Answer the following problems showing all of your work and simplifying your solutions.

1. (36 pts) Evaluate the following integrals. **Show all work!**

   (a) $\displaystyle\int \frac{2x^4+4x^2+2}{x^3+2x}\,\mathrm{d}x$ (b) $\displaystyle\int_0^1 x^3\sqrt{1-x^2}\,\mathrm{d}x$ (c) $\displaystyle\int \sin^{-1}(x)\,\mathrm{d}x$

2. (24 pts) With $I = \displaystyle\int_0^4 \frac{1}{\sqrt{1+x}}\,\mathrm{d}x$, answer the following. **Leave your answer in exact form.**
   (a) Compute T_4 to estimate I. **Do not combine fractions.**
   (b) Find a reasonable bound for the error $|E_T|$ of your calculation in part (a).
   (c) What is the minimum number n of trapezoids required so that T_n is within 10^{-10} of the true solution I?
   (d) Suppose we change the bounds of integration to be $J = \displaystyle\int_{-1}^4 \frac{1}{\sqrt{1+x}}\,\mathrm{d}x$. Compute T_5 to approximate J. What happens to T_5 in this case?

3. (20 pts) Do the following integrals converge or diverge? Fully justify your answers.
   (a) $\displaystyle\int_{-1}^{\infty} \frac{2}{(x-1)^3}\,\mathrm{d}x$
   (b) $\displaystyle\int_1^{\infty} \frac{\ln x}{x^3}\,\mathrm{d}x$

4. (20 pts) Let $\mathcal{R}$ be the region bounded by $y=\tan^{-1}(x)$, $y=0$, and $x=1$.
   (a) Sketch and shade the region $\mathcal{R}$. Label all axes, curves, and intersection points.
   (b) Set up, **but do not evaluate**, integrals to determine each of the following:
      i. The area of $\mathcal{R}$ using integration with respect to x.
      ii. The area of $\mathcal{R}$ using integration with respect to y.

Trigonometric Identities

$\cos^2(x) = \frac{1}{2}(1+\cos 2x) \quad \sin^2 x = \frac{1}{2}(1-\cos 2x) \quad \sin 2x = 2\sin x\cos x \quad \cos 2x = \cos^2 x - \sin^2 x$

Inverse Trigonometric Integral Identities

$$\int \frac{\mathrm{d}u}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C, u^2 < a^2$$
$$\int \frac{\mathrm{d}u}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$
$$\int \frac{\mathrm{d}u}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C, u^2 > a^2$$

Midpoint Rule

$$\int_a^b f(x)\,\mathrm{d}x \approx \Delta x[f(\overline{x}_1)+f(\overline{x}_2)+\cdots+f(\overline{x}_n)],\ \Delta x = \frac{b-a}{n},\ \overline{x}_i = \frac{x_{i-1}+x_i}{2},\ |E_M| \le \frac{K(b-a)^3}{24n^2}$$

Trapezoidal Rule

$$\int_a^b f(x)\,\mathrm{d}x \approx \frac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-1})+f(x_n)],\ \Delta x = \frac{b-a}{n},\ |E_T| \le \frac{K(b-a)^3}{12n^2}$$