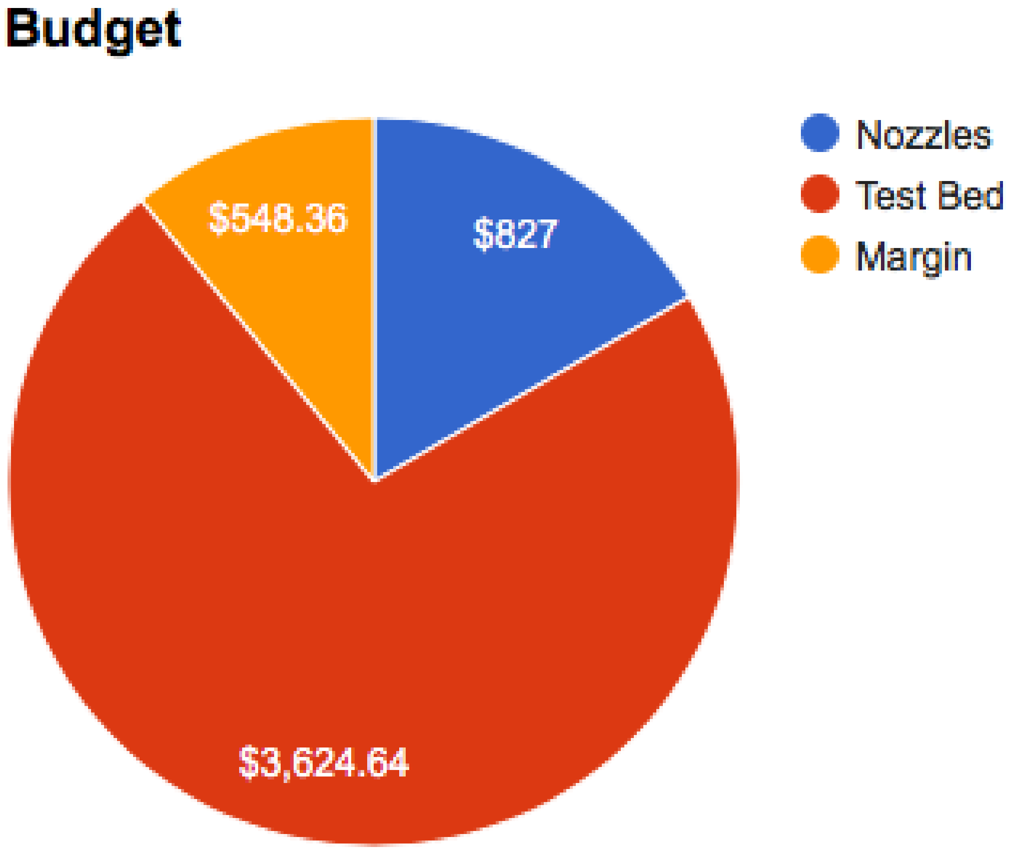
ASEN Labs

Boulder Municipal Airport

Cost Plan

- Two main sources of expenses:
  - Nozzles
  - Test Bed

- Smaller margin, but costs are accurately known

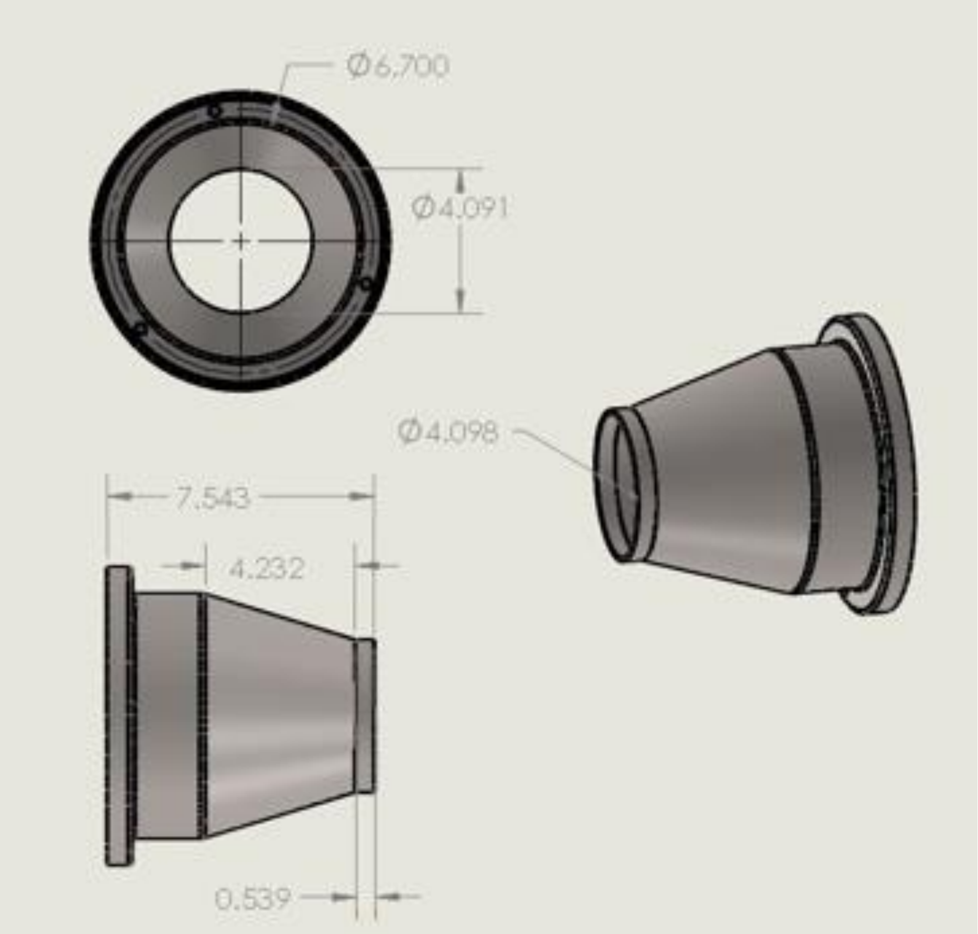


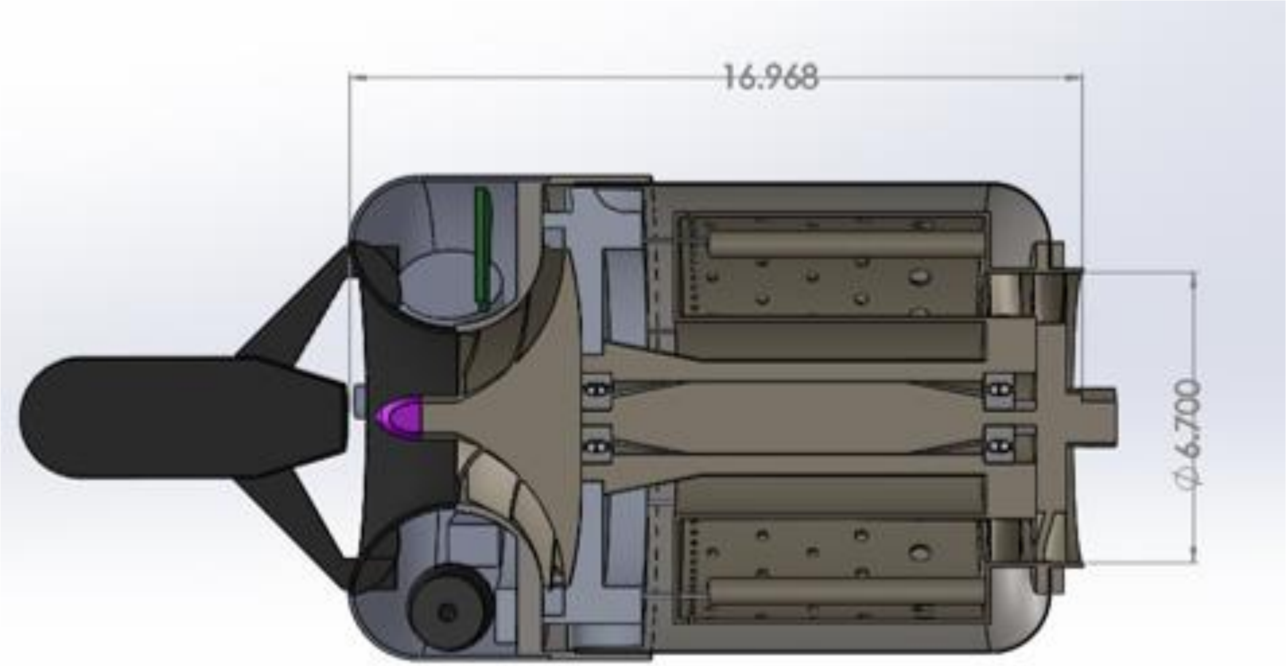
Back-up Slides

Levels of Success

	Model/Simulation	Design/Manufacturing	Testing
Level 1	•Model stock engine exhaust with given parameters (T, P, $\dot{m}$, V) •Model air in nozzle changing from subsonic flow to supersonic flow •No decrease of T/W	•Manufacture convergent-divergent nozzle that attaches to JetCat engine •Material survives the exhaust environment for at least 30 seconds	• Replicate an engine analog that simulates exhaust velocity and temperature, within 15% of stock engine conditions
Level 2	•Increase Thrust to Weight Ratio by 20%	•Nozzle built using additive manufacturing, where material survives testing environment for at least 150 seconds	• Engine analog shall model exhaust pressure within 20% of stock engine
Level 3	•Verification that modelled nozzle and manufactured nozzle have output performance within 20% of one another	•Nozzle built using additive manufacturing that can be reused 3 times and not fail in the testing environment	•Nozzle integrated and tested with the JetCat engine

Design Solution- Nozzle





Nozzle Interchangeability (DR 3.4)

- Nozzle Integration
  - 3 Bolt connection to engine
  - Bolt tolerance of 2 threads per millimeter
  - Dome attaches to nozzle
- Dome Integration
  - 3 Bolt connection to nozzle
  - Bolt tolerance of 2 threads per millimeter
  - Sits flush to engine exit to protect turbine bearing

Thermal Expansion

Assuming: Isentropic Flow Relations to determine static flow temperature, steady state (rate of heat transfer is constant), forced turbulent dry air convection within nozzle, natural convection outside of nozzle, nominal CoCr thermal properties

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$R_{total} = R_{conv,1} + R_{cond} + R_{conv,2}$$

$$= \frac{1}{2\pi r_1 L h_1} + \frac{ln(r_2/r_1)}{2\pi L k} + \frac{1}{2\pi r_2 L h_2}$$

$$h_1 = 13.95 \frac{W}{m^2 K} \qquad h_2 = 11.88 \frac{W}{m^2 K}$$

$$k = 30.8 \frac{W}{m^o C} \qquad \alpha = 15.1 \frac{\mu m}{m^o C}$$

$$\Delta d = d_o \Delta T \alpha$$

Station	Inlet	Throat	Exit
Temperature(K)	871	746	734
Δd (µm)	441 (0.86%)	278 (0.68%)	271 (0.66%)

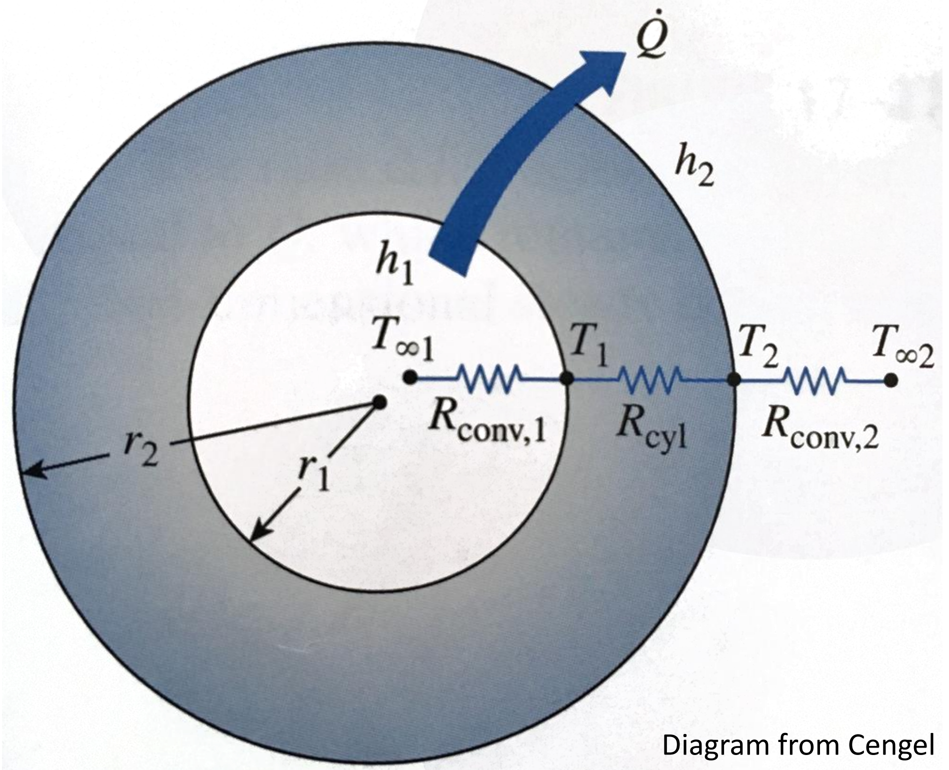
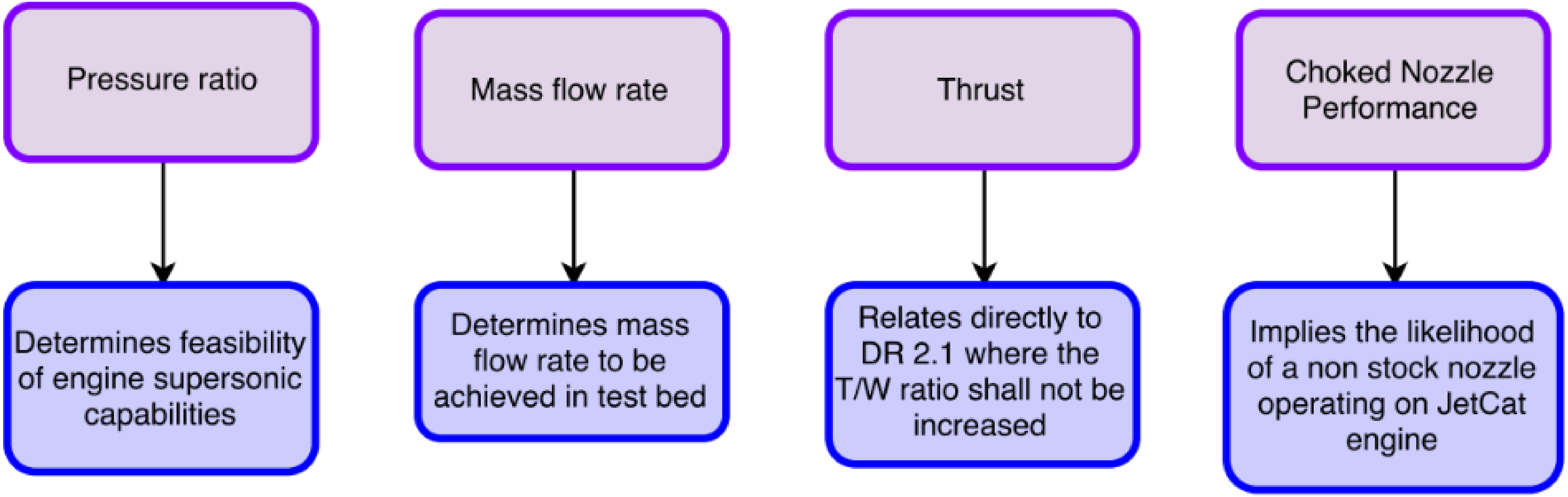


Diagram from Cengel

Engine Testing Objectives



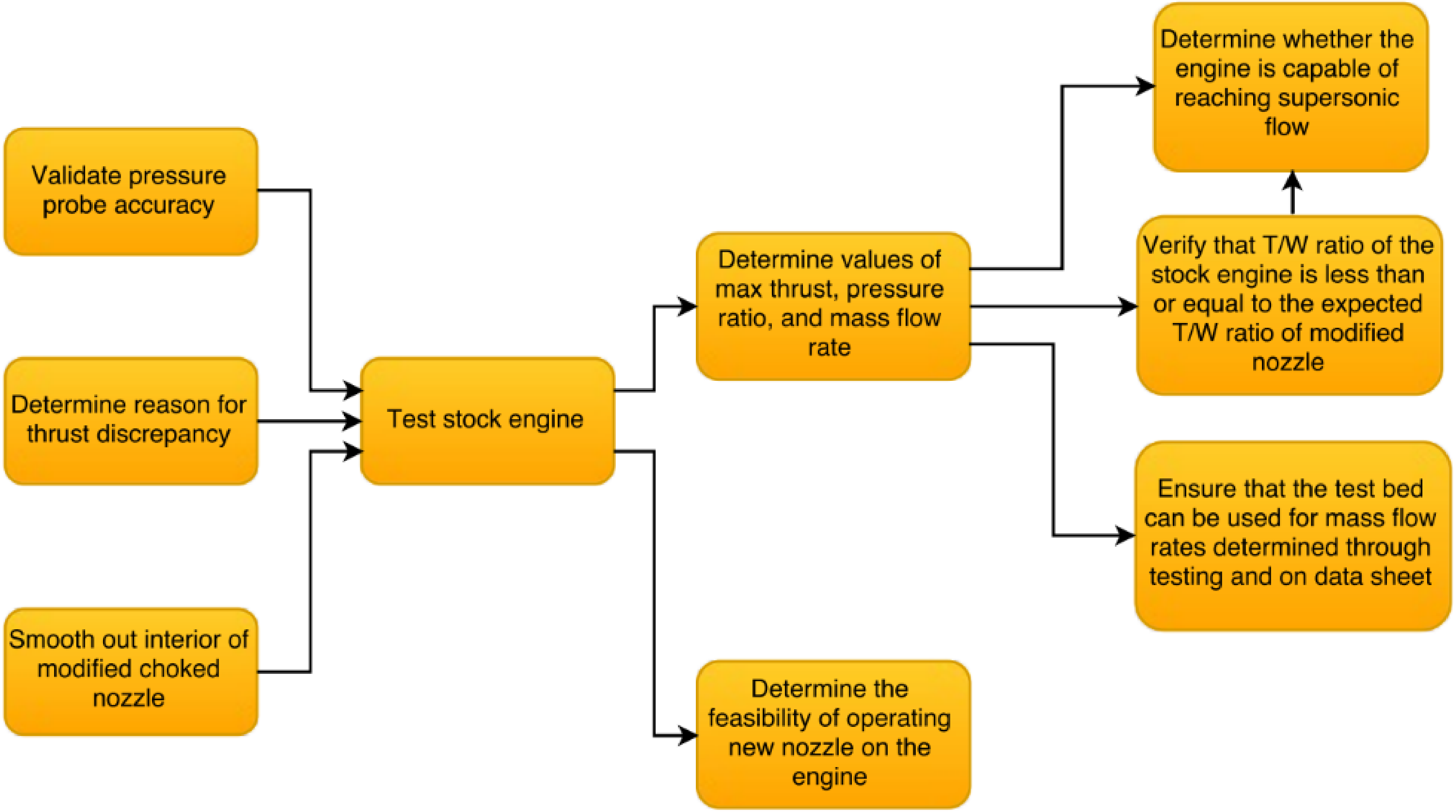
Pressure ratio → Determines feasibility of engine supersonic capabilities

Mass flow rate → Determines mass flow rate to be achieved in test bed

Thrust → Relates directly to DR 2.1 where the T/W ratio shall not be increased

Choked Nozzle Performance → Implies the likelihood of a non stock nozzle operating on JetCat engine

Test Plan: Engine



Validate pressure probe accuracy

Determine reason for thrust discrepancy

Smooth out interior of modified choked nozzle

Test stock engine

Determine values of max thrust, pressure ratio, and mass flow rate

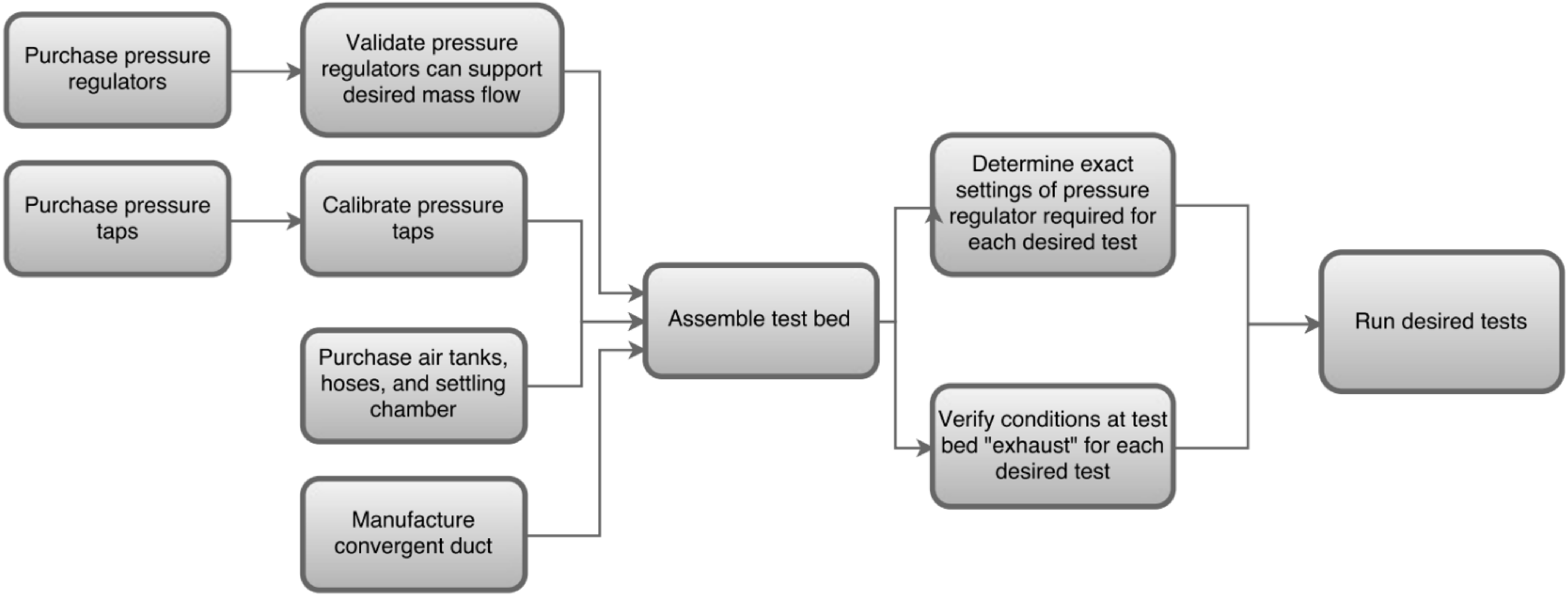
Determine the feasibility of operating new nozzle on the engine

Determine whether the engine is capable of reaching supersonic flow

Verify that T/W ratio of the stock engine is less than or equal to the expected T/W ratio of modified nozzle

Ensure that the test bed can be used for mass flow rates determined through testing and on data sheet

Test Plan: Test Bed



Purchase pressure regulators

Validate pressure regulators can support desired mass flow

Purchase pressure taps

Calibrate pressure taps

Purchase air tanks, hoses, and settling chamber

Manufacture convergent duct

Assemble test bed

Determine exact settings of pressure regulator required for each desired test

Verify conditions at test bed "exhaust" for each desired test

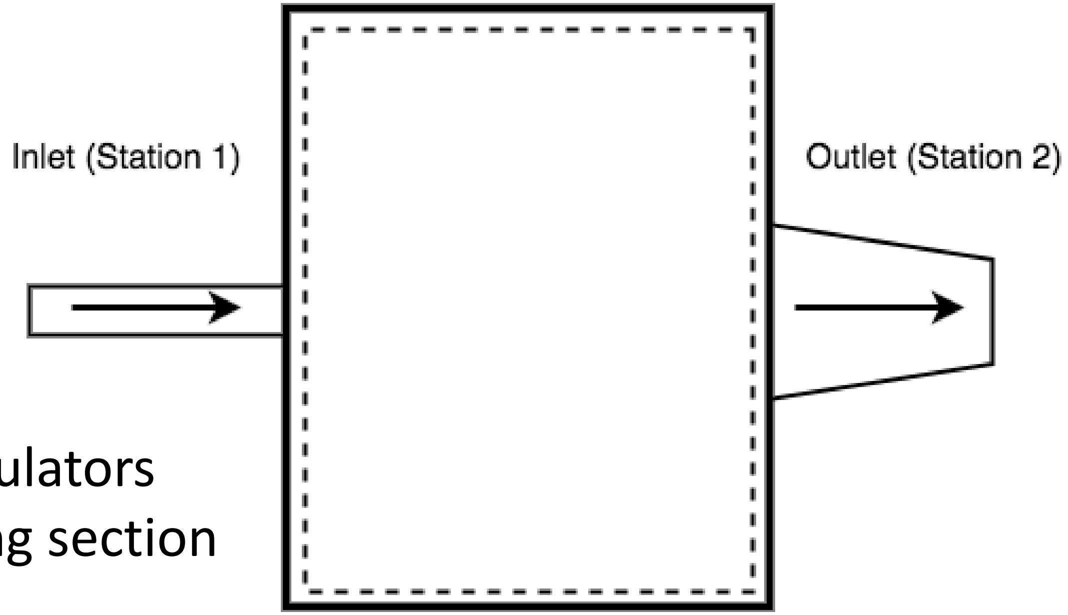
Run desired tests

Testing Schedule

Test	**Date**	**Location**	**Authorization**
Nozzle Design Test on JetCat Engine	Jan. 24, 2017	Boulder Municipal Airport	Tim Head- airport director
Initial Test Bed Testing	Mar. 19, 21, & 23, 2017		
Full System Verification Testing	Apr. 4, 6, 11, & 13, 2017		

Control Volume Analysis

- Control Volume Analysis
  - Using:
    - Conservation of Mass, Energy, and Momentum
    - Ideal Gas Law
- Open system
  - Flow into the settling chamber from pressure regulators
  - Flow out of the settling chamber to the converging section
- **IMPORTANCE:**
  - Solving for inlet parameters
  - Specifically, the static pressure required out of the pressure regulators



Control Volume Analysis

- Assumptions:
  - Steady state: $\dot{m}_1 = \dot{m}_2 = \dot{m}$
  - Potential energy of flow is constant: $z_1 = z_2$
  - No external forces acting on the control volume
  - No heat transfer: $\dot{Q} = 0$
  - No work done on or by the system: $\dot{W} = 0$

Control Volume Analysis

- Equations (final form):
  - Conservation of Mass: $\rho_2 u_2 A_2 - \rho_1 u_1 A_1 = 0$
  - Conservation of Momentum: $\rho_2 u_2^2 A_2 - p_2 A_2 - \rho_1 u_1^2 A_1 - p_1 A_1 = 0$
  - Conservation of Energy: $(c_v T_1 + \frac{u_1^2}{2}) - (c_v T_2 + \frac{u_2^2}{2}) = 0$
  - Equation of State: $p_1 - \rho_1 R T_1 = 0$

Control Volume Analysis

- Knowns:

  $A_1, A_2, \dot{m}, p_2, \rho_2, u_2, p_t, M_2, T_2$

- Unknowns:

  p_1, ρ_1, u_1, T_1

- 4 equations & 4 unknowns
- **RESULTS:** (with $p_t = 167kPa$)

$\rho_1 = 3.62\frac{kg}{s}$

$T_1 = 292K$

$u_1 = 35.5\frac{m}{s}$

$\boxed{\mathbf{p_1 = 44psi}}$

(with $p_t = 233kPa$)

$\rho_1 = 5.05\frac{kg}{s}$

$T_1 = 292K$

$u_1 = 35.5\frac{m}{s}$

$\boxed{\mathbf{p_1 = 61psi}}$

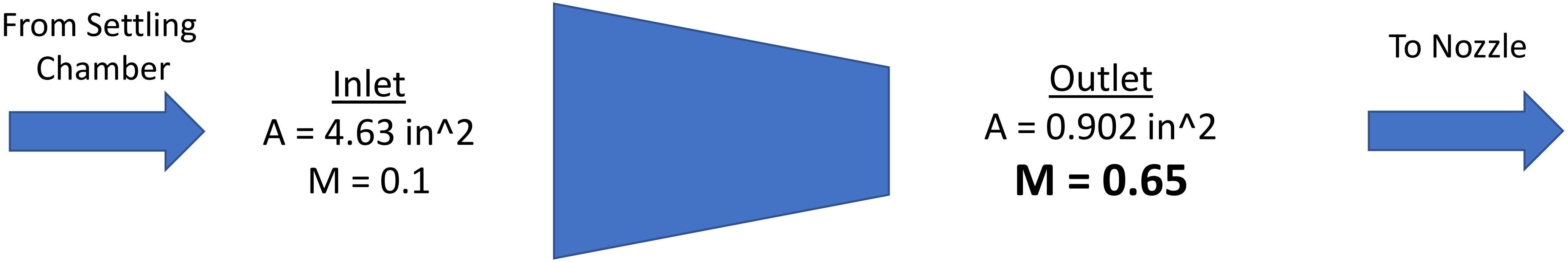
Control Volume Analysis

- With the Control Volume analysis results we are able to produce the desired engine conditions for the cold flow test (using scaled nozzle):

M = 1.06 Test: $\dot{m} = 0.202 \frac{\mathrm{kg}}{\mathrm{s}}$ $M = 0.65$ $p_t = 167\mathrm{kPa}$

M = 1.3 Test: $\dot{m} = 0.281 \frac{\mathrm{kg}}{\mathrm{s}}$ $M = 0.65$ $p_t = 233\mathrm{kPa}$

- Converging Section:



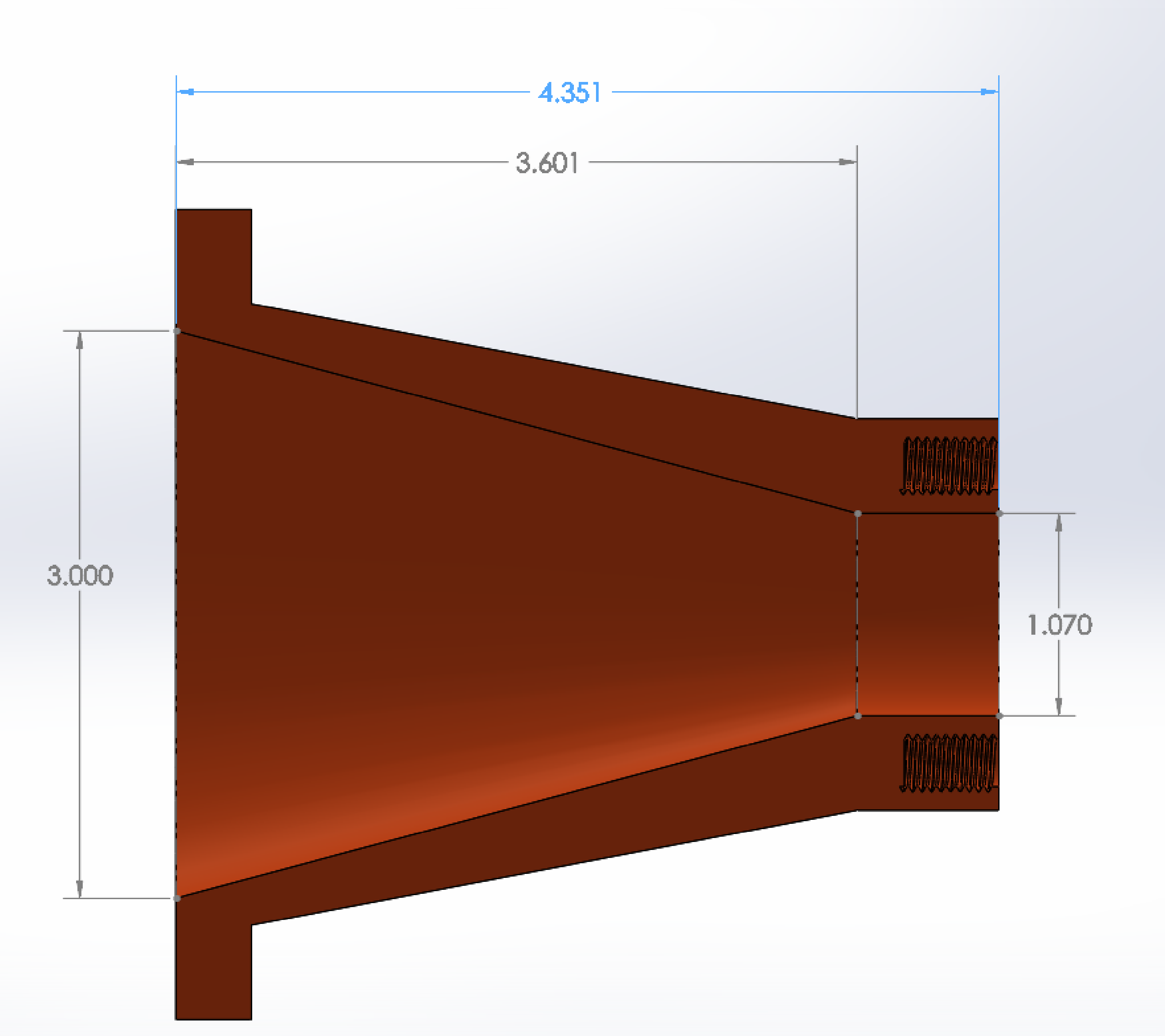
From Settling Chamber

Inlet
A = 4.63 in^2
M = 0.1

Outlet
A = 0.902 in^2
M = 0.65

To Nozzle

Control Volume Analysis



Aluminum
Machined in house

Appendix: Mass-Flow through Nozzle

$$\dot{m} = \frac{p_t A}{\sqrt{T_t}} \sqrt{\frac{\gamma}{R}} M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

- Compression Ratio influences Stagnation Pressure and Temperature, which influence maximum Mach number.

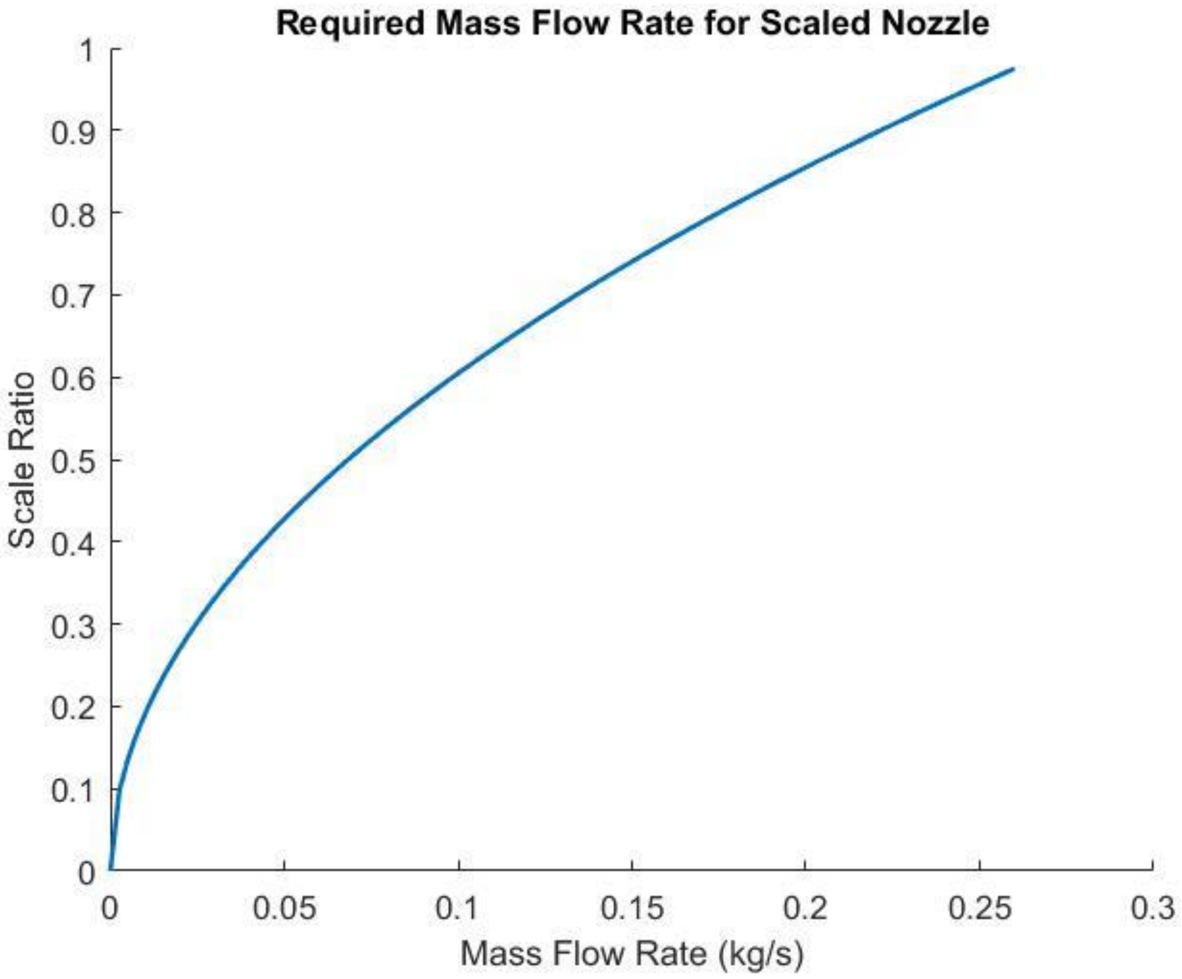
- Critical throat area determined with a Mach number of 1, ideal isentropic Pressure and Temperature conditions, and a fixed maximum mass flow rate.

Appendix: Nozzle Scalability

$$\left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_e^2\right)\right)^{\frac{\gamma+1}{\gamma-1}}$$

$$\dot{m}^* = \frac{p_0 A^*}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

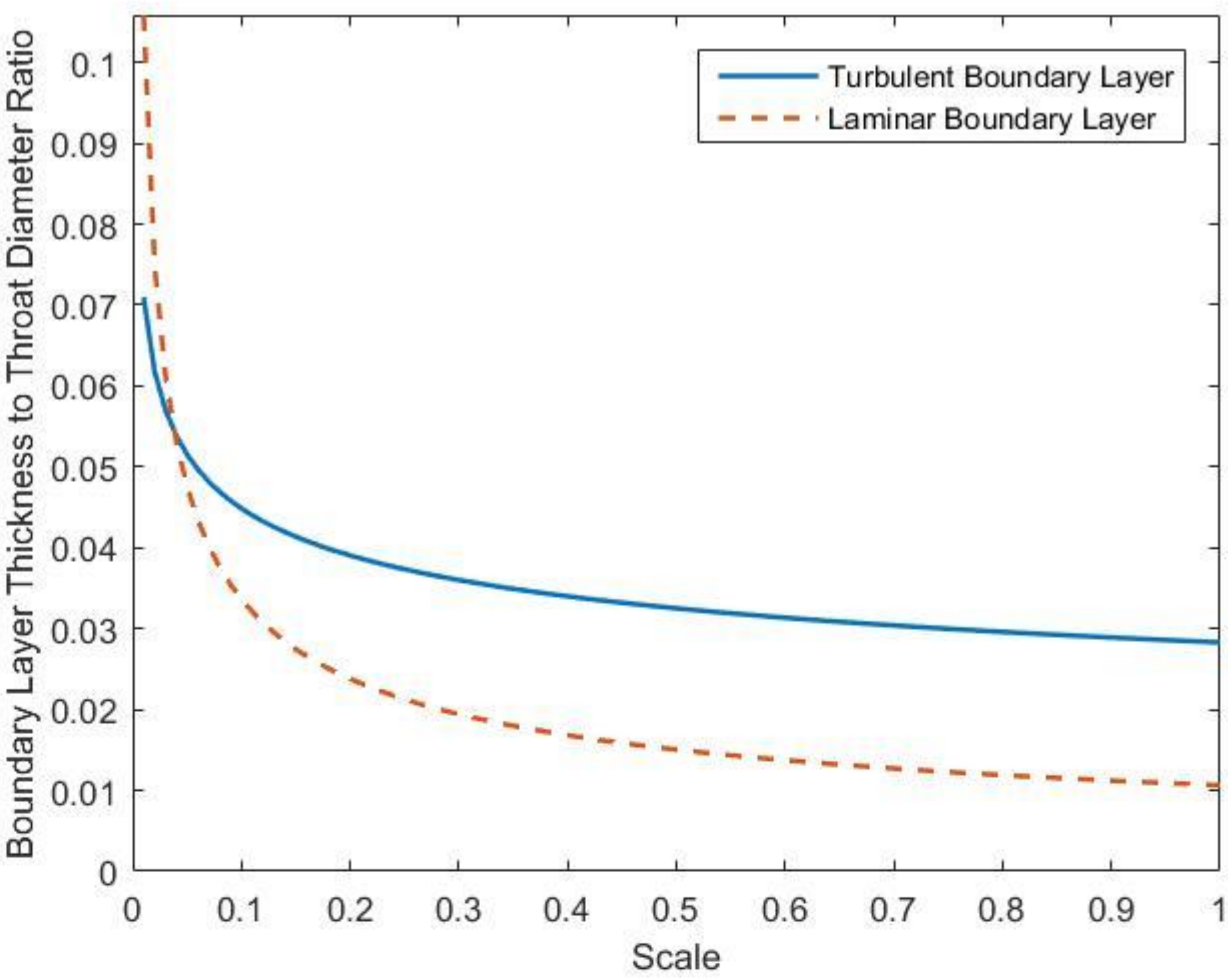
$$A^* = \frac{\dot{m}\sqrt{T_0}}{p_0\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}$$



Appendix: Nozzle Scalability

$$\delta \approx 5.0x/\sqrt{\mathrm{Re}_x}$$

$$\delta \approx 0.37x/{\mathrm{Re}_x}^{1/5}$$



Appendix: Mach # from Pressure

Supersonic Flow

- Rayleigh Pitot Tube Formula:
    1. Holds for supersonic flow, M>1
    2. Accounts for normal shock formed in front of the pitot tube

$$\frac{p_{o_2}}{p_1} = \frac{p_{o_2}}{p_2}\frac{p_2}{p_1} = \left(\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)}\right)^{\gamma/(\gamma-1)} \frac{1-\gamma+2\gamma M_1^2}{\gamma+1}$$

$$\frac{p_{o_1}}{p_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\gamma/(\gamma-1)}$$

A pitot tube measures the stagnation pressure behind the shock. The total pressure before the shock is known from the settling chamber. Therefore, the equation above can be solved for M (the desired value to verify our designed nozzle can achieve supersonic flow).

Appendix: Mach # from Pressure

- Measuring compressible flow (still subsonic)

Compressible Subsonic Flow

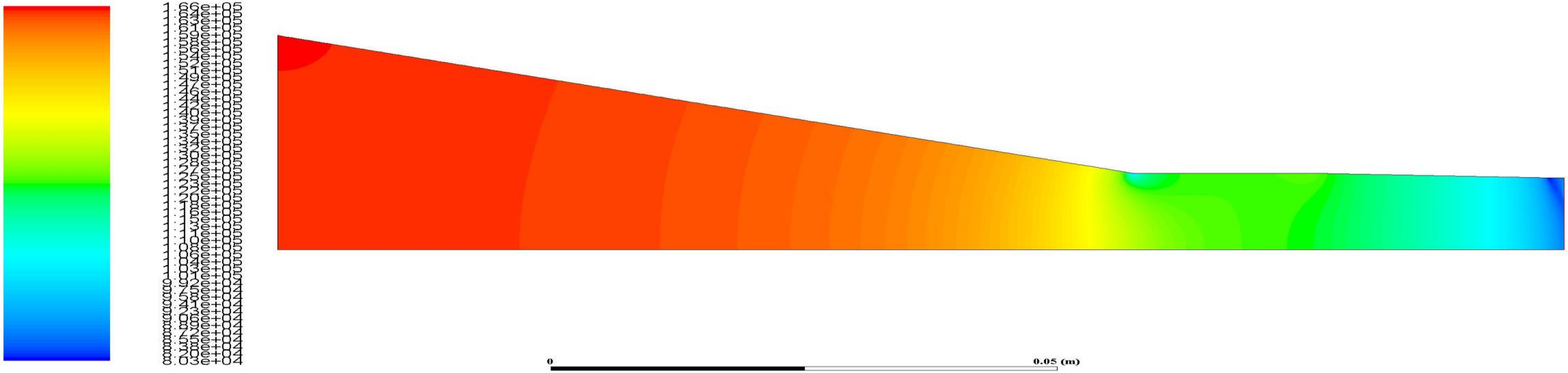
$$M^2 = \frac{2}{\gamma - 1}\left[\left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

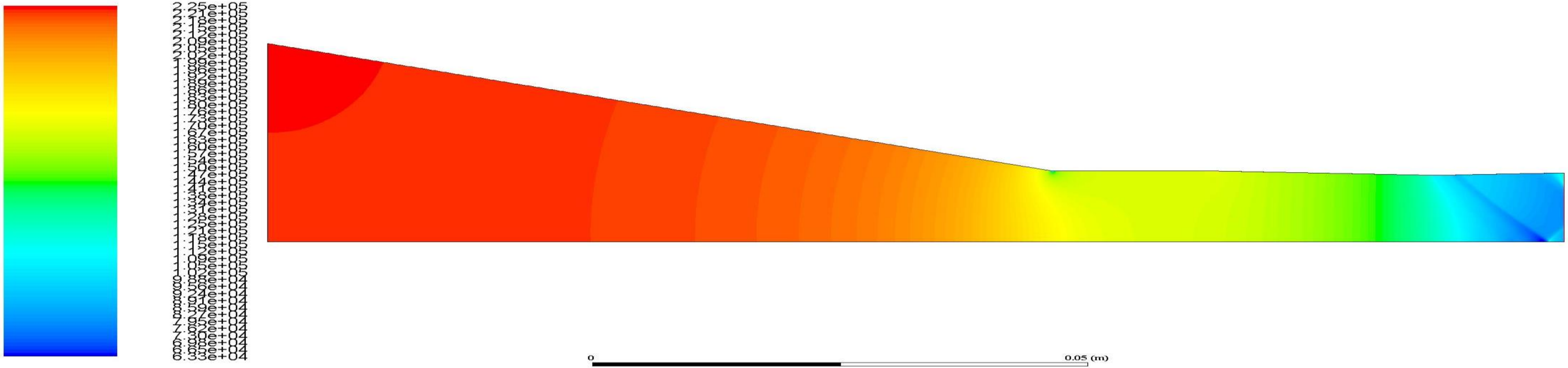
Where:

P_0 - total pressure
P - static pressure

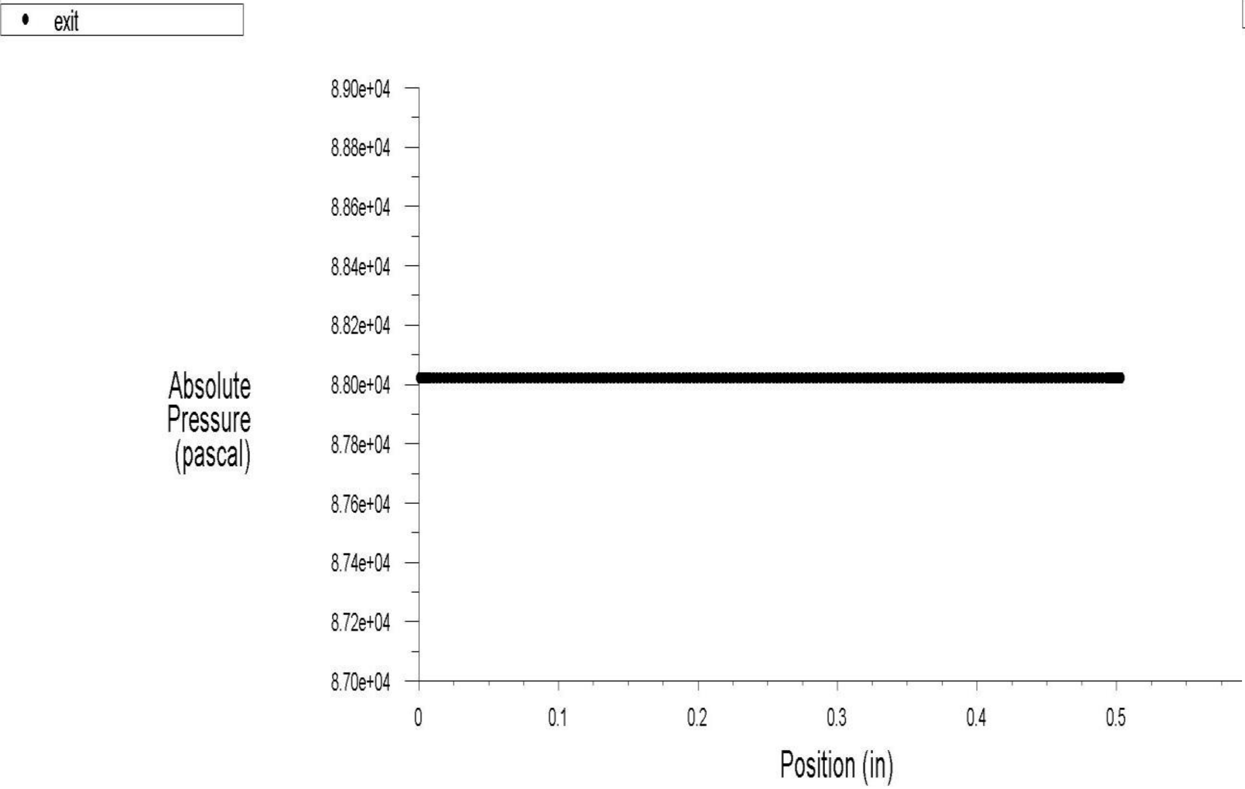
With the pitot tube measuring the total and static pressure, M can be solved for in the above equation.

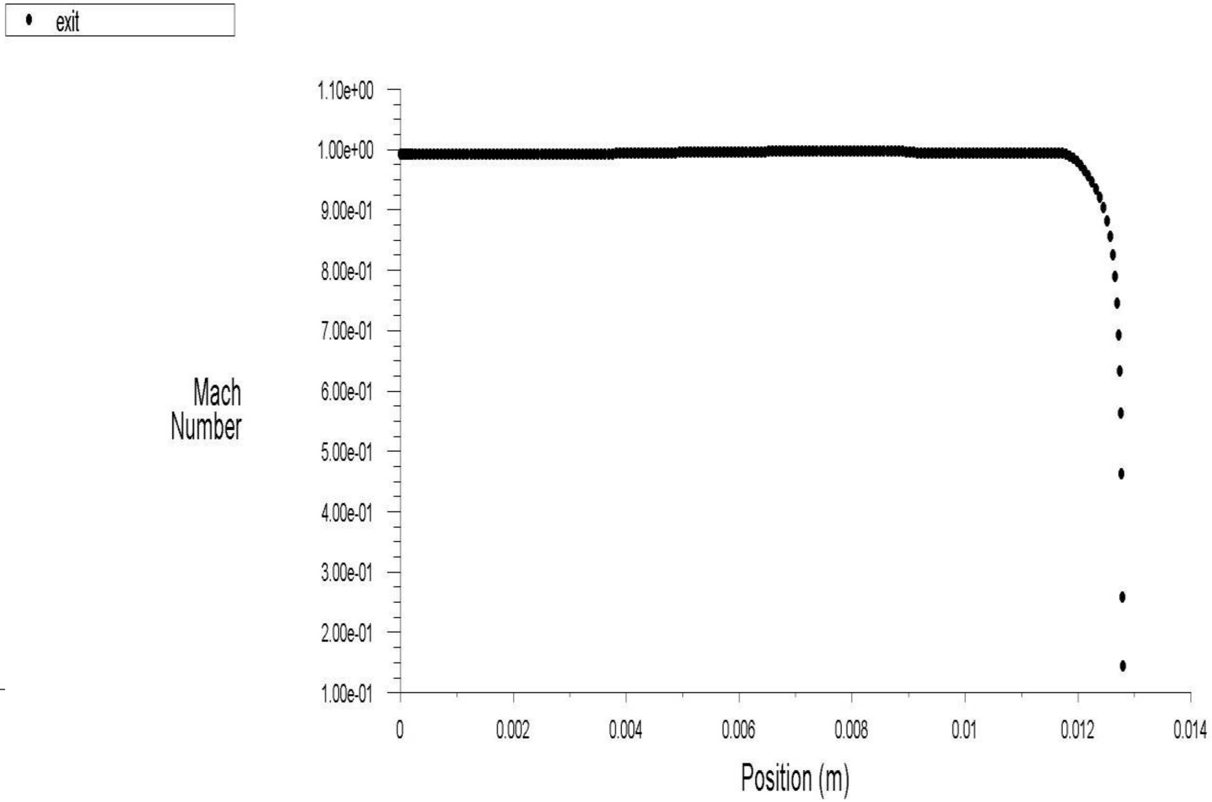
Appendix: Test Bed Pressure Contours



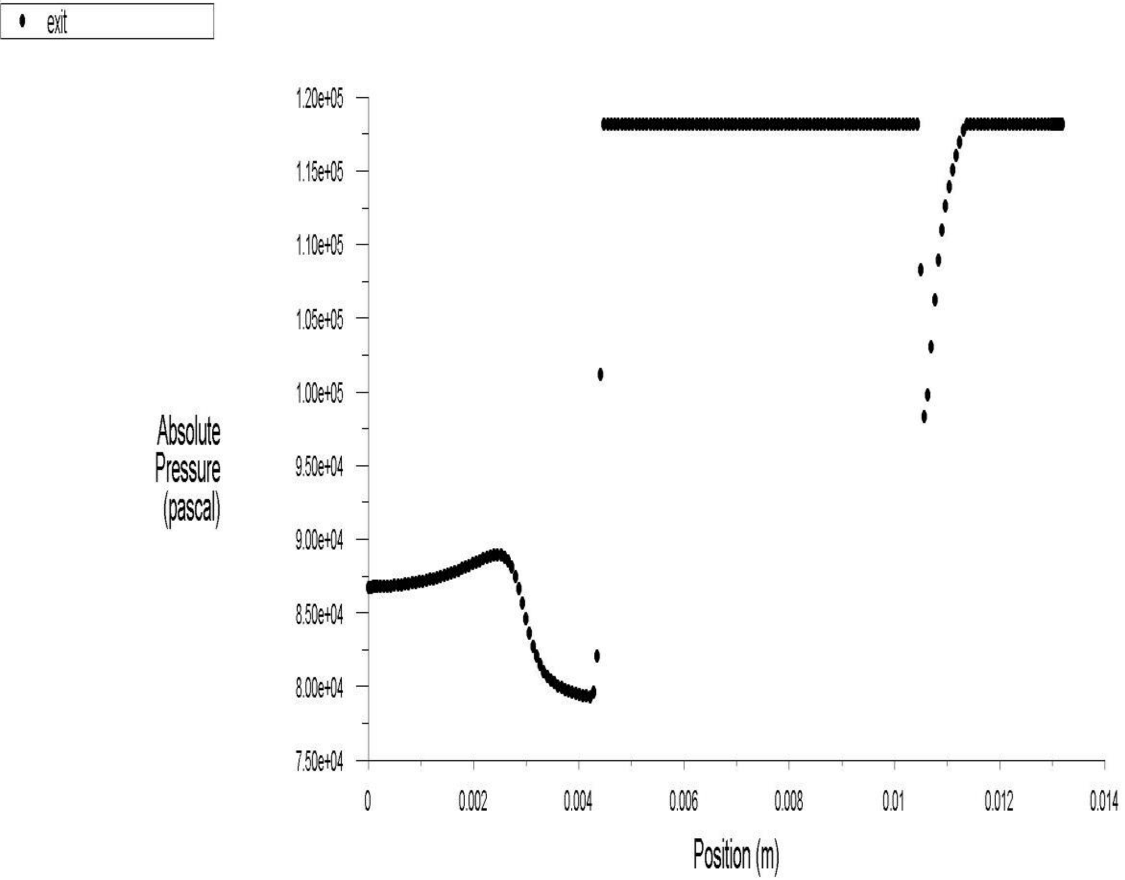


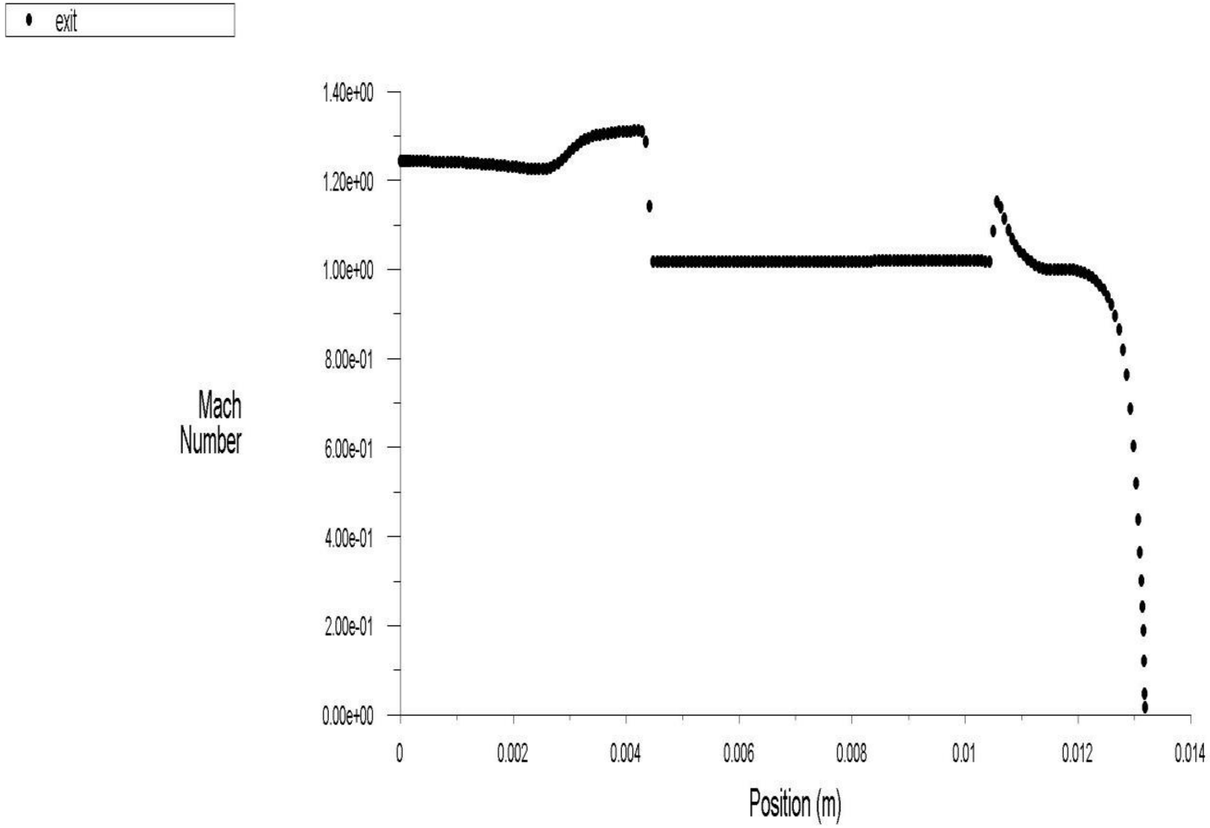
Appendix: Test Bed Pressure and Mach Number at Exit Plane (Mach 1.06 Design)



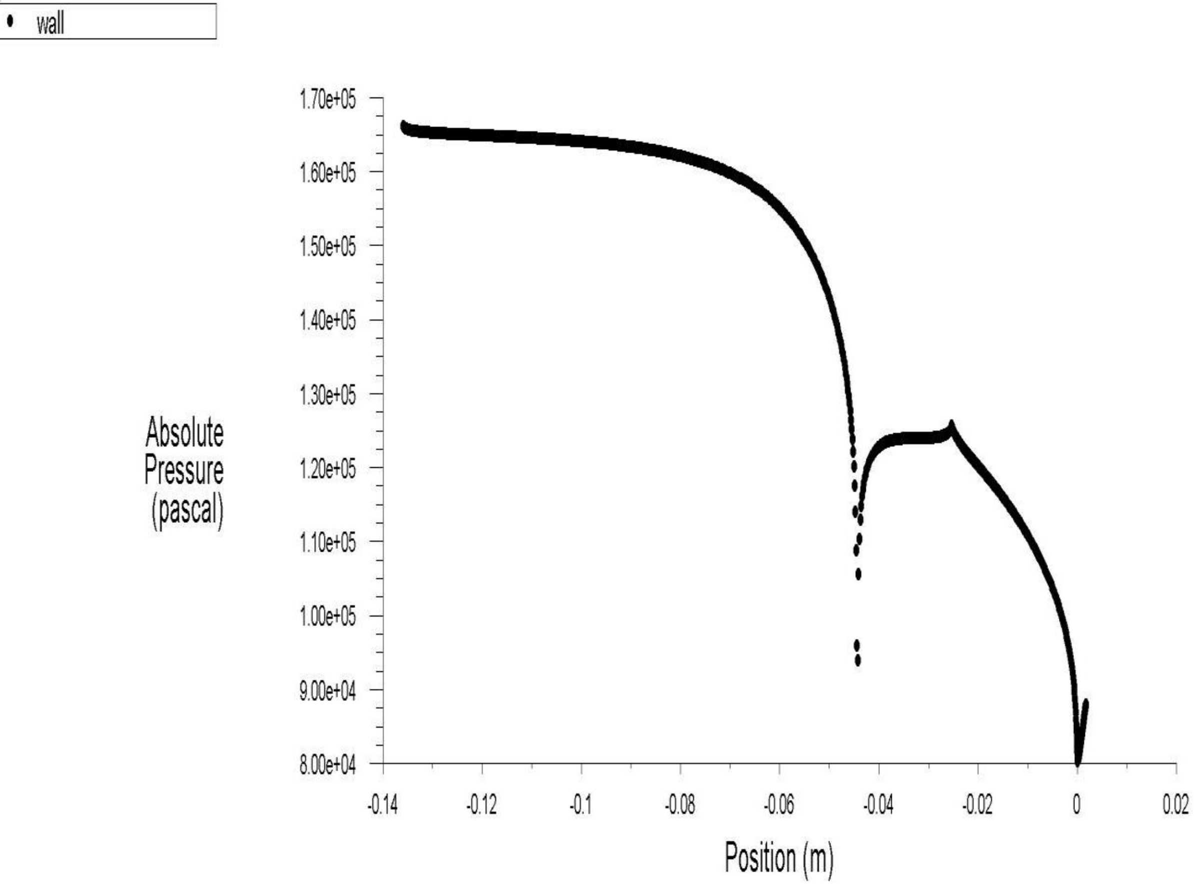


Appendix: Test Bed Pressure and Mach Number at Exit Plane (Mach 1.3 Design)

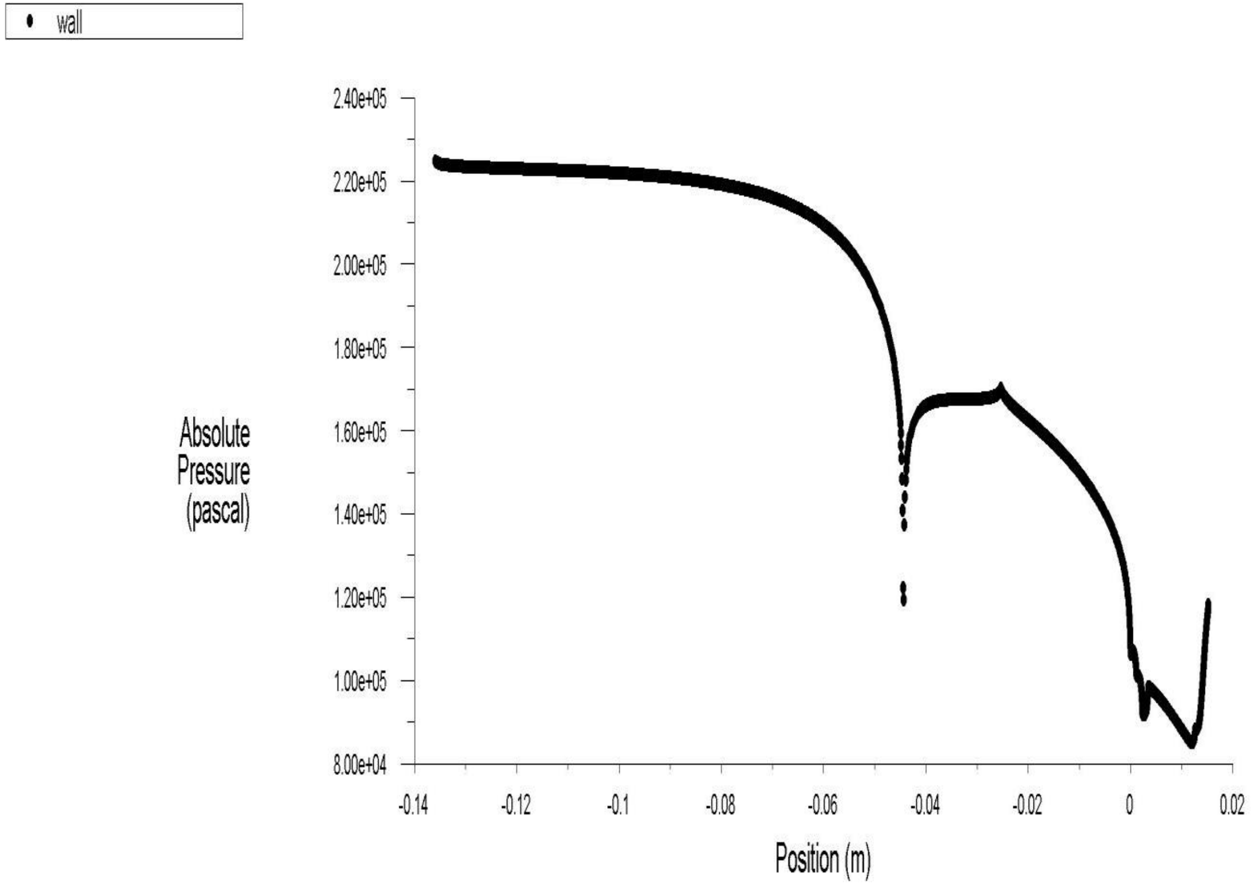




Appendix: Test Bed Wall Pressure Values

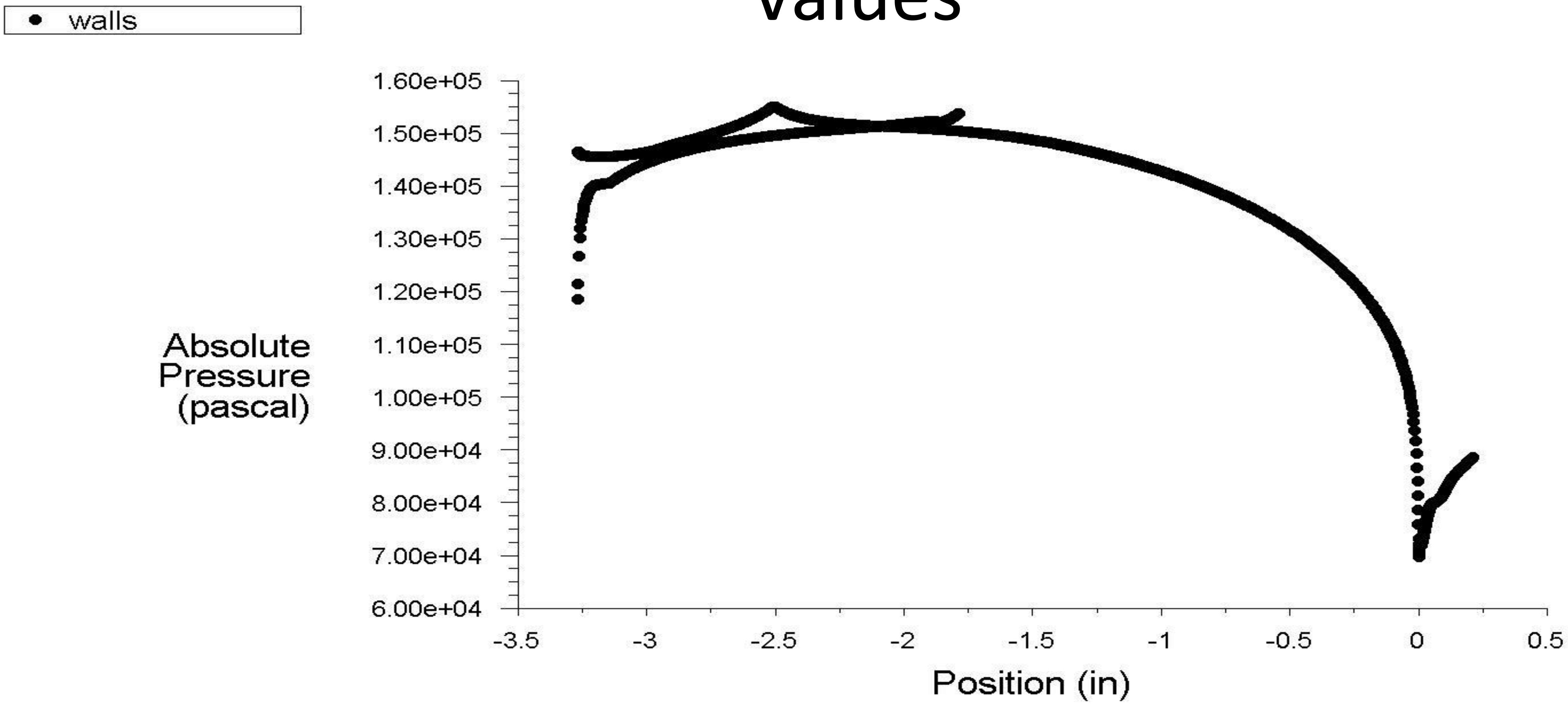


Mach 1.06



Mach 1.3

Appendix: SABRE-Nozzle Wall Pressure Values

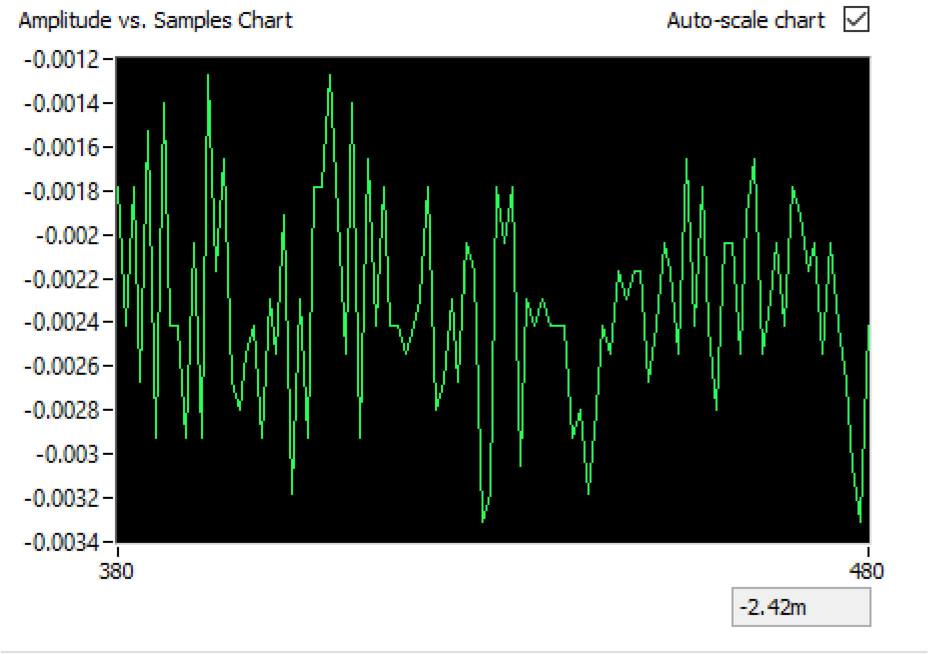


K-Type Thermocouple

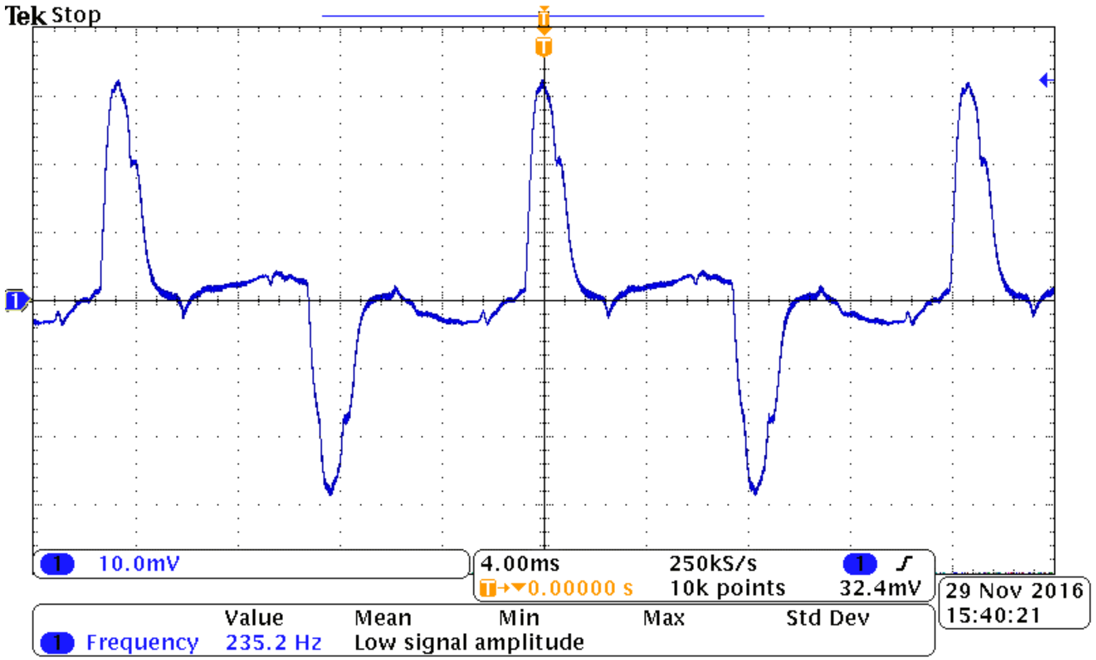
- Chromel-alumel
- Low cost, easy to acquire
- Wide temperature range: -200°C to 1350°C
- Operates well in an oxidizing environment

Sensor Noise

NI-USB-6009 Noise



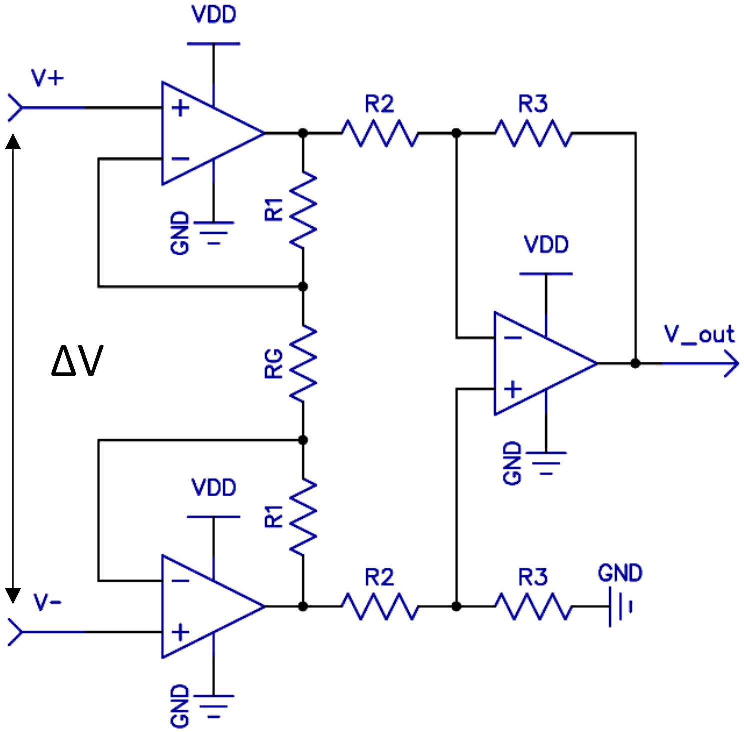
PX137-030DV Noise



	Value	Mean	Min	Max	Std Dev
1 Frequency	235.2 Hz	Low signal amplitude			

Signal Processing

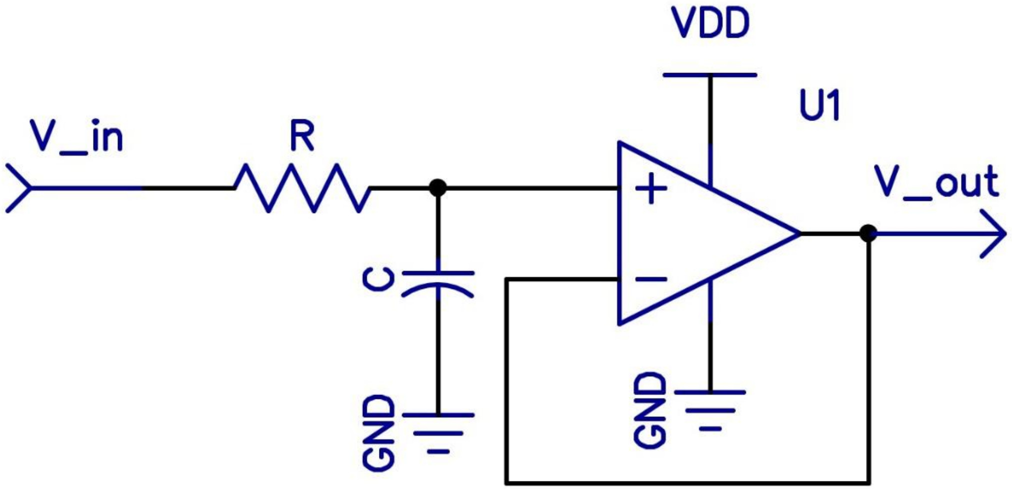
Instrumentation Amplifier



- Gain Calculation

$$\frac{V_{out}}{\Delta V} = \left(1 - \frac{2R_1}{R_G}\right)\left(\frac{R_3}{R_2}\right)$$

Low Pass Filter



- Cutoff Calculation

$$f_c = \frac{1}{2\pi RC}$$

Test Bed Budget