

Preliminary Design Review

SABRE



NOZZLE

Supersonic Air-Breathing Redesigned Engine Nozzle

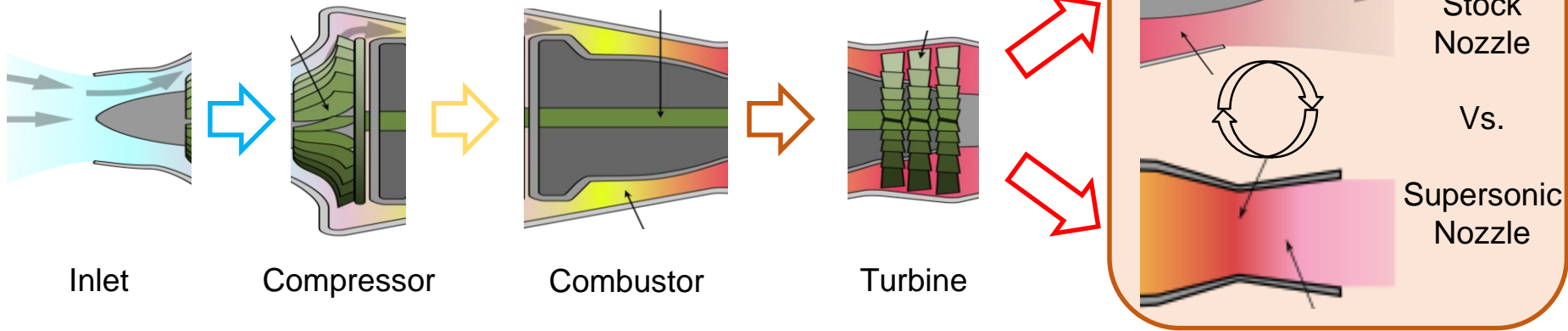
Customer: Air Force Research Lab

Advisor: Brian Argrow

Team Members: Corrina Briggs, Jared Cuteri, Tucker Emmett, Alexander Muller,
Jack Oblack, Andrew Quinn, Andrew Sanchez, Grant Vincent, Nathaniel Voth

Project Description

Model, manufacture, and **verify** an **integrated nozzle** capable of accelerating subsonic exhaust to **supersonic exhaust** produced from a **P90-RXi JetCat** engine for **increased thrust and efficiency** from its stock configuration.



Objectives/Requirements

- **FR 1:** The Nozzle Shall accelerate the flow from subsonic to supersonic conditions.
- **FR 2:** The Nozzle shall not decrease the Thrust-to-Weight Ratio.
- **FR 3:** The Nozzle shall be designed and manufactured such that it will integrate with the JetCat Engine.
 - **DR 3.1:** The Nozzle shall be manufactured using additive manufacturing.
 - **DR 3.4:** Successful integration of the nozzle shall be reversible such that the engine is operable in its stock configuration after the new nozzle has been attached, tested, and detached.
- **FR4:** The Nozzle shall be able to withstand engine operation for at least 30 seconds.

Project
Description

Baseline
Design

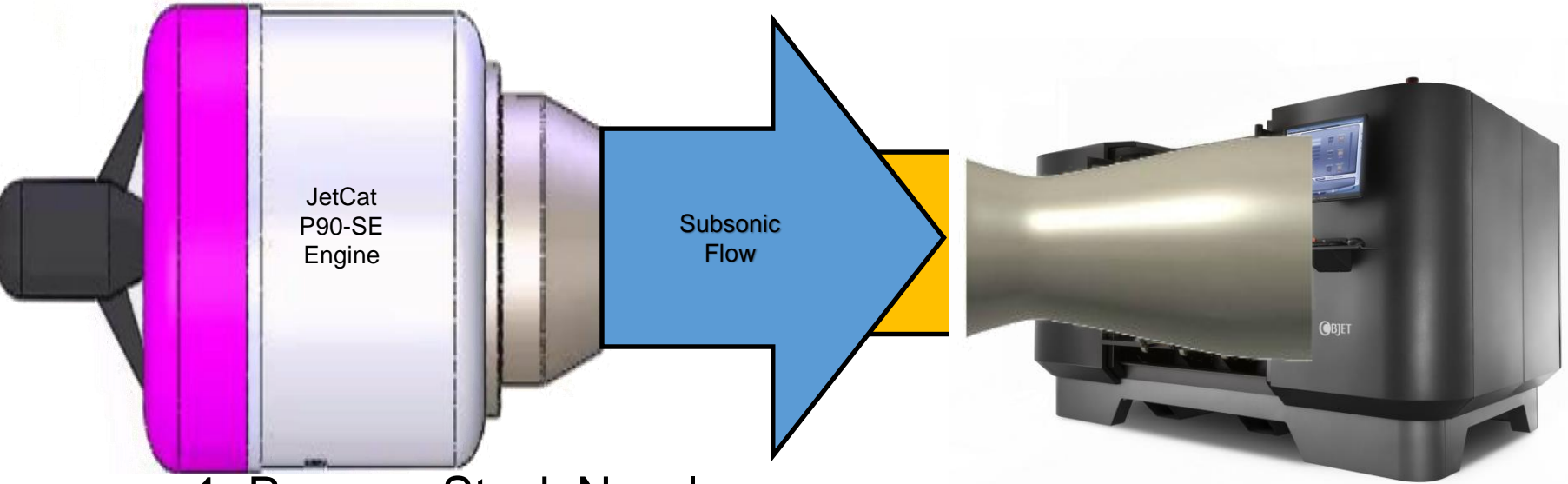
Nozzle
Aerodynamics

Nozzle Test
Bed

Nozzle
Integration

Project
Summary

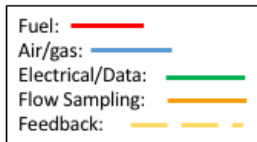
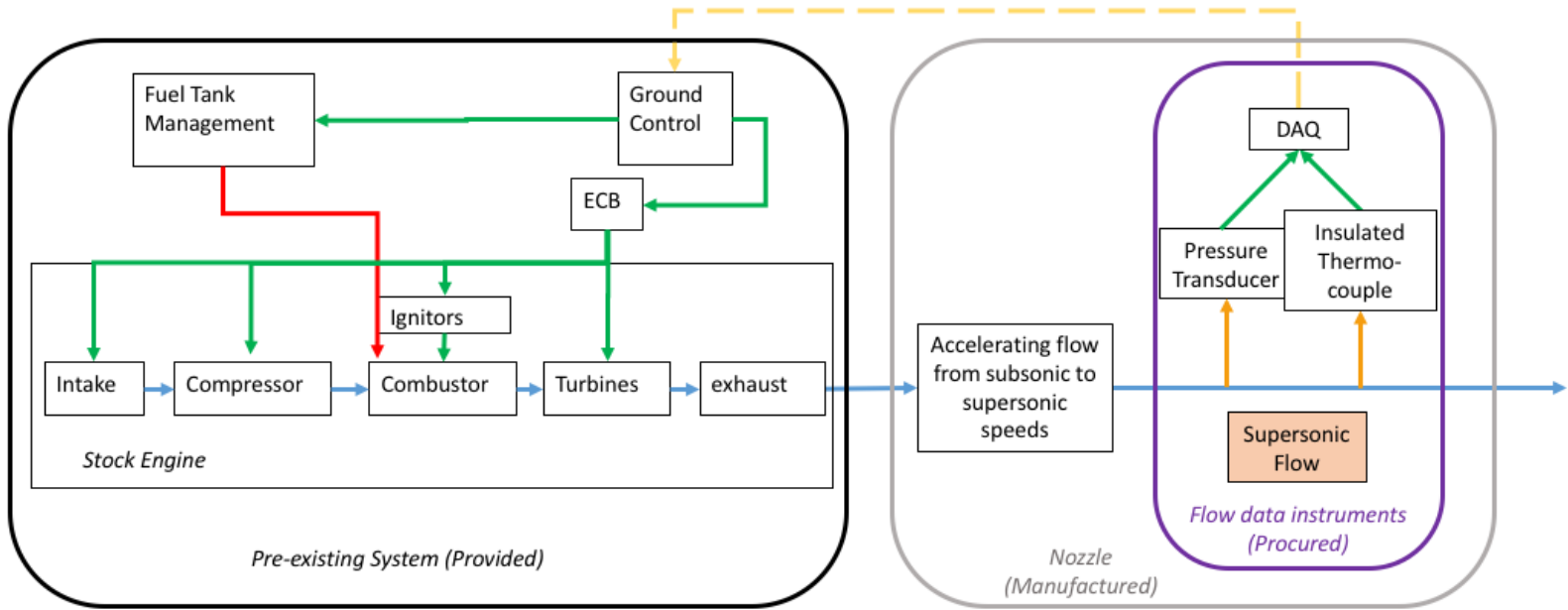
Concept of Operations



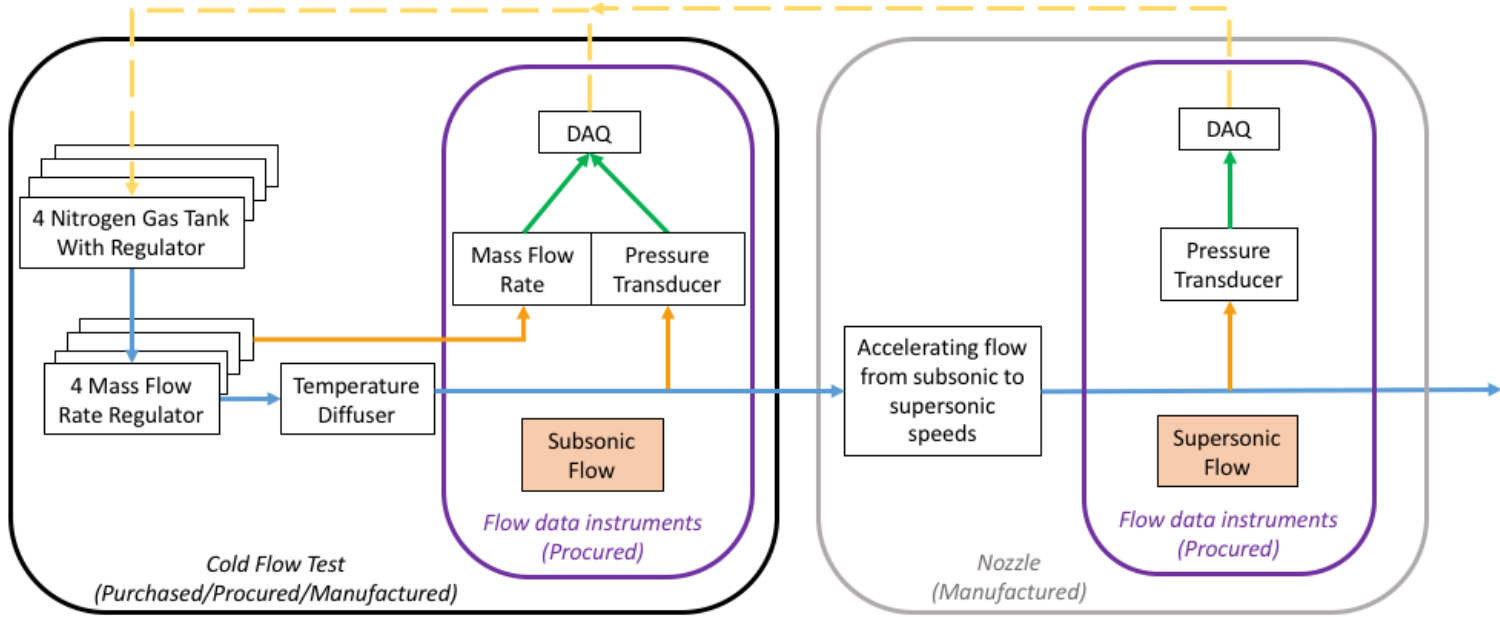
1. Remove Stock Nozzle
2. Additive Manufactured 3-D Nozzle
3. Integrate Nozzle to JetCat
4. Operate Supersonic Engine



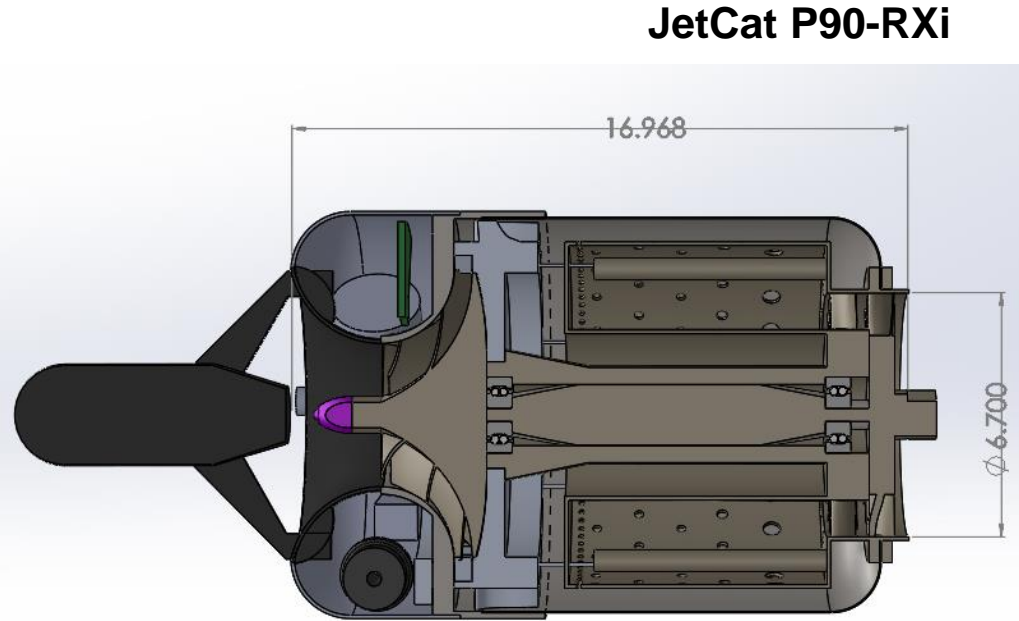
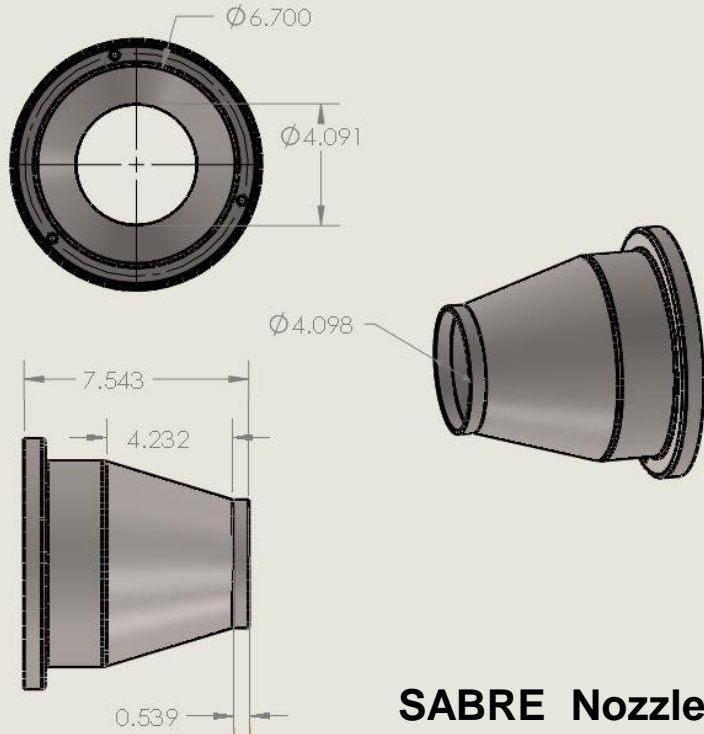
Engine FBD



Cold Flow Analog FBD



Nozzle Aerodynamics Baseline Design



All units in cm

Project Description

Baseline Design

Nozzle Aerodynamics

Nozzle Test Bed

Nozzle Integration

Project Summary

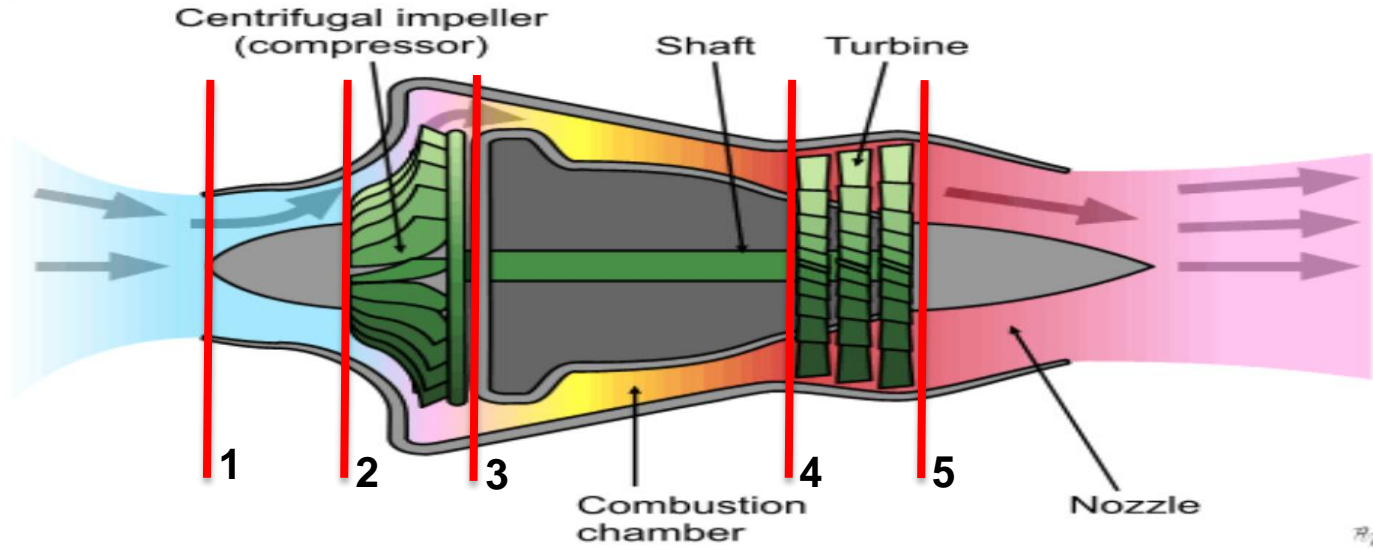
Nozzle Aerodynamics Feasibility

- Nozzle design concept trade study:
 - Most Critical Considerations:
 1. Weight
 2. Cost
 3. Complexity
 - Highest scoring designs:
 1. Minimum Length Nozzle(MLN)
 2. De Laval

| | Weighting | Minimum Length | de Laval |
|--------------------------|-----------|----------------|------------|
| Weight | 0.4 | 5 | 4 |
| Cost | 0.3 | 5 | 4 |
| Complexity | 0.25 | 4 | 5 |
| Altitude Envelope | 0.05 | 1 | 1 |
| Total | 5 | 4.55 | 4.1 |



Nozzle Aerodynamics Feasibility

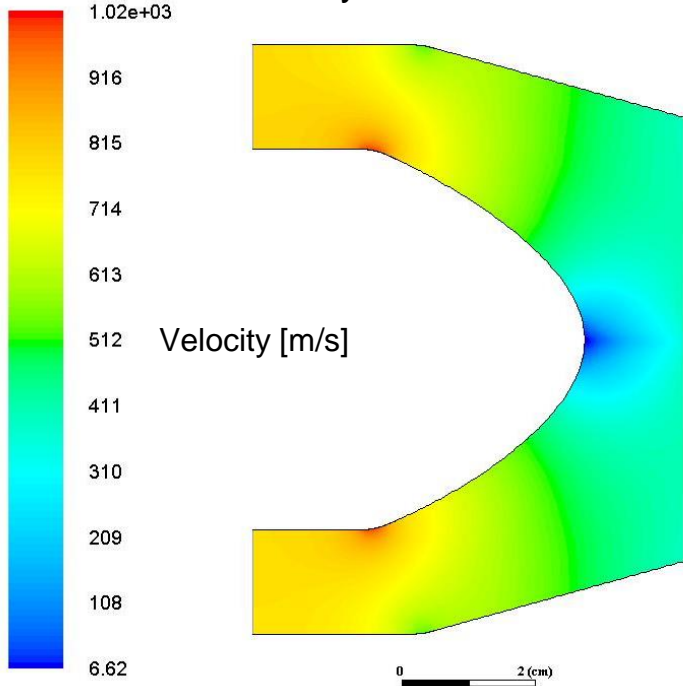


| | | | | |
|---------------------------|---------------------------|------------------------------|------------------------------|-----------------------------|
| $T_{T1} = 291 \text{ K}$ | $T_{T2} = 291 \text{ K}$ | $T_{T3} = 382.3 \text{ K}$ | $T_{T4} = 1214.8 \text{ K}$ | $T_{T5} = 1125.2 \text{ K}$ |
| $P_{T1} = 84 \text{ kPa}$ | $P_{T2} = 84 \text{ kPa}$ | $P_{T3} = 218.4 \text{ kPa}$ | $P_{T4} = 218.4 \text{ kPa}$ | $P_{T5} = 167 \text{ kPa}$ |

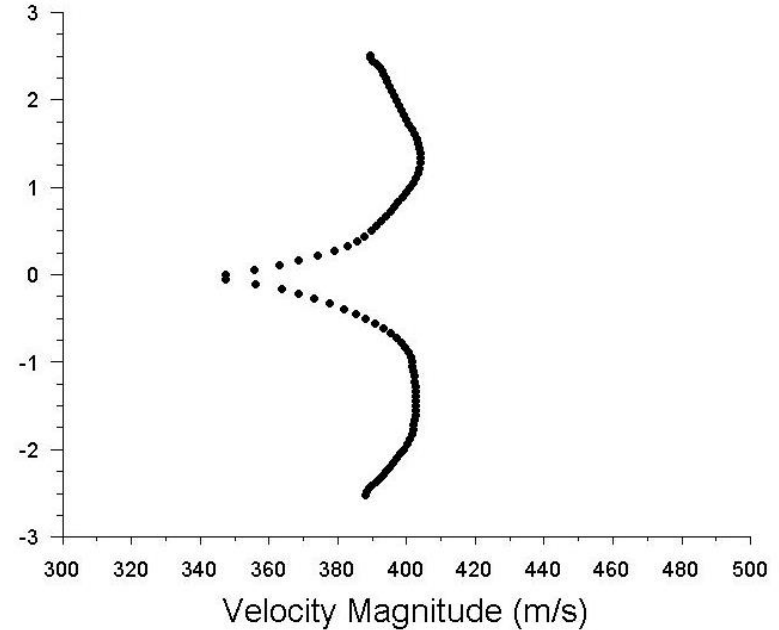
Nozzle Aerodynamics Feasibility

Stock Nozzle with Plug

Velocity Contour



Velocity Profile at Exit



Project Description

Baseline Design

Nozzle Aerodynamics

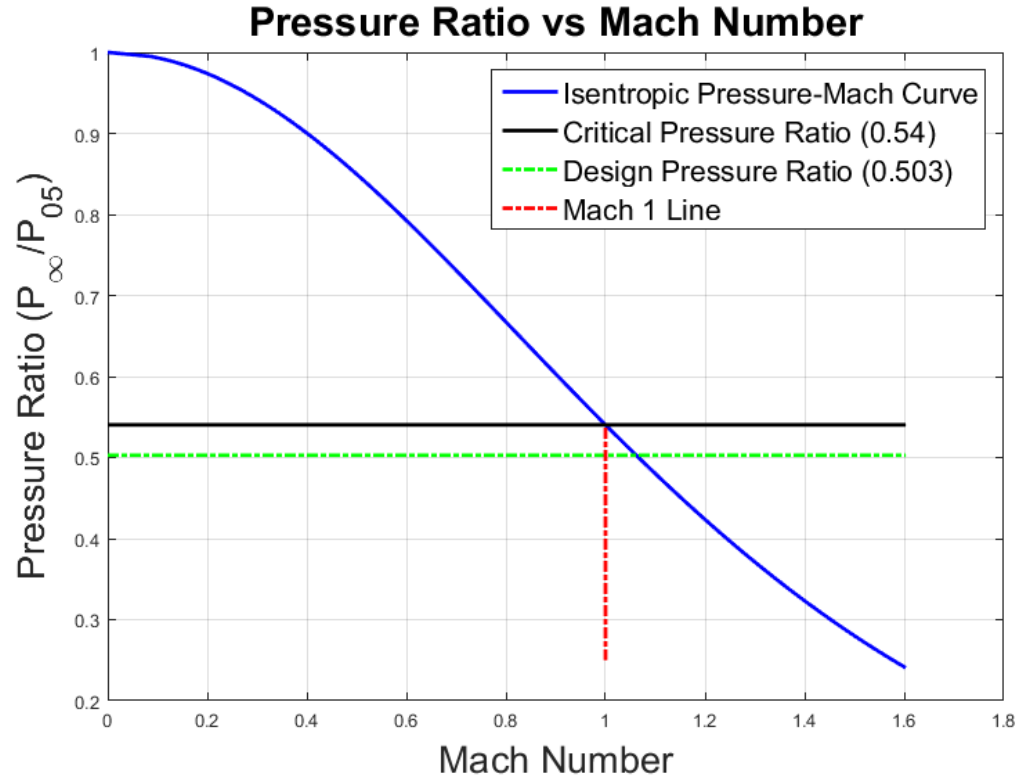
Nozzle Test Bed

Nozzle Integration

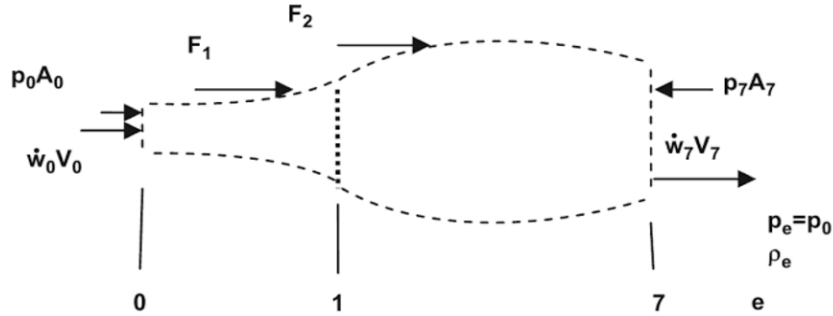
Project Summary

Nozzle Aerodynamics Feasibility

- Quasi 1-D Analysis shows tolerances are very small
- Critical Pressure Ratio for Sonic Flow: **0.540**
- Ambient Pressure to Turbine Exit Pressure Ratio: **0.503**
- Maximum Mach: **1.06**

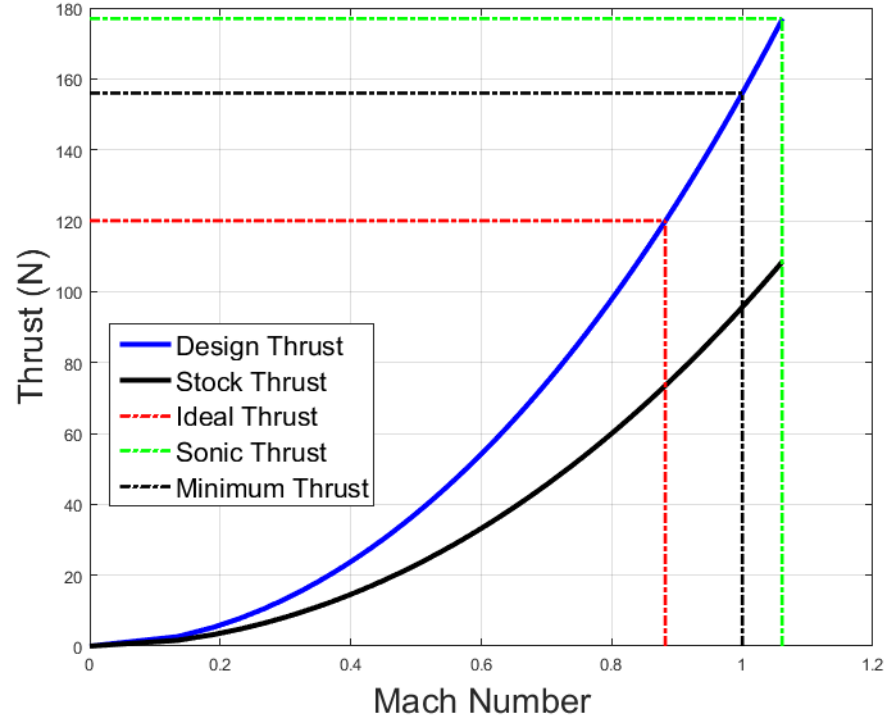


Nozzle Aerodynamics Feasibility



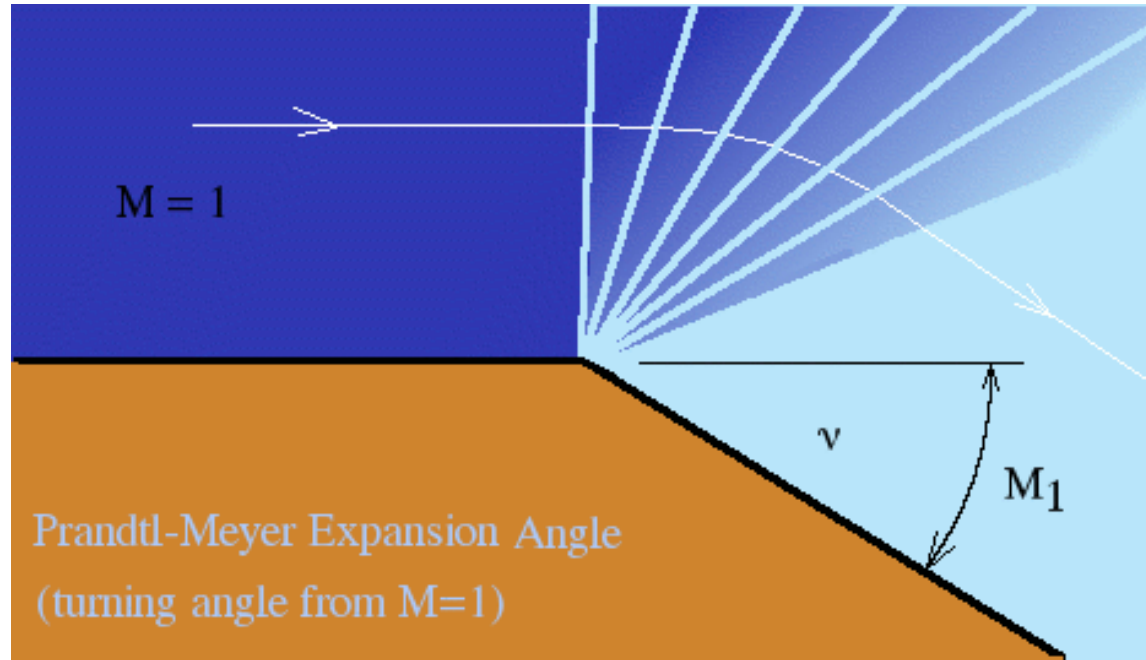
$$F = \left[\frac{\dot{w}_0 + \dot{w}_f}{g} V_7 - \frac{\dot{w}_0}{g} V_0 + A_7(p_7 - p_0) \right] - [p_0(A_0 - A_7) + F_1 + F_4]$$

Thrust vs Mach Number



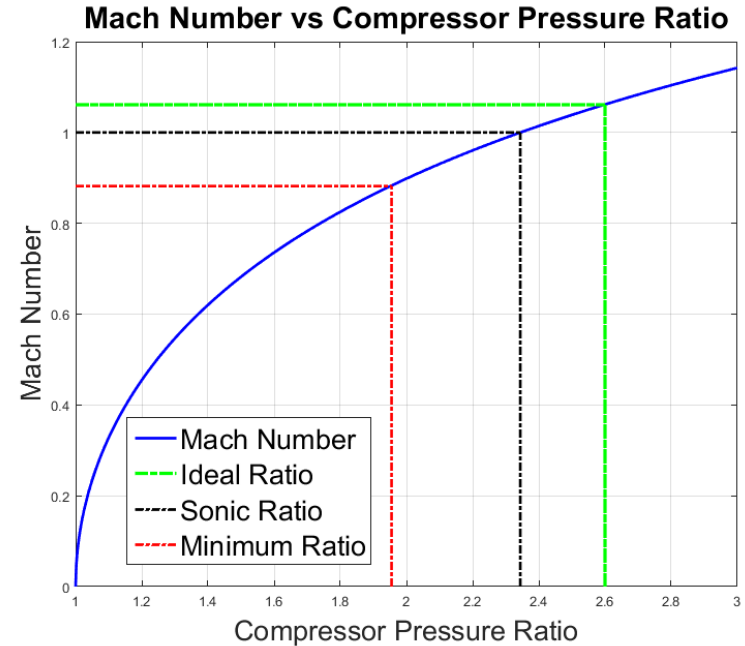
Nozzle Aerodynamics Feasibility

- Prandtl-Meyer Expansion Fan Angle (ν) = **0.65 deg.**
- Entrance Mach (M) = 1
- Exit Mach (M_1) = **1.06**



Nozzle Aerodynamics Feasibility

- Ideal Compressor Ratio = **2.6**
Total pressure at compressor exit = **218.40 kPa**
- Critical (sonic) Compressor Ratio = **2.34**
Total pressure at compressor exit = **196.56 kPa**
- Small Margin: A drop in total pressure greater than **21.84 kPa** causes for the flow to never reach sonic or supersonic speeds
- **Minimum for Thrust Requirement = 1.95**



Project
Description

Baseline
Design

Nozzle
Aerodynamics

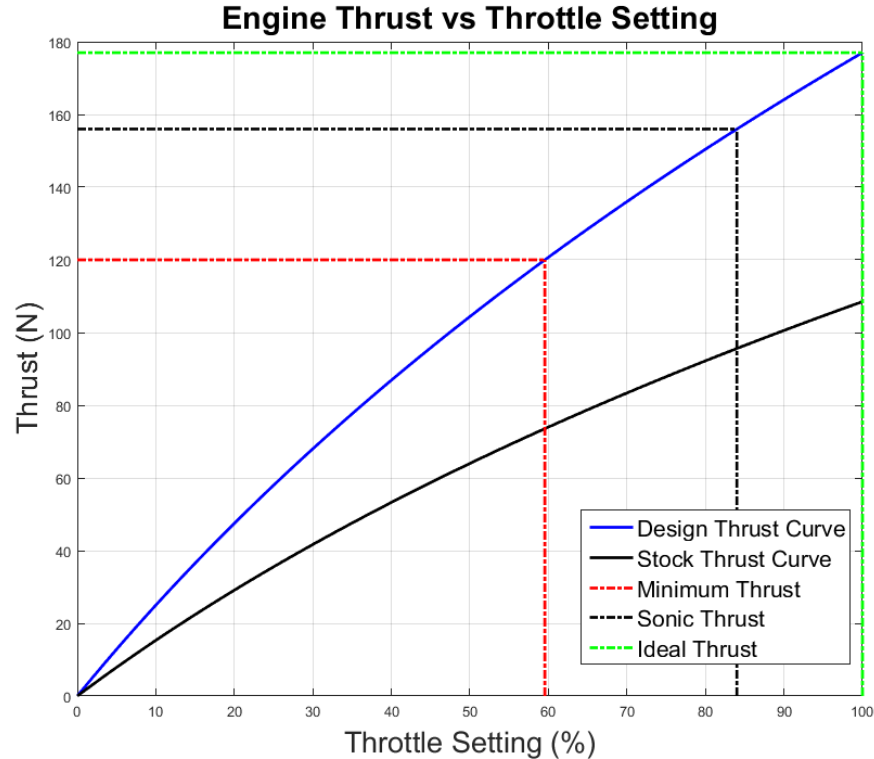
Nozzle Test
Bed

Nozzle
Integration

Project
Summary

Nozzle Aerodynamics Feasibility

- Old Max Thrust: **105 N**
- New Max Thrust: **177 N**
- Design Point: **120 N**
(FR2)



Test Bed CPE's

Simulate flow conditions from engine at the turbine exit

Conditions to model:

- Mass flow rate (0.26 kg/s from engine data sheet)
- Exit pressure of the turbine (167 kPa based off of Brayton cycle analysis)
- Mach number, which determines velocity
- Area is constrained by the above conditions $\dot{m} = \rho AV$

Assumptions

- Flow is laminar through the pipe
- No acceleration of fluid in hoses
- Steady flow

Project
Description

Baseline
Design

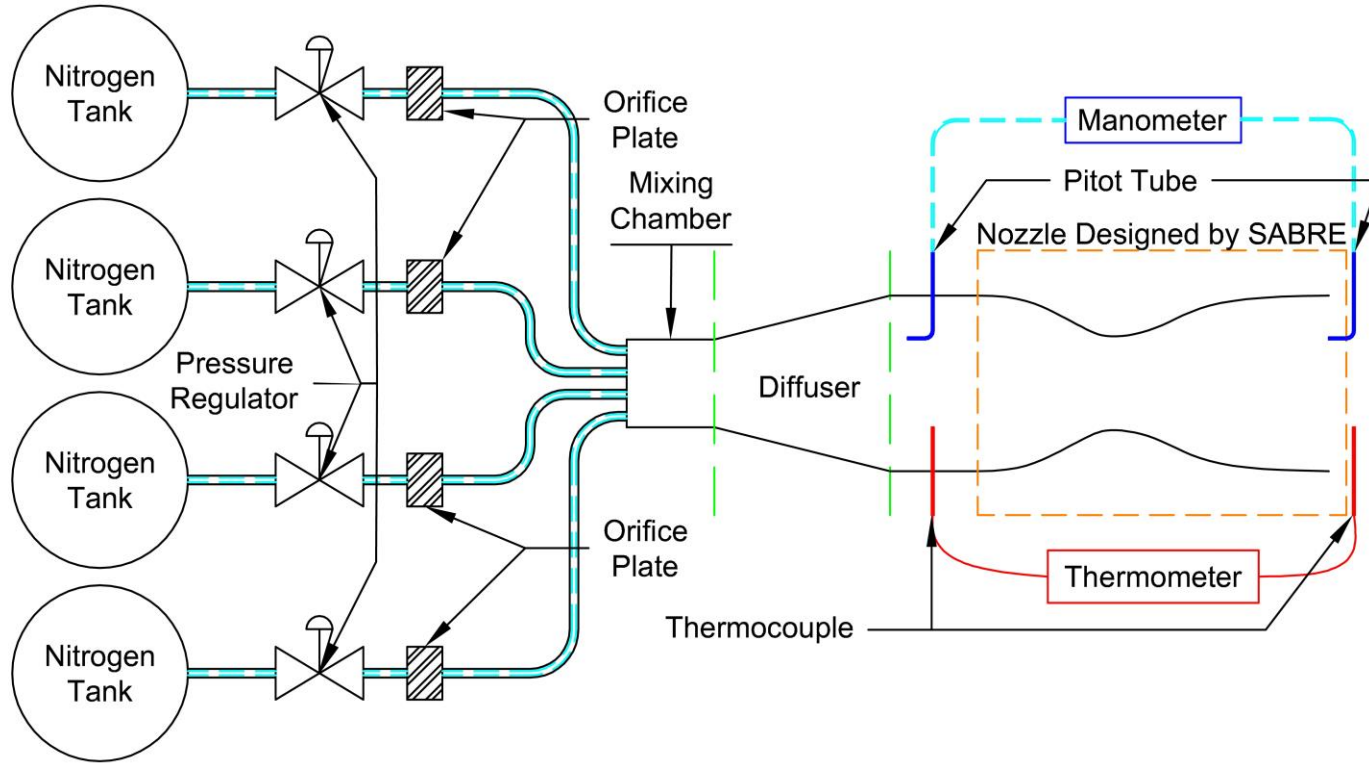
Nozzle
Aerodynamics

Nozzle Test
Bed

Nozzle
Integration

Project
Summary

Test Bed Design



Test Bed Feasibility

Necessity:

$\dot{m} = 0.26 \text{ kg/s}$ and $D_{\text{orifice}} < 1.905 \text{ cm}$ \longrightarrow High p_1 \longrightarrow Temp through hose too extreme ($> 500 \text{ K}$)
 ($\frac{3}{4}$ in.)

$$\dot{m} = CAp_1 \sqrt{\frac{\gamma}{RT_1} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} \quad \frac{0.528p_1}{\rho R}$$

When $\dot{m} = 0.065$ and $D_{\text{orifice}} < 1.905 \text{ cm}$ \longrightarrow $p_1 = 46 \text{ psi}$ \longrightarrow Highest temp through hose $< 500 \text{ K}$
 ($\frac{3}{4}$ in.)

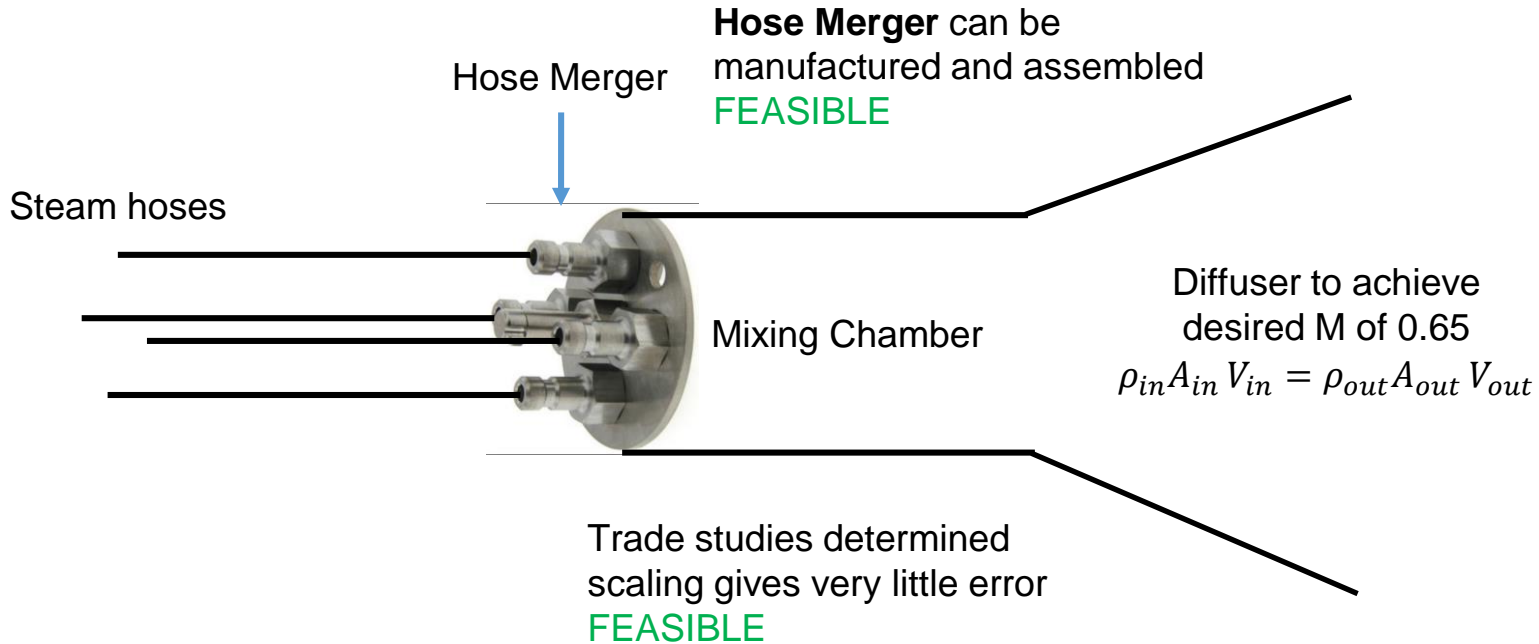
- If area is multiplied by X, mass flow rate is multiplied by X
- Multiple hoses coming together would sum mass flow rates to get total mass flow rate

$$\dot{m} = \rho AV$$

FEASIBLE



Test Bed Feasibility

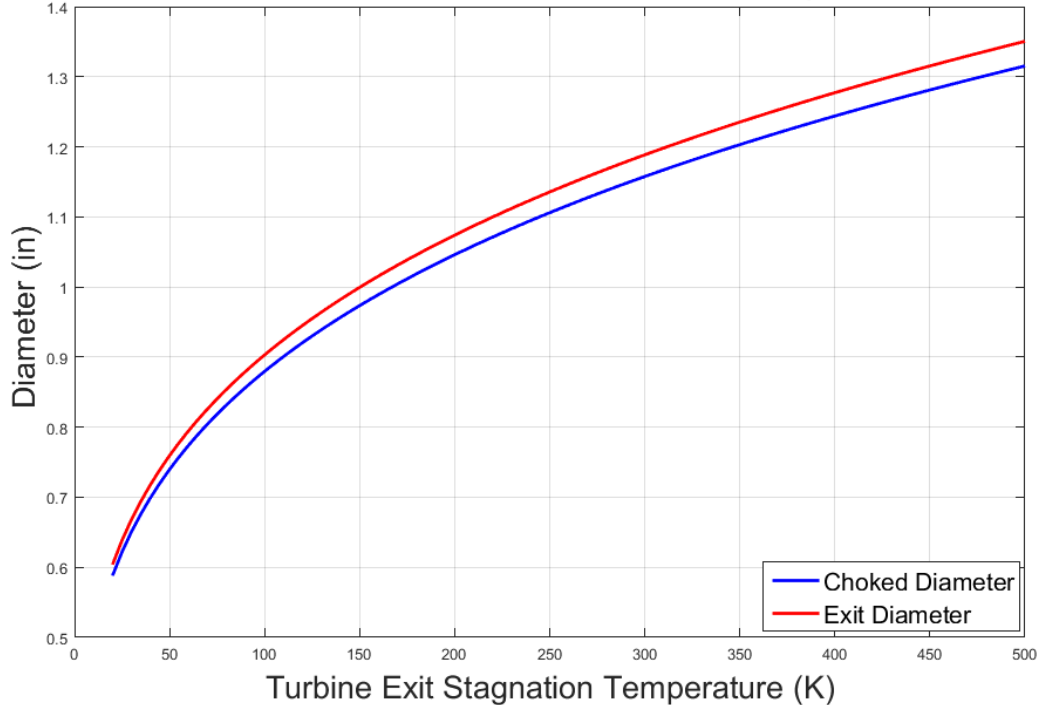


Test Bed Feasibility

- Stagnation temperature affects the optimal throat and exit areas of the nozzle.
- Cold flow test requires nozzle designed to operate at cold flow stagnation temperature
- Same design method can be used to design cold flow nozzle

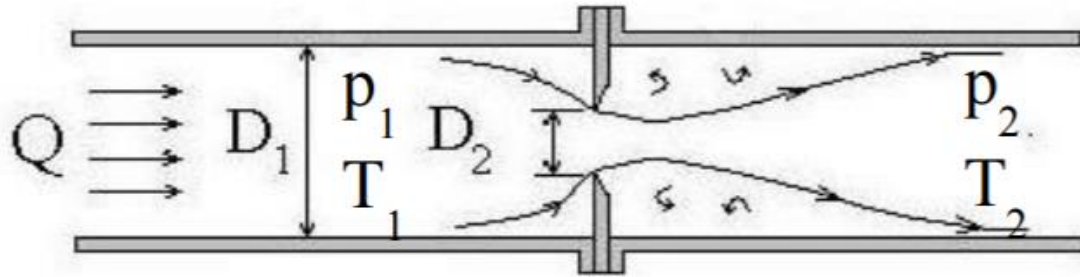
FEASIBLE

Nozzle Throat and Exit Diameter vs Turbine Exit Stagnation Temperature



Test Bed Feasibility

- Forces a mass flow rate
- Works similarly to a convergent divergent nozzle



$\dot{m} = 0.065 \text{ kg/s}$
 $p_1 = 317 \text{ kPa (46 psi)}$
 $C \text{ (orifice plate coefficient)} = 0.77$
 (common value)
 $T_1 = 293 \text{ K (room)}$



For choked flow:

$$\dot{m} = CAp_1 \sqrt{\frac{\gamma}{RT_1} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}}$$

Solving:

D_{orifice} is 1.21 cm, which is **less than** a common hose diameter of 1.905 cm (3/4 in.)

FEASIBLE

Nozzle Integration CPE's

- Additive Manufacturing Process
 - Tolerances within 25.4 μm (0.001 in)
- Nozzle Mass
 - Less than 291 g at 120 N design point to maintain T/W ratio
- Interfacing with Engine
 - No extensive modifications to stock engine and its parts
 - No further flow impedance than that of the stock design
 - Interchangeability between nozzle designs for engine

Project
Description

Baseline
Design

Nozzle
Aerodynamics

Nozzle Test
Bed

Nozzle
Integration

Project
Summary

Nozzle Integration Feasibility

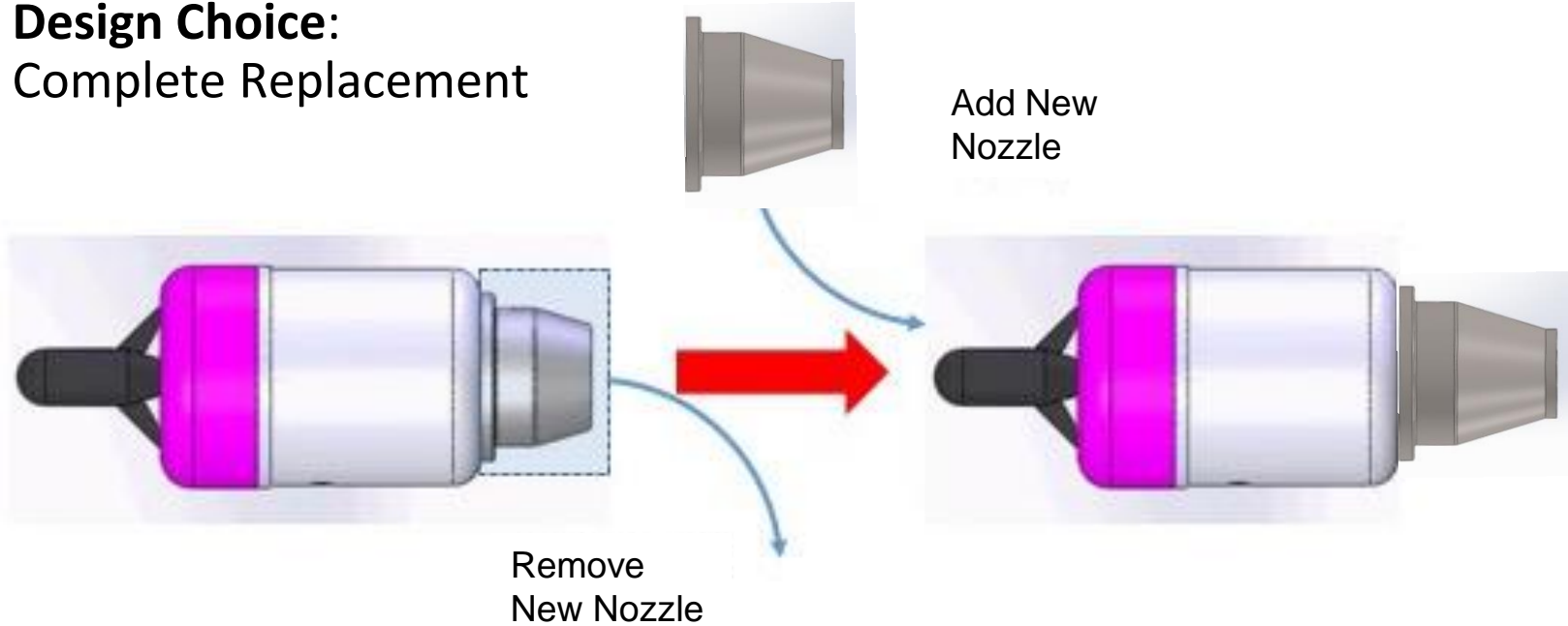
- Nozzle Integration Trade Study:
 - Highest scoring designs:
 1. Complete Replacement
 2. Dome Replacement

| | Weighting | Replacement | Attachment | Sock | Dome Replacement |
|---------------------------|-----------|-------------|-------------|------------|------------------|
| Work Cost | 0.25 | 5 | 2 | 3 | 5 |
| Mass | 0.3 | 5 | 3 | 1 | 4 |
| Flow Impedance | 0.15 | 5 | 3 | 3 | 5 |
| Interchangeability | 0.2 | 5 | 1 | 4 | 5 |
| Tolerance | 0.1 | 4 | 2 | 3 | 2 |
| Total | 5 | 4.9 | 2.25 | 2.6 | 4.4 |



Nozzle Integration Feasibility

Design Choice:
Complete Replacement



Project
Description

Baseline
Design

Nozzle
Aerodynamics

Nozzle Test
Bed

Nozzle
Integration

Project
Summary

Nozzle Integration Feasibility

Design Choice: Complete Replacement

- One piece Design
 - Additional 44 g anticipated with stock material, 130 g nozzle design
 - Very low complexity of integration; identical to stock integration
- No permanent alteration/damage to stock engine components
 - Stock Nozzle can be interchanged with SABRE-Nozzle
 - No added mounting equipment

Project
Description

Baseline
Design

Nozzle
Aerodynamics

Nozzle Test
Bed

Nozzle
Integration

Project
Summary

Nozzle Integration Feasibility

Integration Dome Transfer

- 3 screw-Bolt system
 - Low Complexity
 - Tolerance of 0.5 mm (0.02")
- Velocity profile at exit uniform despite presence
- Main purpose to cover turbine bearings



Project
Description

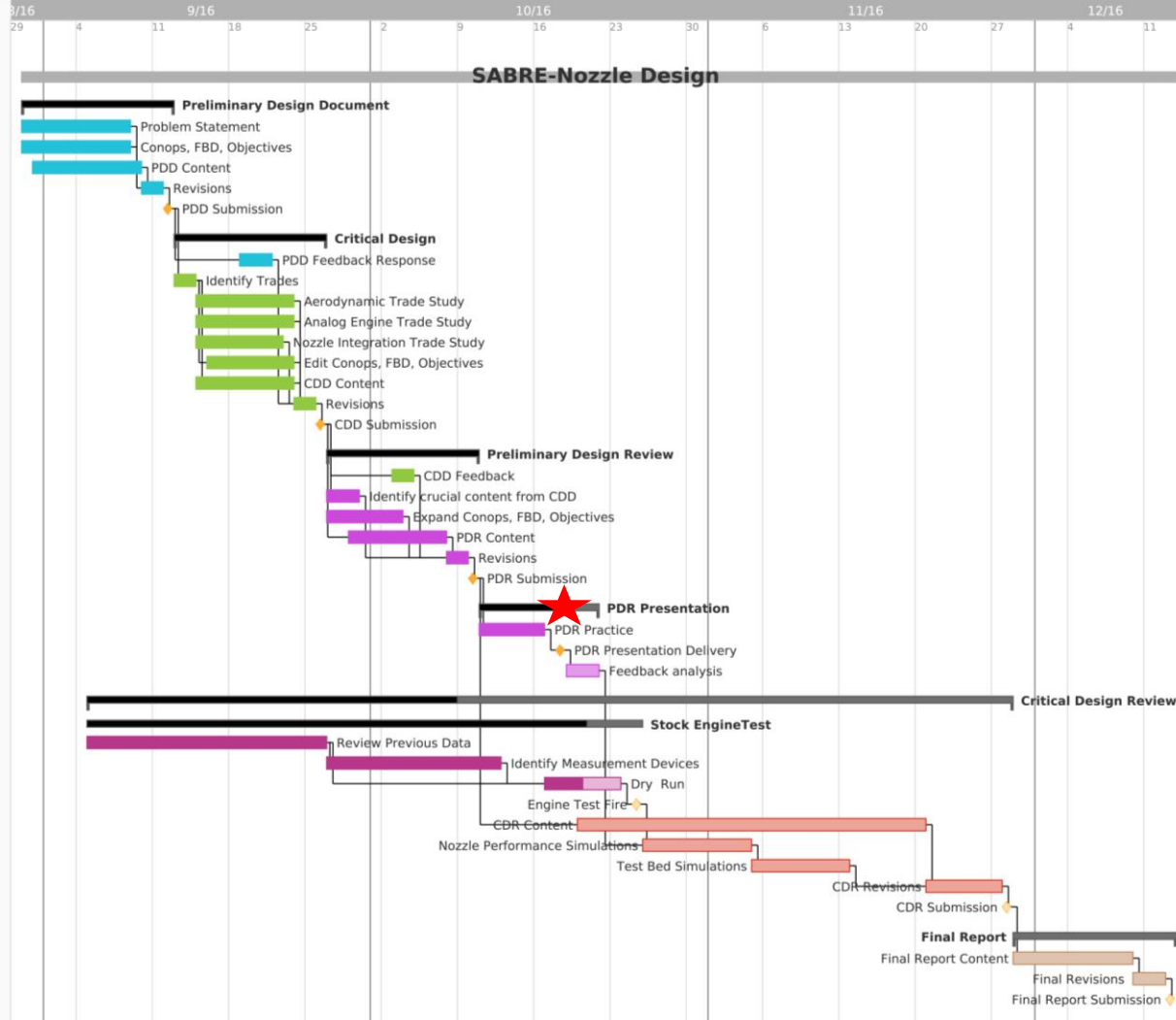
Baseline
Design

Nozzle
Aerodynamics

Nozzle Test
Bed

Nozzle
Integration

Project
Summary



SABRE-Nozzle Design

Preliminary Design Document

- Problem Statement
- Conops, FBD, Objectives
- PDD Content
- Revisions
- PDD Submission

Critical Design

- PDD Feedback Response
- Identify Trades
- Aerodynamic Trade Study
- Analog Engine Trade Study
- Nozzle Integration Trade Study
- Edit Conops, FBD, Objectives
- CDD Content
- Revisions
- CDD Submission

Preliminary Design Review

- CDD Feedback
- Identify crucial content from CDD
- Expand Conops, FBD, Objectives
- PDR Content
- Revisions
- PDR Submission

PDR Presentation

- PDR Practice
- PDR Presentation Delivery
- Feedback analysis

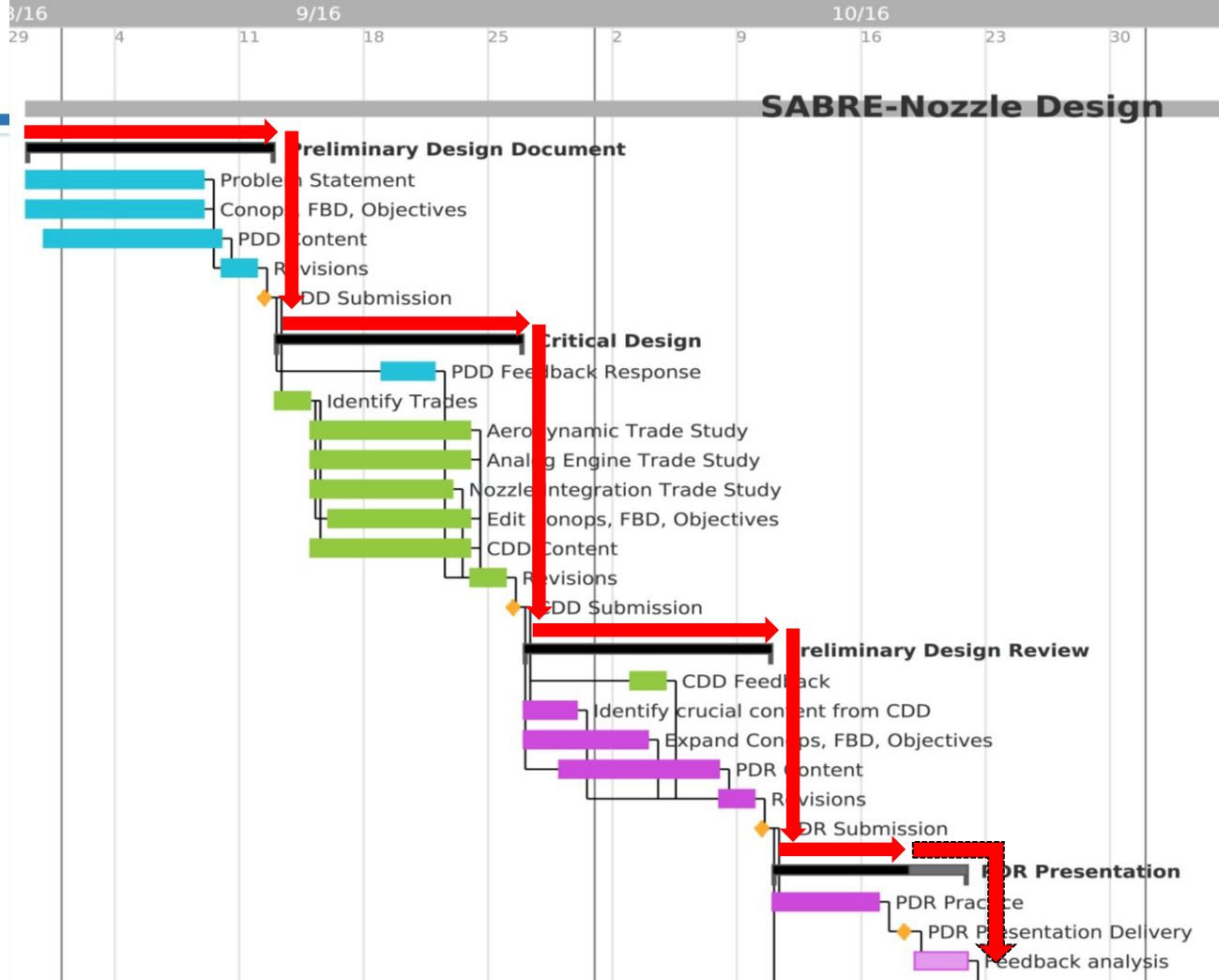
Critical Design Review

Stock EngineTest

- Review Previous Data
- Identify Measurement Devices
- Dry Run
- Engine Test Fire
- CDR Content
- Nozzle Performance Simulations
- Test Bed Simulations
- CDR Revisions
- CDR Submission

Final Report

- Final Report Content
- Final Revisions
- Final Report Submission



Nozzle Manufacturing

- Direct Metal Laser Sintering (DMLS)
 - Additive Manufacturing Technique
 - Laser binds layers of sinter powdered material together
 - Accuracy: 0.0005-0.001"
 - Manufactured: 20-30 μ m
 - Finished: down to 1 μ m
 - **Feasible** (Budget, Time, Accuracy)

| DMLS | Cobalt Chrome | Inconel 718 |
|-------------------------------|-----------------------|------------------------|
| Price | \$727 | \$662 |
| Temperature Rating | 2100 °F (1422 K) | 1200 °F (922 K) |
| Density | 8.5 g/cm ³ | 8.22 g/cm ³ |
| Mass of Current Nozzle Design | 137.2 g | 132.7 g |

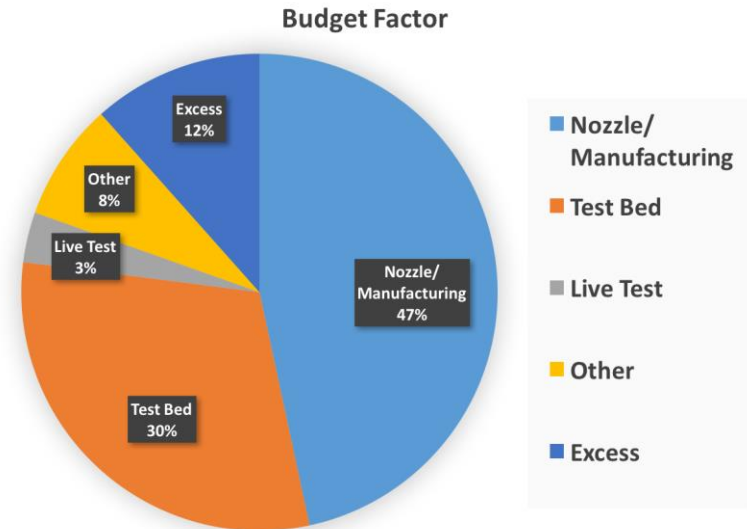


Status Report

Budget/Financial

- Includes cost of 3 nozzles and 2 testing nozzles

| Category | Cost |
|----------------------|---------------|
| Nozzle/Manufacturing | \$2331 |
| Test Bed | \$1520 |
| Live Testing | \$170 |
| Other/Miscellaneous | \$400 |
| Grand Total | \$4421 |



Project Description

Baseline Design

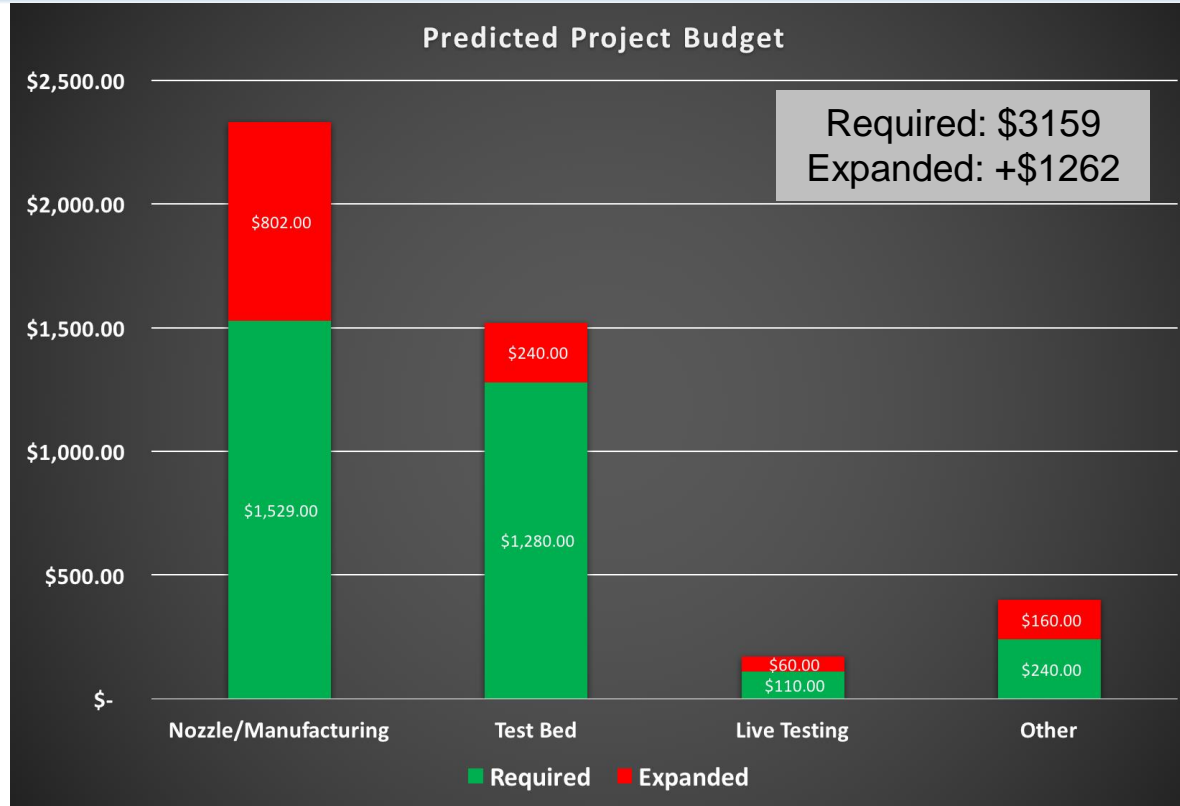
Nozzle Aerodynamics

Nozzle Test Bed

Nozzle Integration

Project Summary

Status Report cont.



Moving Forward

- Test current stock engine Nozzle
- Nozzle Simulations (CFD: Thermal and Aerodynamic)
- Test Bed functionality/Design
 - Conduct critical design experiments

Project
Description

Baseline
Design

Nozzle
Aerodynamics

Nozzle Test
Bed

Nozzle
Integration

Project
Summary

References

1. Cengel, Yunus A., and Robert H. Turner. *Fundamentals of Thermal-fluid Sciences*. 4th ed. Boston: McGraw-Hill, 2010. Print.
2. StratasysDirect. "Direct Metal Laser Sintering | Materials | Stratasys Direct Manufacturing." *Stratasys Direct Manufacturing*. N.p., n.d. Web. 10 Oct. 2016.
3. Mitchell, Michelle. "DMLS - Direct Metal Laser Sintering." *GPI Prototype & Manufacturing Services*. N.p., n.d. Web. 10 Oct. 2016.
4. "MP1 Cobalt Chrome," GPI Prototype & Manufacturing Services. MatWeb Database. Web. Accessed 1 Oct. 2016. <http://gpiprototype.com/images/PDF/EOS_CobaltChrome_MP1_en.pdf>.
5. "Inconel 718," Stratasys Direct Manufacturing. MatWeb Database. Web. Accessed 1 Oct. 2016. <https://www.stratasysdirect.com/wp-content/themes/stratasysdirect/files/material-datasheets/direct_metal_laser_sintering/DMLS_Inconel_718_Material_Specifications.pdf>.
6. Moran, M.J., Shapiro, H.N., Munson, B.R., DeWitt, D.P, *Introduction to Thermal Systems Engineering: Thermodynamics, Fluid Mechanics, and Heat Transfer*, 1st ed., Wiley, New York, 2003.
7. "Engine Data Sheet," JetCat USA, 14 Aug. 2015. Web. 10 Oct. 2016.
8. Patil, Shivanand. "Cold Flow Testing of Laboratory Scale Hybrid Rocket Engine Test Apparatus." Ryerson University. Toronto, Ontario, Canada. 2014. Web. 5 Oct 2016.
9. Anderson, John D. *Fundamentals of Aerodynamics*. 3rd Ed. Boston: McGraw-Hill, 2001. Print.
10. Reil, B. R., "About HybridBurners," *Hybridburners.com home page* Available: <http://hybridburners.com>.
11. Stratasys, "Objet30," *3D Printer* Available:<http://www.stratasys.com/3d-printers/design-series/objet30>.
12. "Fluid Statics, Dynamics, and Airspeed Indicators" Virginia Tech. Blacksburg, Virginia.
13. Sforza, P. M. *Theory of Aerospace Propulsion*. Waltham, MA: Butterworth-Heinemann, 2012. Print.

Back-Up Slides

Appendix: Data Sheet

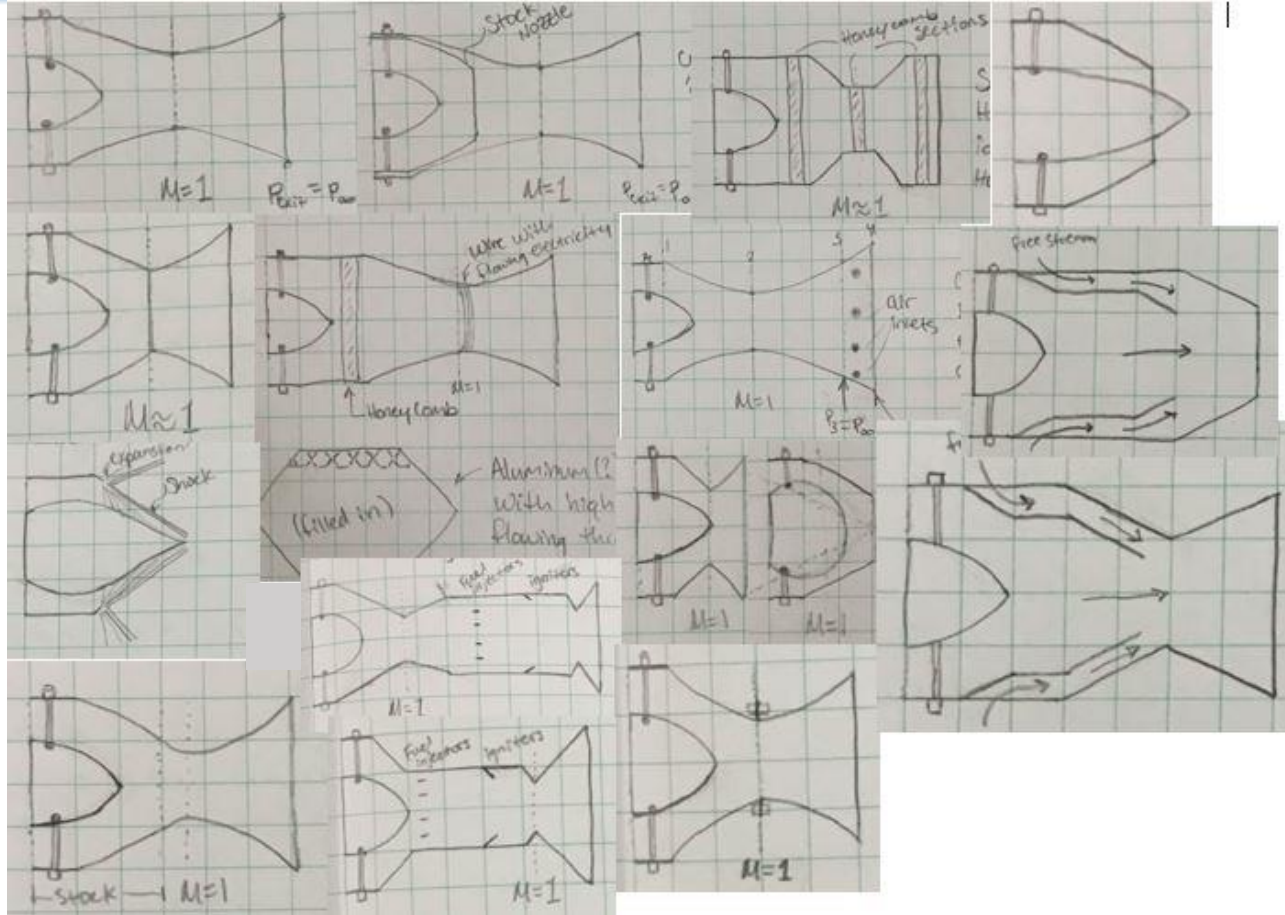


Data converted to ISA: 15°C and 101,325 Pa

Turbine Data Sheet

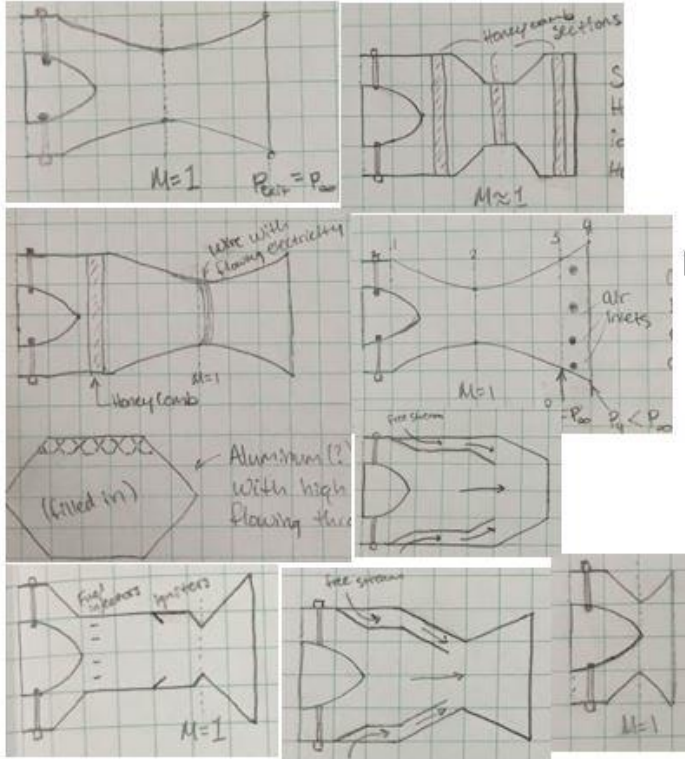
| | P20-SX | P60-SE | P90-RXi | P100-RX | P140-RXi | P180-Rxi | P200 RX | P300-RX | P300-RXG | P400-RX | P400-RXG |
|-------------------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----------|----------|
| Idle RPM (1/min) | 85000.00 | 50000.00 | 35000.00 | 40000.00 | 32000.00 | 32000.00 | 33000.00 | 35000.00 | 35000.00 | 30000.00 | 30000.00 |
| Max ROM (1/min) | 245000.00 | 165000.00 | 130000.00 | 154000.00 | 125000.00 | 128000.00 | 112000.00 | 108000.00 | 108000.00 | 98000.00 | 98000.00 |
| Idle thrust (N) | 0.30 | 1.00 | 3.00 | 2.00 | 6.00 | 6.00 | 9.00 | 14.00 | 14.00 | 13.00 | 13.00 |
| Idle thrust (lbs) | 0.07 | 0.22 | 0.67 | 0.45 | 1.35 | 1.35 | 2.02 | 3.15 | 3.15 | 2.92 | 2.92 |
| Max thrust (N) | 24.00 | 63.00 | 105.00 | 100.00 | 142.00 | 175.00 | 230.00 | 300.00 | 300.00 | 395.00 | 395.00 |
| Max thrust (lbs) | 5.40 | 14.16 | 23.81 | 22.48 | 31.92 | 39.34 | 51.71 | 67.44 | 67.44 | 88.80 | 88.80 |
| EGT (°C) | 690.00 | 730.00 | 690.00 | 720.00 | 720.00 | 750.00 | 750.00 | 750.00 | 750.00 | 750.00 | 750.00 |
| EGT (°F) [calc] | 1274.00 | 1348.00 | 1274.00 | 1292.00 | 1292.00 | 1382.00 | 1382.00 | 1382.00 | 1382.00 | 1382.00 | 1382.00 |
| Pressure ratio | 1.50 | 2.00 | 2.80 | 2.90 | 3.40 | 3.50 | 4.00 | 3.55 | 3.55 | 3.80 | 3.80 |
| Mass-flow (kg/s) | 0.05 | 0.16 | 0.26 | 0.29 | 0.34 | 0.36 | 0.45 | 0.50 | 0.50 | 0.67 | 0.67 |
| Exhaust gas velocity (km/h) | 1674.00 | 1417.50 | 1454.00 | 1565.00 | 1504.00 | 1658.00 | 1840.00 | 2160.00 | 2160.00 | 2122.00 | 2122.00 |
| Exhaust gas velocity (mph) | 1039.55 | 880.27 | 902.93 | 971.87 | 933.98 | 1029.62 | 1142.64 | 1341.36 | 1341.36 | 1317.78 | 1317.78 |
| Power output (thrust) (kW) | 5.60 | 12.40 | 21.20 | 20.80 | 29.70 | 40.30 | 58.78 | 90.00 | 90.00 | 116.40 | 116.40 |
| Fuel consumption @maxRpm (ml/min) | 90.00 | 240.00 | 370.00 | 390.00 | 510.00 | 585.00 | 730.00 | 980.00 | 980.00 | 1300.00 | 1300.00 |
| Fuel consumption @maxRpm (oz/min) | 3.04 | 8.12 | 12.51 | 13.18 | 17.25 | 19.78 | 24.68 | 33.14 | 33.14 | 43.98 | 43.98 |
| Fuel consumption @idle (ml/min) | 12.00 | 70.00 | 95.00 | 80.00 | 115.00 | 120.00 | 129.00 | 179.00 | 179.00 | 200.00 | 200.00 |
| Fuel consumption @idle (fl oz/min) | 0.41 | 2.37 | 3.21 | 2.71 | 3.89 | 4.08 | 4.36 | 6.05 | 6.05 | 6.78 | 6.78 |
| Specific fuel consumption @ maxRpm (kg/N) | 0.19 | 0.18 | 0.17 | 0.19 | 0.17 | 0.16 | 0.15 | 0.16 | 0.16 | 0.16 | 0.16 |
| Weight (g) | 350.00 | 845.00 | 1435.00 | 1090.00 | 1590.00 | 1530.00 | 2370.00 | 2630.00 | 2630.00 | 3550.00 | 3550.00 |
| Weight (oz) | 12.35 | 29.81 | 50.62 | 38.10 | 56.09 | 53.97 | 83.80 | 92.77 | 92.77 | 125.22 | 125.22 |
| Diameter (mm) | 60.00 | 83.00 | 112.00 | 97.00 | 112.00 | 112.00 | 132.00 | 132.00 | 132.00 | 147.00 | 147.00 |
| Diameter (in) | 2.36 | 3.27 | 4.41 | 3.82 | 4.41 | 4.41 | 5.20 | 5.20 | 5.20 | 5.79 | 5.79 |
| Length (incl. Starter) (mm) | 180.00 | 245.00 | 300.00 | 235.00 | 285.00 | 285.00 | 350.00 | 365.00 | 365.00 | 350.00 | 350.00 |
| Length (incl. Starter) (in) | 7.09 | 9.65 | 11.81 | 9.25 | 11.22 | 11.22 | 13.78 | 14.37 | 14.37 | 13.78 | 13.78 |

Appendix: Nozzle Designs

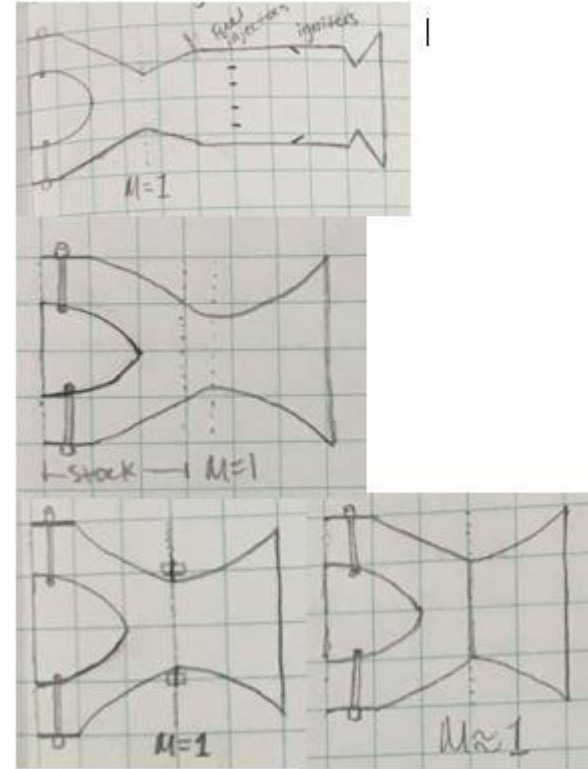


Appendix: Nozzle Integration

Replacement

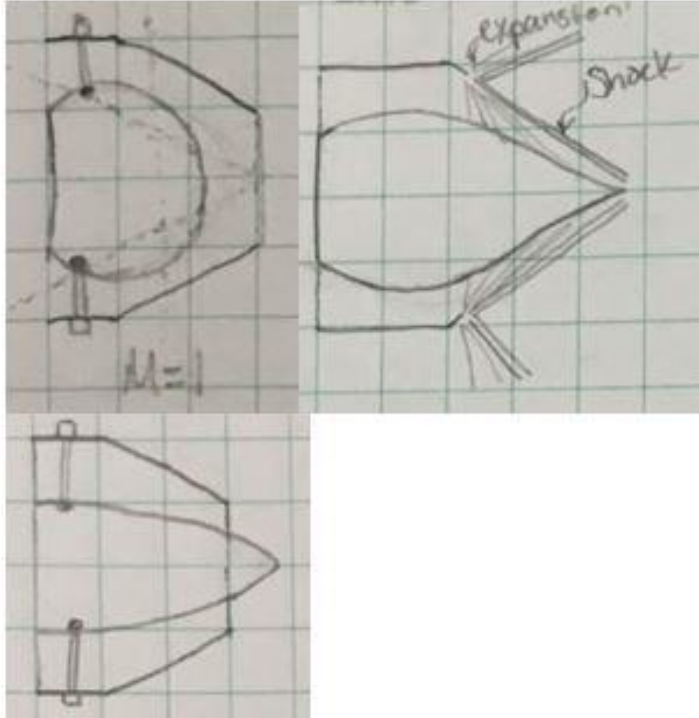


Attachment

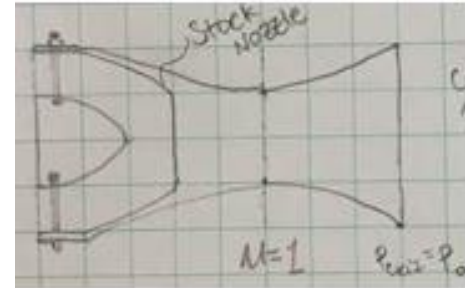


Appendix: Nozzle Integration

Dome Replacement



Sock

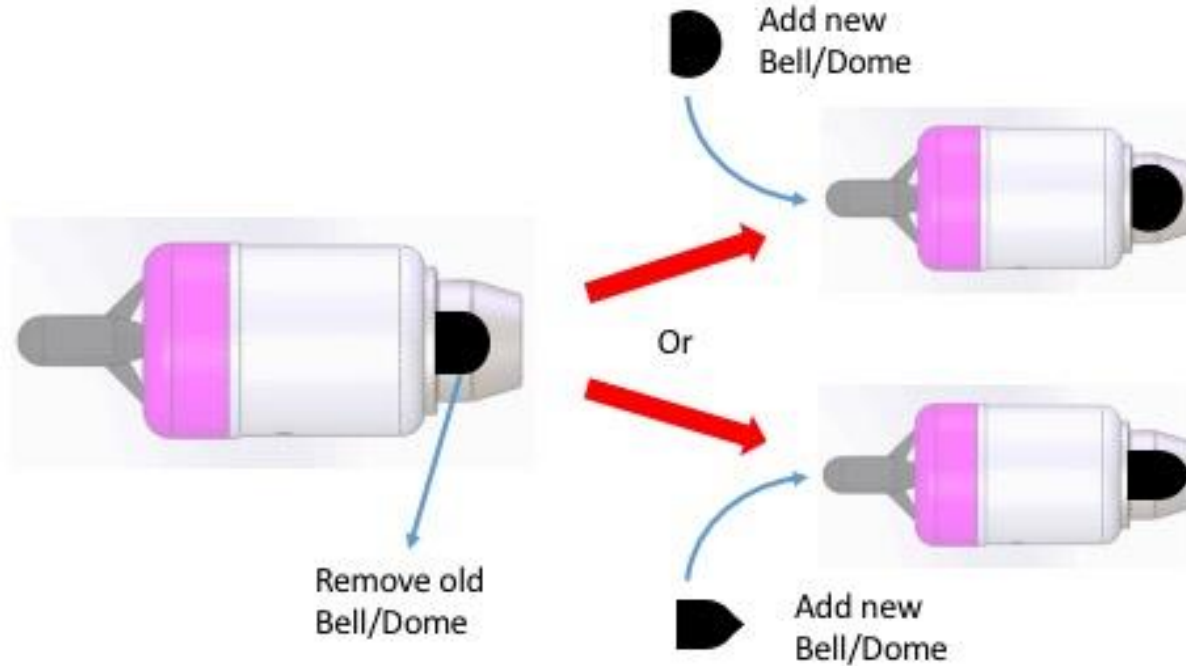


Appendix: Nozzle Integration Feasibility

Design consideration: Dome Replacement

- Stock nozzle maintained
- Small Size
 - Low mass cost; additional 9 g anticipated
 - Cost Savings in manufacturing
 - Easy to implement in large scale product change
- Aerodynamic Complexity
 - Small length to work with (stock nozzle)

Appendix: Nozzle Integration Feasibility



Appendix: Nozzle Integration Feasibility

Design Consideration: Nozzle 'Sock'

- Stock nozzle maintained
- Manufacturing complexity
- Tolerances of integration design
- Additional 130 g anticipated

Design Consideration: Nozzle Extension

- Stock engine not maintained
- Connection Vulnerability
- Less manufacturing material required
- Additional 44 g anticipated for nozzle, additional supports required

Appendix: Pressure Regulator

- Pressure output of Nitrogen tank: 15.168 MPa (2200 psi)
- Ideal pressure output of regulator: 317 kPa (46 psi)



Praxair 4095 Pressure Regulator
Inlet Range: up to 27.58 MPa (4000 psi)
Output Range: up to 2.068 MPa (300 psi)

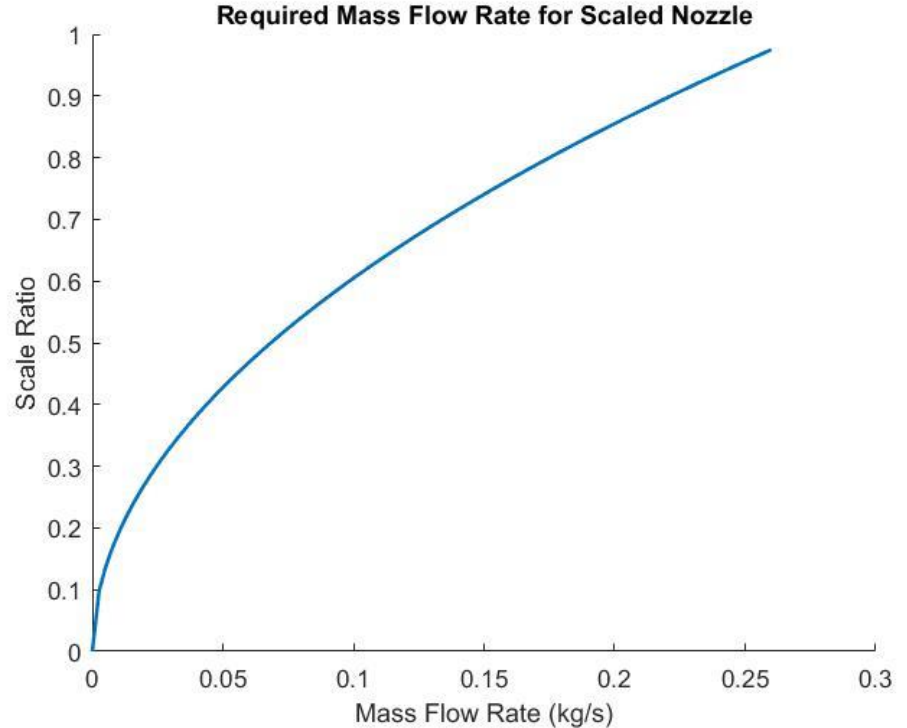
FEASIBLE

Appendix: Nozzle Scalability

$$\left(\frac{A_e}{A^*}\right)^2 = \frac{1}{M_e^2} \left(\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M_e^2 \right) \right)^{\frac{\gamma+1}{\gamma-1}}$$

$$\dot{m}^* = \frac{p_0 A^*}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}}$$

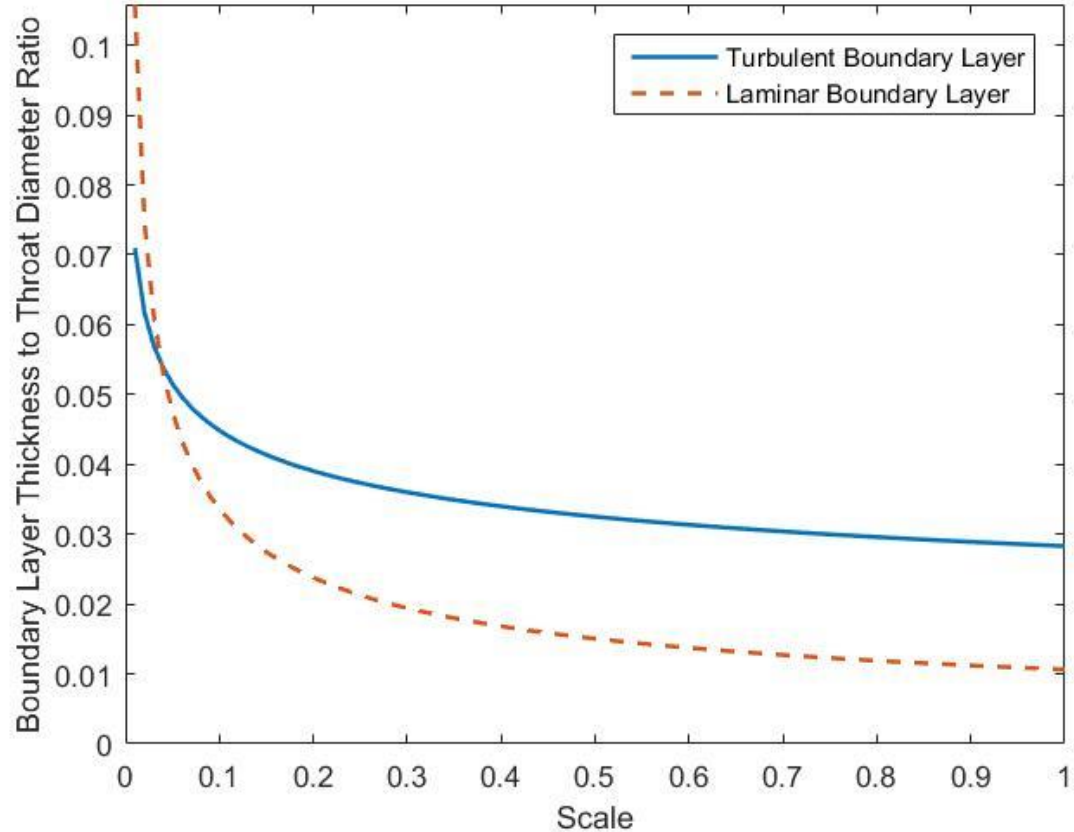
$$A^* = \frac{\dot{m} \sqrt{T_0}}{p_0 \sqrt{\frac{\gamma}{R} \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}}}$$



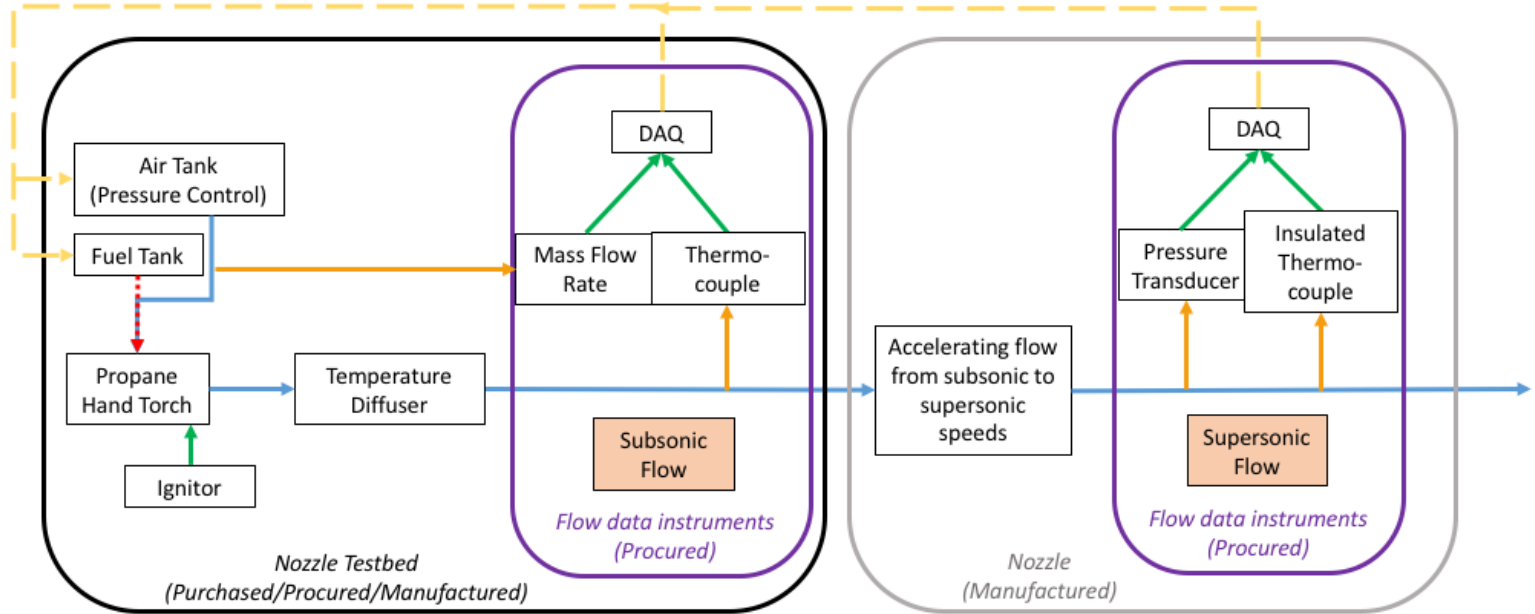
Appendix: Nozzle Scalability

$$\delta \approx 5.0x / \sqrt{\text{Re}_x}$$

$$\delta \approx 0.37x / \text{Re}_x^{1/5}$$



Appendix: FBD Hot Flow (Level 2 Success)



Appendix: Hot Flow Testing

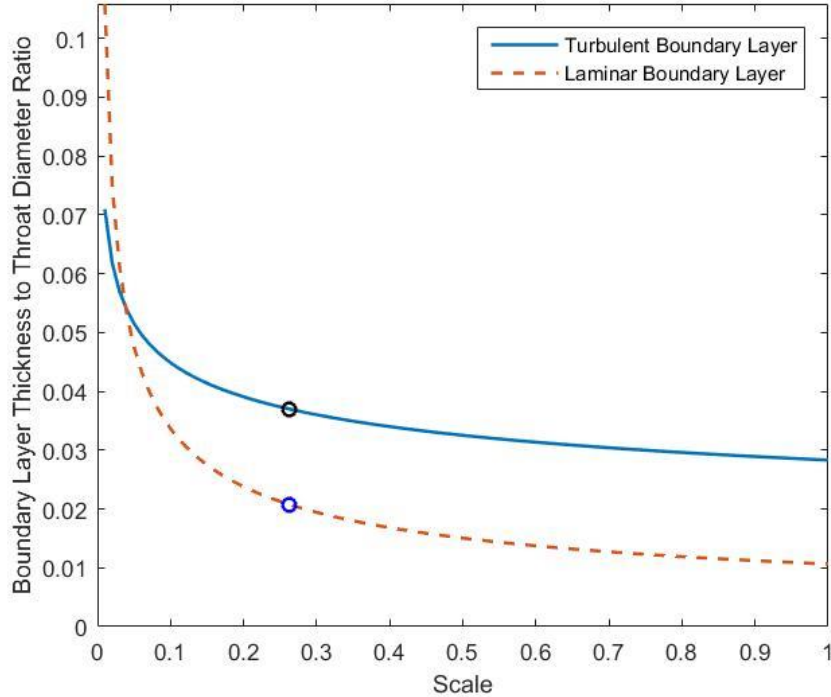
| | Burner | JetCat P-90 xi |
|----------------------|--------|----------------|
| Pressure (atm) | 1.42 | 1.6248 |
| Temperature (K) | 1258.3 | 963.15 |
| Mach | 0.95 | 0.66 |
| Mass flow rate(kg/s) | 0.0183 | 0.26 |



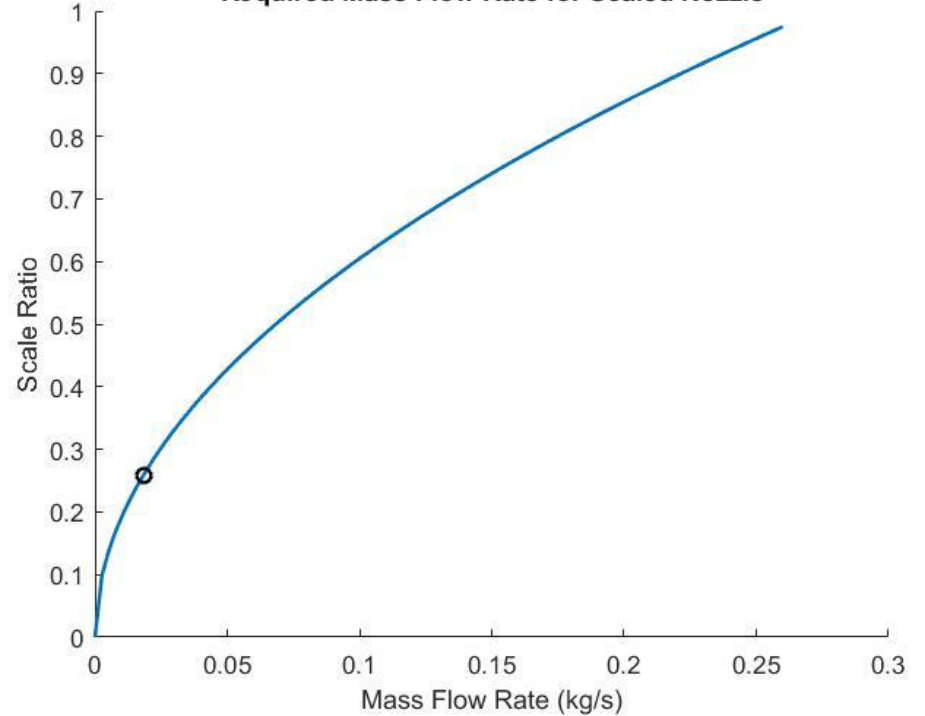
•Hybrid burners' 1.25" foundry and kiln burner

Appendix: Hot Flow Testing

Nozzle Scaling Boundary Layer



Required Mass Flow Rate for Scaled Nozzle



Appendix: Thermal Expansion

Assuming: Isentropic Flow Relations to determine static flow temperature, steady state (rate of heat transfer is constant), forced turbulent dry air convection within nozzle, natural convection outside of nozzle, nominal CoCr thermal properties

$$h_1 = 13.95 \frac{W}{m^2K} \quad h_2 = 11.88 \frac{W}{m^2K}$$

$$k = 30.8 \frac{W}{m^{\circ}C} \quad \alpha = 15.1 \frac{\mu m}{m^{\circ}C}$$

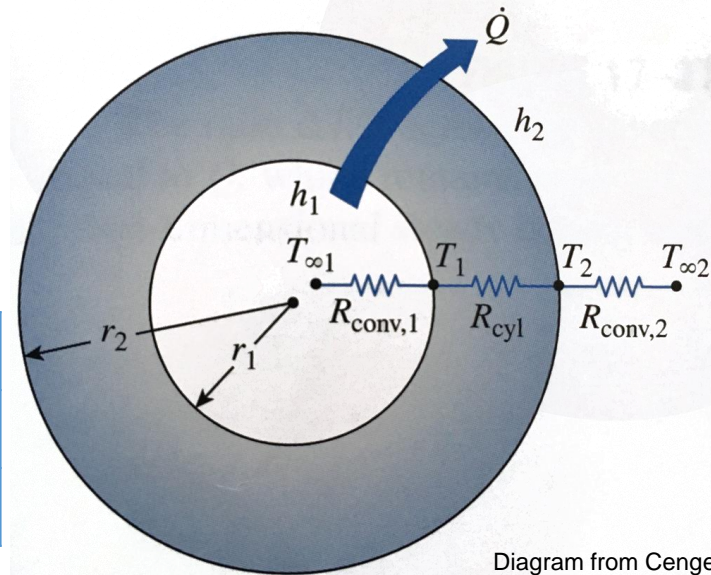
$$\Delta d = d_o \Delta T \alpha$$

| Station | Inlet | Throat | Exit |
|------------------------|-------------|-------------|-------------|
| Temperature(K) | 871 | 746 | 734 |
| Δd (μm) | 441 (0.86%) | 278 (0.68%) | 271 (0.66%) |

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$R_{total} = R_{conv,1} + R_{cond} + R_{conv,2}$$

$$= \frac{1}{2\pi r_1 L h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{2\pi r_2 L h_2}$$



Appendix: Ideal Turbo-jet Analysis

- **Station 1-2: Inlet**

Total pressure and total temperature are **conserved**

$$P_{t,2} = P_{t,1}$$

$$T_{t,2} = T_{t,1}$$

- **Station 2-3: Compressor**

Isentropic Compression-
Compressor Pressure Ratio - $\pi_c = \frac{P_{t,3}}{P_{t,2}}$

$$P_{t,3} = \pi_c * P_{t,2}$$

$$T_{t,3} = T_{t,2} (\pi_c)^{\frac{\gamma-1}{\gamma}}$$

- **Station 3-4: Combustor**

Constant Pressure Combustion:

$$P_{t,4} = P_{t,3}$$

$$\frac{f}{a} = \frac{\dot{m}_{fuel}}{\dot{m}_0 - \dot{m}_{fuel}}$$

$$T_{t,4} = \frac{\left(\frac{f}{a} * HV + c_p * T_{t,3}\right)}{c_p + c_p * \frac{f}{a}}$$

- **Station 4-5: Turbine**

$$\frac{P_{t,5}}{P_{t,4}} = \left[1 - \frac{T_{t,2}}{T_{t,4}} * \frac{1}{\left(1 + \frac{f}{a}\right)} * \left(\pi_c^{\frac{\gamma-1}{\gamma}} - 1\right)\right]^{\frac{\gamma}{\gamma-1}}$$

$$P_{t,5} = \frac{P_{t,5}}{P_{t,4}} * P_{t,4}$$

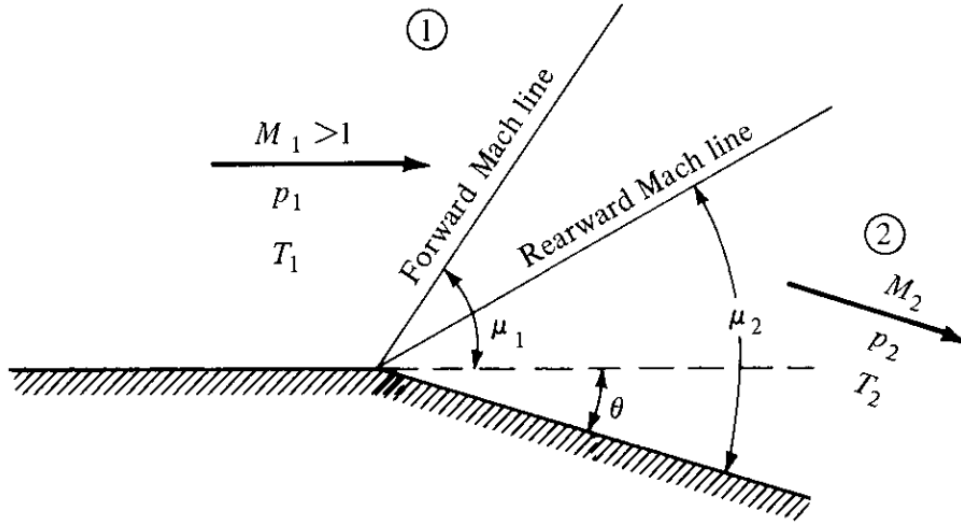
$$T_{t,5} = T_{t,4} * \left(\frac{P_{t,5}}{P_{t,4}}\right)^{\frac{\gamma-1}{\gamma}}$$

Appendix: Mass-Flow through Nozzle

$$\dot{m} = \frac{p_t A}{\sqrt{T_t}} \sqrt{\frac{\gamma}{R}} M \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{-\frac{\gamma + 1}{2(\gamma - 1)}}$$

- Compression Ratio influences Stagnation Pressure and Temperature, which influence maximum Mach number.
- Critical throat area determined with a Mach number of 1, ideal isentropic Pressure and Temperature conditions, and a fixed maximum mass flow rate.

Appendix: Prandtl-Meyer Expansion



$$1: \theta = v(1.06) - v(1)$$

$$2: \theta = 0.011 - 0 \text{ (rad)}$$

$$3: \theta = 0.654 \text{ (deg)}$$

$$v(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \sqrt{\frac{\gamma - 1}{\gamma + 1} (M^2 - 1)} - \tan^{-1} \sqrt{M^2 - 1}$$

$$\theta = v(M_2) - v(M_1)$$

Appendix: Mach # from Pressure

Supersonic Flow

- Rayleigh Pitot Tube Formula:
 1. Holds for supersonic flow, $M > 1$
 2. Accounts for normal shock formed in front of the pitot tube

$$\frac{P_{02}}{P} = \left[\frac{(\gamma + 1)^2 M^2}{4\gamma M^2 - 2(\gamma - 1)} \right]^{\frac{\gamma}{\gamma - 1}} \left(\frac{1 - \gamma + 2\gamma M^2}{\gamma + 1} \right)$$

Where:

- P_{02} = stagnation pressure after the shock wave
 P = static pressure (same before or after shock wave)
 M = Mach number before the shock wave

A pitot tube measures both stagnation pressure and static pressure behind the shock. Therefore, the equation above can be solved for M (the desired value to verify our designed nozzle can achieve supersonic flow).

Appendix: Mach # from Pressure

- Measuring compressible flow (still subsonic)

Compressible Subsonic Flow

$$M^2 = \frac{2}{\gamma - 1} \left[\left(\frac{P_0}{P} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]$$

Where:

P_0 - total pressure

P - static pressure

With the pitot tube measuring the total and static pressure, M can be solved for in the above equation.



Appendix: Expanded Budget Chart

| | | | |
|----------------------|-----------------------|-----------|--------|
| Nozzle/Manufacturing | Nozzle | 3 x \$727 | \$2181 |
| | Test Nozzle (Plastic) | 2 x \$75 | \$150 |
| Test Bed | Regulator | 4 x \$20 | \$80 |
| | Oriface | 4 x \$250 | \$1000 |
| | Pitot Probe | 2 x \$20 | \$40 |
| | Hose | 1 x \$400 | \$400 |
| Live Testing | Plexiglass | 1 x \$50 | \$50 |
| | Nitrogen Tanks | 8 x \$15 | \$120 |
| Other | Project Management | - | \$360 |
| | Extra Piping/Tubes | - | \$40 |
| Total | | | \$4421 |