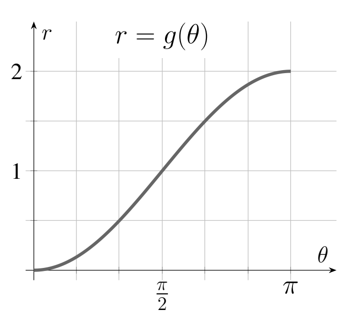
1. (38 pts) Decide whether the following expressions are convergent or divergent. If convergent, find the value the expression converges to. Explain your reasoning and name any test you use.
   (a) The sequence given by $a_n = \dfrac{5 \cdot 10 \cdot 15 \cdots (5n)}{n!}$ for $n = 1, 2, 3, \ldots$
   (b) $\displaystyle\sum_{k=2}^{\infty} [\ln(k/2) - \ln(k)]$
   (c) $\displaystyle\sum_{n=1}^{\infty} (\cos(3))^n$
   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1/2}}$

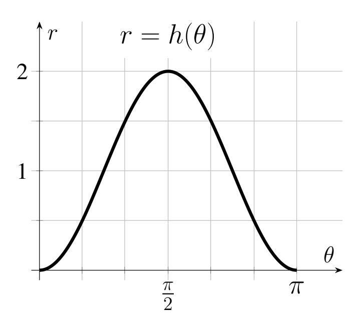
2. (26 pts) Let $f(x) = e^{-x/3}$.
   (a) Find a power series representation for f centered at $a = 3$.
   (b) Find T_2, the second order Taylor polynomial for f, centered at $a = 3$.
   (c) Use the Taylor Remainder Formula to find an error bound if T_2 is used to approximate f for x in the interval $[3, 3.3]$. Express your answer in terms of e.

3. (28 pts) The following two problems are not related.
   (a) Evaluate $\displaystyle\int x \sec^2 x \, dx$
   (b) i. Find the first 3 nonzero terms of the Maclaurin series for the function $\sqrt{1+x}$.
   ii. Use series to evaluate $\displaystyle\lim_{x \to 0} \frac{\sqrt{1+x} - 1 - \frac{x}{2}}{x \arctan(2x)}$.

4. (24 pts) Consider the region $\mathcal{R}$ bounded by the hyperbola $y^2 - \dfrac{x^2}{9} = 1$ and the lines $x = 0$, $x = 3$.
   (a) Sketch and shade the region $\mathcal{R}$. Label all intercepts.
   (b) Set up integrals to find the following quantities. Simplify integrands but otherwise do not evaluate the integrals.
   i. Volume of the solid generated by rotating the region $\mathcal{R}$ about the y-axis
   ii. Area of the surface generated by rotating the upper half of the hyperbola, for $0 \leq x \leq 3$, about the line $x = -1$

5. (34 pts)





(a) Sketch the polar curves $r = g(\theta)$ and $r = h(\theta)$, shown above as r-θ graphs, in the xy-plane. Label the curves and their x, y intercepts.

(b) Consider the intersection of the regions bounded by the polar curves $r = g(\theta)$ and $r = h(\theta)$ in quadrants I and II. The area of the intersection can be represented as the sum of two integrals. Set up, but do not evaluate, the integrals. Express your answer in terms of $g(\theta)$ and $h(\theta)$.

(c) The curve $r = g(\theta)$ can be represented in parametric form as

$$x = \cos\theta - \cos^2\theta, \quad y = \sin\theta - \frac{1}{2}\sin(2\theta).$$

Find an equation for the line tangent to the polar curve at $\theta = \frac{\pi}{2}$.