**INSTRUCTIONS:** Books, notes, and electronic devices are **not** permitted. Write (1) **your full name**, (2) **1340/Exam 2**, (3) **lecture number/instructor name** and (4) **FALL 2021** on the front of your bluebook. Make a **grading table** for 4 problems and a total. Do all problems. **Start each problem on a new page.** **Box** **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **Justify your answers, show all work.**

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1. (28pts) The following problems are not related.

   (a)(12pts) Suppose we know that the function $f(x)$ is an *even* function. Show that the function $g(x) = \sin(x) + xf(x)$ is an *odd* function. Justify your answer.

   (b)(12pts) Evaluate the limit: $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{x^2+x}\right)$

   (c)(4pts) The function $h(x) = \dfrac{3x+1}{\sqrt[3]{8x^3+5}}$ has a *horizontal asymptote* at which choice below? (**No justification necessary** - *Choose only one answer, copy down the entire answer.*)

   (A) $y=0$ (B) $y=\dfrac{3}{2}$ (C) $y=0$ and $y=3/2$ (D) $y=-3/2$ and $y=3/2$ (E) None of these

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2. (24pts) Start this problem on a **new** page. The following problems are not related.

   (a)(12pts) Use the *Squeeze Theorem* to evaluate the following limit: $\lim_{x\to 0^+} \sqrt{x}\cos^2\left(\frac{1}{x}\right)$. Show all work, explain your answer.

   (b)(12pts) Find the limit $\lim_{x\to 0} \dfrac{\sin(3x)\sin(5x)}{x^2}$. Justify your answer, show all work.

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**PROBLEMS #3 & #4 ON THE OTHER SIDE**

3. (28pts) Start this problem on a **new** page. The following problems are not related.

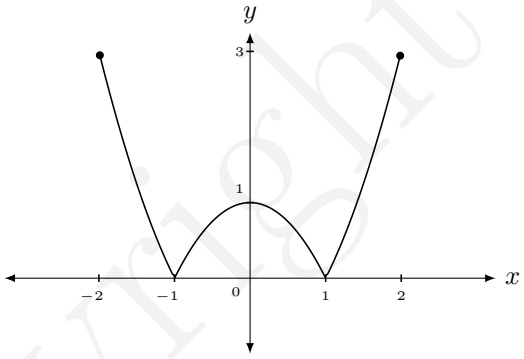
(a)(12pts) Evaluate the limit: $\lim_{x\to 0} \dfrac{\sqrt{3+x}-\sqrt{3}}{x}$. Show all work.

(b)(12pts) Suppose $g(x) = \begin{cases} x^2+x, & \text{if } x<0, \\ 1-\cos(x), & \text{if } x=0, \\ \sin(x), & \text{if } x>0. \end{cases}$ $(i)$(6pts) Find the $\lim_{x\to 0} g(x)$. $(ii)$(6pts) Show that $g(x)$ is continuous at $x=0$. Be sure to show that all the conditions of continuity have been satisfied.

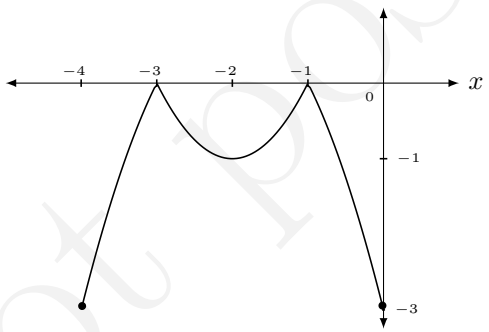
(c)(4pts) Consider the graph of the function below labeled as Graph $A$. If this function is $y=f(x)$ then which of the following choices given below correctly represents the graph labeled as Graph $B$? **No justification necessary** - *Choose only <u>one</u> answer, copy down the entire answer.*)

(A) $y=-f(x)-2$ (B) $y=f(-x+2)$ (C) $y=f(-x)-2$ (D) $y=-f(x+2)$ (E) $y=f(-x)+2$

Graphs for Problem 3(c)




(a) Graph A – the function $y=f(x)$




(b) Graph B

4. (20pts) Start this problem on a **new** page. The following problems are not related.

(a)(12pts) Use limits to classify all discontinuities of: $f(x)=\dfrac{x-2}{x^3-2x^2}$. Justify with limits.

(b)(8pts) The function $g(x)=\dfrac{x+10}{|x|+2}$ has two horizontal asymptotes. They are $y=1$ and $y=-1$. Use the Intermediate Value Theorem to show that $g(x)$ crosses one of its horizontal asymptotes on the interval $[-10,0]$. Clearly explain your answer.

**— END —**