APPM 2360 - Introduction to Differential Equations with Linear Algebra

Course Objectives:

This course provides an introduction to ordinary differential equations and linear algebra. The main objectives are to:

- Understand qualitative and quantitative methods of solving differential equations.
- Understand linear algebra and utilize to solve differential equations.
- Improve problem solving and critical thinking skills.

Textbook: *Differential Equations and Linear Algebra*, 2^{nd} Edition by Farlow, Hall, McDill and West. You will also need an access code for WebAssign's online homework system. The access code can also be purchased separately.

Schedule and Topics Covered

Day	Section	Topics
1	1.2	Introduction/ODE Solutions/Direction Fields
2	1.2/1.3	ODE Solutions/Direction Fields cont'd; Separation of Variables
3	1.3/1.4	Separation of Variables cont'd; Numerical Methods
4	1.4/1.5	Numerical Methods cont'd/Picard's Theorem
5	2.1	Linear ODE Solutions
6	2.2	Solving First Order Linear ODE
7	2.3	Growth and Decay
8	2.4	Mixing and Cooling
9	2.5	Nonlinear ODE/Logistic Eqn.
10	2.6	Systems of ODE
11	3.1	Matrices
12	3.1/ 3.2	Matrices cont'd; Systems of Linear Eqns.
13	Exam 1 Review	1.2-2.6
14	3.2/3.3	Systems of Linear Eqns cont'd /Matrix Inverse
15	3.3	Matrix Inverse cont'd.
16	3.4	Determinants
17	3.5	Vector Spaces
18	3.5	Vector Spaces cont'd.
19	3.6	Basis and Dimension
20	3.6	Basis and Dimension cont'd.
21	5.3	Eigenvalues
22	5.3	Eigenvalues cont'd.
23	4.1	Harmonic Oscillator
24	4.2	Real Characteristic Roots
25	Exam 2 Review	3.1-3.6, 5.3
26	4.3	Complex Characteristic Roots
27	4.4	Undetermined Coefficients
28	4.4/4.5	Undetermined Coefficients cont'd/Variation of Parameters
29	4.5	Variation of Parameters cont'd.
30	4.6	Forced Oscillations
31	4.7	Conservation and Conversion
32	8.1	Laplace Transform
33	8.2	Solving IVP with Laplace Transforms
34	8.2	Solving IVP with Laplace Transforms
35	8.3	Step Function & Delta Function

36	8.3	Step Function & Delta Function
37	Exam 3 Review	4.1-8.3
38	6.1/6.2	Theory of Linear ODE Systems/ Linear ODE Systems with Real Eigenvalues
39	6.2	Linear ODE Systems with Real Eigenvalues cont'd.
40	6.3	Linear ODE Systems with Nonreal Eigenvalues
41	6.3	Linear ODE Systems with Nonreal Eigenvalues
42	6.4	Stability and Linear Classification
43	6.4	Stability and Linear Classification/Nonlinear Systems and Chaos (as time allows)
44	Review/Misc	

Prerequisites:

- Any ONE of the following courses (minimum grade C-): APPM 1360 or MATH 2300

Equivalent Courses: Duplicate Degree Credit Not Granted:

- both MATH 2130 and MATH 3430

Learning Objectives by Section

Section	Topics	Learning Objectives – After completing this section, students should be able to do the following:
1.2	Solutions and Direction Fields	• Understand what a differential equation is • What is a solution to a differential equation • State and Solve Initial Value Problems • Analyze behavior of solutions with direction fields • Find equilibrium solutions • Classify stability of equilibrium solutions • Draw direction fields for various differential equations • Answer what the long term behavior of solutions are based on the direction fields alone
1.3	Separation of Variables	• Identify separable ODE's • Perform separation of variables to find explicit solutions to ODEs
1.4	Numerical Methods	• Implement and understand Euler's Method for approximating solutions • Understand error propagation in Euler's Method • Various Other Methods Dicussed such as Runge-Kutta and Adams Bashforth

1.5	Picard's Theorem	• Determine when a solution might exist to an ODE • Employ Picard's Existence and Uniqueness theorem to state that a sol. does exist
2.1	Linear ODE Solutions	• Define linearity for ODE's • Recognize linear ODE's • Properties of linear operators and the superposition principle • Nonhomogeneous Principle • Solving nonhomogeneous ODEs
2.2	Solving First Order Linear Differential Equations	• Employ either variable coefficient techniques or integrating factor to solve Linear first order ODEs • Analyze solutions in terms of transient and steady-state behavior
2.3	Growth and Decay	• Apply linear differential equations to radioactive decay or compounding interest problems • Solve the "savings" problem • Apply the same model to various other real world problems
2.4	Mixing and Cooling	• Understand and set up the mixing problem. • Be able to solve the mixing problem for concentrations of substance • Solve Newton's law of cooling for various different initial conditions
2.5	Nonlinear ODE/Logistic Eqn.	• Qualitative Analysis of nonlinear DEs • Identify equilibria and determine stability • Identify autonomous and non-autonomous DEs • Solve and interpret IVPs for the Logistic Eqn. • Solve and interpret IVPs for the Threshold Eqn. • Simple bifurcations and their impact on solutions to DEs

2.6	Systems of ODEs	• Define what a system of DEs is • Understand what a solution to a system of DEs is • Define Phase Planes for DE Systems of two variables • Define and find equilibria and stability for systems of DEs • Define and sketch nullclines • Understand uniqueness of solutions and its relation to trajectories • Analyze various different models of systems of DE

3.1	Matrices	• Understand basic properties of matrices including
		  – Associativity
		  – Zero Element
		  – Inverse Element
		  – Distributivity
		  – Transpose
		  – Addition and Multiplication
		• Understand similar basic properties of vectors
		• Understand length and orthogonality

3.2	Systems of Linear Equations	• Convert systems of equations into matrix form • Understand the affect of row operations and be able to perform them to solve an equation • Determine when solutions for linear systems exist • Determine when solutions to a linear system are unique • Identify the rank of a matrix
3.3	Matrix Inverse	• Compute the inverse of a matrix • Determine when a matrix inverse exists • Determine properties of an invertible matrix

3.4	Determinants	• Compute determinants of 2x2 matrices • Compute determinants of an $n \times n$ matrix • Use Cramer's rule to determine unique solutions via determinants • Relate determinants with invertability of a matrix
3.5	Vector Spaces	• Understand the concept of vector spaces and the associated properties • Identify various vector spaces • Understand the vector subspace theorem and apply it • Be able to give several examples of common vector spaces • Identify when a space is not a vector space
3.6	Basis and Dimension	• Understand a spanning set in various vector spaces • Define the column space of a matrix • Determine when two vectors are linearly independent • Use the Wronskian to determine if functions are linearly independent • Define basis for a vector space • Define dimension of a vector space and compute it
5.3	Eigenvalues	• Set up and solve eigenvalue/eigenvector problems • Interpret the importance of eigenvectors and eigenvalues • How to handle multiple eigenvalues and the associated eigenvector • Basic properties of eigenvalues • Understand the connection with the characteristic equation
4.1	Harmonic Oscillator	• Understand the properties of the DE called the harmonic oscillator • Motivate and understand the importance of the harmonic oscillator equation in specific relation to physics • Solve the second order differential equation • Understand the impact of various coefficient choices in the system • Convert the second order equation to a system of equations

4.2	Real Characteristic Roots	• Solve constant coefficient second order linear DE's via the characteristic equation • How to handle repeated roots to the characteristic equation • Determine when solutions to second order non-constant DE's exist and are unique • Use the Wronskian to determine if solutions of DE's are linearly independent
4.3	Complex Characteristic Roots	• Determine general form of solutions to second order DEs when the characteristic polynomial has complex roots • Use the characteristic polynomial technique to solve higher order DEs • Be able to determine what DE a solution came from (work backwards)
4.4	Undetermined Coefficients	• Learn and utilize the method of undetermined coefficients to solve non-constant coefficient non-homogenous second order DEs • Predict forms of particular solution knowing common families of solutions and the homogeneous solution as well as inferring from the forcing term.
4.5	Variation of Parameters	• Learn and utilize the method of undetermined coefficients to solve non-constant coefficient non-homogenous second order DEs • Determine in which scenario to use variation of parameters over method of undetermined coefficients
4.6	Forced Oscillations	• Apply the solution techniques learned above to real world models • Determine the affect of damping on the mass spring problem and observe the impact on solutions • Identify resonance within the mathematical model and observe the affect it has on solutions • Determine a general solution to the damped, forced, mass-spring system
4.7	Conservation and Conversion	• Determine how to convert high order DE to first order systems • Determine how to convert back from a system of DEs to higher order DE

8.1	Laplace Transform	• Understand how a transform is helpful in solving problems • Define the Laplace transform • Properties of the Laplace transform • Finding inverse Laplace transforms
8.2	Solving IVP with Laplace Transforms	• Derivative Theorem for Laplace Transform • Relationship between multiplication and differentiation for Laplace transforms • Shifting and Translation properties of Laplace transforms
8.3	Step Function & Delta Function	8.3 The Step Function and the Delta Function • How to find Laplace transforms of discontinuous functions without working from the definition • Using the Delta function to solve DEs via the Laplace transform
6.2	Linear ODE Systems with Real Eigenvalues	• Solving systems of first order DEs using eigenvalues/eigenvectors • Providing general solution for $n \times n$ systems of differential eqns. with no repeated roots • Interpreting the behavior of solutions based on the sign of the eigenvalues • Learn how to handle repeated eigenvalues • Determine a generalized eigenvector • Utilize generalized eigenvectors in solutions to systemswhen necessary
6.3	Linear ODE Systems with Nonreal Eigenvalues	• Use complex conjugates to determine second eigenvalue/eigenvector with no work and understand why it works • How to choose constants such that the solution remains real-valued • Classify the behavior of solutions with complex roots via the phase portrait. For example determine if solutions have the following properties: – Spirals – Direction of spirals – Tilt/skew of the spirals portrait

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<tr><td>6.4</td><td>/ Stability and Linear Classification</td><td>Determine stability, asymptotic stability, and instability of equilibrium solutions under varying conditions namely
Distinct Eigenvalues
Repeated Eigenvalues
Positive real part/Negative real part of eigenvalue
No real part of eigenvalue
Determine degenerate and star nodes.</td></tr>
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