1. Find the slant asymptote the following rational function: (5 pts)

$$r(x) = \frac{4x^3 + 16x^2 + 12x}{2x^2 - 6x}$$

Solution:
First we notice that

$$r(x) = \frac{4x^3 + 16x^2 + 12x}{2x^2 - 6x} = \frac{2x^3 + 8x^2 + 6x}{x^2 - 3x}$$

Then, we use long division to find the quotient and the remainder

$$\begin{array}{r|l} & 2x + 14 \\ \hline x^2 - 3x & 2x^3 + 8x^2 + 6x \\ & -(2x^3 - 6x^2) \\ \hline & 14x^2 + 6x \\ & -(14x^2 - 42x) \\ \hline & 48x \end{array}$$

Using this we can write

$$2x^3 + 8x^2 + 6x = (x^2 - 3x)(2x + 14) + 48x \tag{1}$$

$$\therefore \frac{2x^3 + 8x^2 + 6x}{x^2 - 3x} = (2x + 14) + \frac{48x}{x^2 - 3x} \tag{2}$$

Hence the end behavior of the rational function can be written as:
$r(x) \to 2x + 14$ as $x \to \pm\infty$
Thus the slant asymptote is given by

$$\boxed{y = 2x + 14}$$

2. For $R(x) = \dfrac{3x^2 - 3x - 18}{x^2 - 2x - 3}$ (8 pts)

   (a) Find the (x-coordinate(s)) of any hole(s). If there are none state NONE.
   Solution:
   First, we factor the numerator and the denominator of the rational function:

$$R(x) = \frac{3x^2 - 3x - 18}{x^2 - 2x - 3} \tag{3}$$

$$= \frac{3\left(x^2 - x - 6\right)}{x^2 - 2x - 3} \tag{4}$$

$$= \frac{3(x - 3)(x + 2)}{(x - 3)(x + 1)} \tag{5}$$

   We notice that this rational function simplifies to $y = \dfrac{3(x + 2)}{(x + 1)}$ when $x \neq 3$

   Hence a hole is located at $\boxed{x = 3}$

(b) Find the (y-coordinate(s)) of any hole(s). If there are none state NONE.
Solution:
We plug in $x = 3$ into this simplified function to find the y-coordinate of the hole

$$y = \frac{3(3+2)}{(3+1)} = \boxed{\frac{15}{4}}$$

(c) Determine the end behavior of $R(x)$.
Solution:
We notice that the Numerator and the Denominator of $R(x)$ both have degree 2. Hence as $x \to \pm\infty$ the function approaches the ratio of the leading coefficients 3 and thus has a Horizontal Asymptote given by

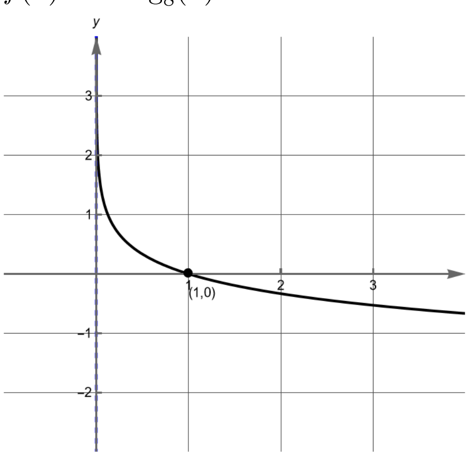
$$\boxed{y = 3}$$

(d) Find any vertical asymptote(s). If there are none state NONE.
Solution:
Looking at the reduced function $y = \dfrac{3(x+2)}{(x+1)}$, there is Vertical Asymptote when the denominator is zero. It is given by the vertical line

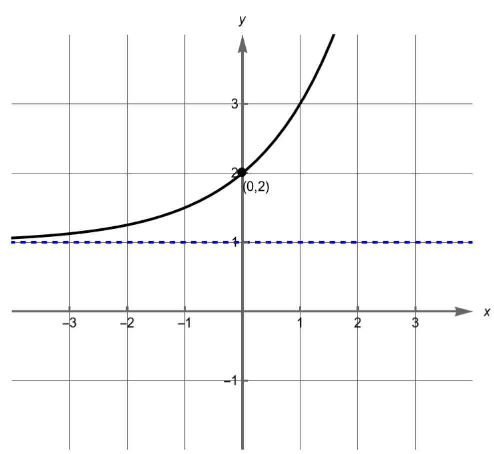
$$\boxed{x = -1}$$

3. Sketch the following graphs: Be sure to label any asymptotes and intercepts for each graph. (10 pts)

(a) $f(x) = -\log_8(x)$



(b) $g(x) = 2^x + 1$



(c) For $f(x)$ given in part (a) find $f\left(8^{2x}\right)$.
Solution:

$$f\left(8^{2x}\right) = -\log_8\left(8^{2x}\right) \quad (6)$$
$$= \boxed{-2x} \quad (7)$$

4. The following are unrelated.

(a) Simplify (rewrite without logs): $5\log(1) - e^{3\ln(t)} + \log_4(64) + \log_3(30) - \log_3(10)$ (4 pts)
Solution:

$$5\log(1) - e^{3\ln(t)} + \log_4(64) + \log_3(30) - \log_3(10) = 0 - e^{\ln(t^3)} + \log_4(4^3) + \log_3\left(\frac{30}{10}\right) \quad (8)$$
$$= -t^3 + 3 + \log_3(3) \quad (9)$$
$$= -t^3 + 3 + 1 \quad (10)$$
$$= \boxed{-t^3 + 4} \quad (11)$$

(b) Rewrite as a single logarithm without negative exponents (as usual, simplify your final answer): $-4\log_3(x) + \log_3(y) + 3\log_3(\sqrt{x})$ (5 pts)

Solution:

$$\begin{aligned}
-4\log_3(x) + \log_3(y) + 3\log_3(\sqrt{x}) &= \log_3(x^{-4}) + \log_3(y) + 3\log_3(x^{\frac{1}{2}}) && (12)\\
&= \log_3(x^{-4}) + \log_3(y) + \log_3(x^{\frac{3}{2}}) && (13)\\
&= \log_3(x^{-4}y) + \log_3(x^{\frac{3}{2}}) && (14)\\
&= \log_3\left(x^{-4}yx^{\frac{3}{2}}\right) && (15)\\
&= \log_3\left(yx^{-\frac{5}{2}}\right) && (16)\\
&= \boxed{\log_3\left(\frac{y}{x^{\frac{5}{2}}}\right)} && (17)
\end{aligned}$$

5. Solve the following equations for x. If there are no solutions write "no solutions" (as usual, be sure to justify answer for full credit). (16 pts)

(a) $\log_x(27) = 3$

Solution:

Converting to exponential form, we obtain

$$\begin{aligned}
x^3 &= 27 && (18)\\
x^3 &= 3^3 && (19)\\
x &= \boxed{3} && (20)
\end{aligned}$$

(b) $\ln(4) - \ln(x+1) = \ln(3)$

Solution:

$$\begin{aligned}
\ln(4) - \ln(x+1) &= \ln(3) && (21)\\
\ln\left(\frac{4}{x+1}\right) &= \ln(3) && (22)\\
\frac{4}{x+1} &= 3 && (23)\\
x + 1 &= \frac{4}{3} && (24)\\
x &= \boxed{\frac{1}{3}} && (25)
\end{aligned}$$

(c) $3^{x+1} = 9^{x-1}$

Solution:

$$\begin{aligned}
3^{x+1} &= 9^{x-1} && (26)\\
3^{x+1} &= (3^2)^{x-1} && (27)\\
3^{x+1} &= 3^{2(x-1)} && (28)\\
x + 1 &= 2x - 2 && (29)\\
x &= \boxed{3} && (30)
\end{aligned}$$

(d) $7 + 4x = xe^2 - 3$

Solution:

$$\begin{aligned}
7 + 4x &= xe^2 - 3 && (31)\\
xe^2 - 4x &= 10 && (32)\\
x(e^2 - 4) &= 10 && (33)\\
x &= \boxed{\frac{10}{e^2 - 4}} && (34)
\end{aligned}$$

6. The velocity of a sky diver t seconds after jumping is modeled by $v(t) = 50\left(1 - e^{-0.2t}\right)$.

   (a) After how many seconds is the velocity 5 ft/s? (Give your answer as an exact value, do not attempt to approximate). (4 pts)
   Solution:

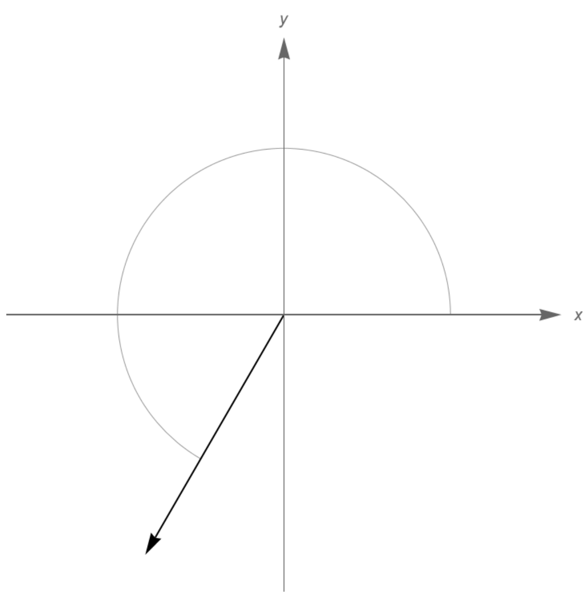
$$\begin{aligned} 5 &= 50\left(1 - e^{-0.2t}\right) && (35) \\ \frac{5}{50} &= \left(1 - e^{-0.2t}\right) && (36) \\ e^{-0.2t} &= 1 - \frac{1}{10} && (37) \\ e^{-0.2t} &= 0.9 && (38) \\ -0.2t &= \ln(0.9) && (39) \\ t &= \boxed{-\frac{\ln(0.9)}{0.2}} && (40) \end{aligned}$$

   (b) After a very long time, the velocity reaches an approximately constant value, known as the terminal velocity. What is the terminal velocity of the sky diver ? (3 pts)
   Solution:
   We know that $e^{-0.2t} \to 0$ as $t \to \infty$. Hence the terminal velocity is given by

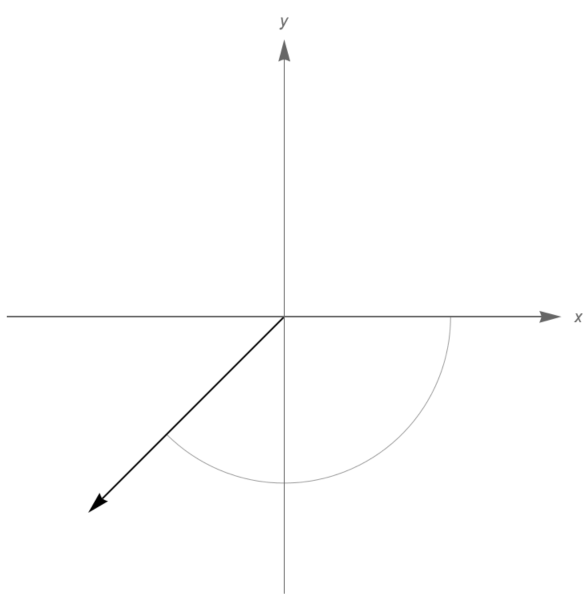
$$\begin{aligned} v(t \to \infty) &= 50(1 - 0) && (41) \\ &= \boxed{50 \text{ ft/s}} && (42) \end{aligned}$$

7. Sketch each angle in standard position on the unit circle.

   (a) $\frac{4\pi}{3}$ (2 pts)



   (b) $-\frac{3\pi}{4}$ (2 pts)



8. The point $(-3, -2)$ is on the terminal side of an angle, θ, in standard position. Determine the exact values of the following.

   (a) $\sin\theta$ (3 pts)
   Solution:
   $r = \sqrt{(-2)^2 + (-3)^2} = \sqrt{13}$ so $\sin\theta = \frac{y}{r} = \boxed{-\frac{2}{\sqrt{13}}}$

(b) $\csc\theta$ (2 pts)
Solution:
$\csc\theta = \frac{1}{\sin\theta} = \boxed{-\frac{\sqrt{13}}{2}}$

(c) $\tan\theta$ (2 pts)
Solution:
$\tan\theta = \frac{y}{x} = \boxed{\frac{2}{3}}$

9. My friend and I are watching a football game at my place, and ordering pizza. I want to order a 10 inch diameter circular pizza, but my friend thinks ordering two 7 inch diameter circular pizzas will give us more food (in terms of surface area of pizza). Is my friend correct? Please show work to get points for this question, just a "yes" or a "no" won't suffice. (4 pts)

Solution:
We know that the area inside a circle of radius r is given by πr^2.
Hence the area of 10 inch diameter circular pizza is

$$A_1 = \pi\left(\frac{10}{2}\right)^2 \tag{43}$$
$$= \pi\frac{10^2}{4} \tag{44}$$
$$= \frac{100\pi}{4} \tag{45}$$

The combined area of two 7 inch diameter circular pizzas is

$$A_2 = 2\times\pi\left(\frac{7}{2}\right)^2 \tag{46}$$
$$= 2\times\pi\frac{7^2}{4} \tag{47}$$
$$= \frac{98\pi}{4} \tag{48}$$

So the combined area of two 7 inch diameter circular pizzas is less than the area of 10 inch diameter circular pizza. Hence my friend is $\boxed{\text{NOT CORRECT}}$

10. Simplify the following:

(a) $\cos(30°) + 2\sin^2(30°)$ (4 pts)
Solution:

$$\cos(30°) + 2\sin^2(30°) = \frac{\sqrt{3}}{2} + 2\left(\frac{1}{2}\right)^2 \tag{49}$$
$$= \frac{\sqrt{3}}{2} + \frac{1}{2} \tag{50}$$
$$= \boxed{\frac{\sqrt{3}+1}{2}} \tag{51}$$

(b) For a particular angle θ in standard position suppose we know $\tan\theta < 0$ and $\cos\theta > 0$. What quadrant is θ in? You do not need to justify your answer. (3 pts)
Solution:
$\tan\theta < 0$ in quadrants II and IV. $\cos\theta > 0$ in quadrants I and IV. So $\boxed{\theta \text{ is in quadrant IV.}}$

11. Find the following. If a value does not exist write DNE. (18 pts)

(a) $\sin\left(\frac{2\pi}{3}\right)$

Solution:

$\boxed{\frac{\sqrt{3}}{2}}$

(b) $\cos\left(\frac{\pi}{2}\right)$

Solution:

$\boxed{0}$

(c) $\cot(0^\circ)$

Solution:

$\boxed{\text{DNE}}$

(d) $\csc\left(\frac{7\pi}{6}\right)$

Solution:

$\boxed{-2}$

(e) $\tan(-45^\circ)$

Solution:

$\boxed{-1}$

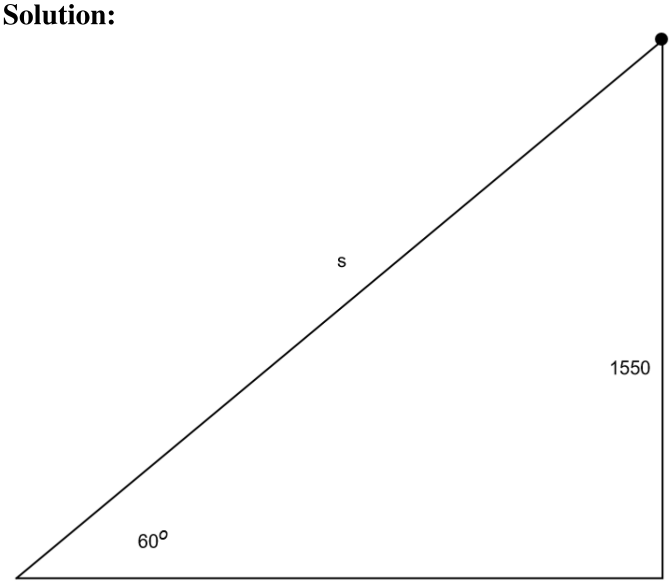
(f) $\sec\left(\frac{\pi}{6}\right)$

Solution:

$\boxed{\frac{2}{\sqrt{3}}}$

12. Last night, I was standing near the base of Flagstaff mountain in a flat field. The peak of the mountain is 1550 ft above the field. I took out my laser pointer, and pointed it at the top of the mountain. If the angle of elevation of the laser beam is 60°, how far does the laser beam travel to reach the top of the mountain? (5 pts)

Solution:



From the right triangle above, we see that

$$\sin(60^o) = \frac{1550}{s} \tag{52}$$

$$\frac{\sqrt{3}}{2} = \frac{1550}{s} \tag{53}$$

$$s = \frac{(2)1550}{\sqrt{3}} \tag{54}$$

$$s = \frac{3100}{\sqrt{3}} \tag{55}$$

Hence the distance traveled by the beam is given by $\boxed{\frac{3100}{\sqrt{3}} \text{ ft}}$