Answer the following problems and simplify your answers.

1. (22 pts) Let $\mathcal{R}$ be the region bounded by the curves $y = x^2 - 2x$ and $y = 2x$.
   (a) Sketch and shade the region $\mathcal{R}$, labeling the axes, intersection points, and curves.
   (b) Set up, **but do not evaluate**, an integral with respect to x to find the volume of the solid generated when $\mathcal{R}$ is revolved about the line $x = -1$.
   (c) Set up, **but do not evaluate**, an integral with respect to x to find the volume of the solid generated when $\mathcal{R}$ is revolved about the line $y = 8$.

2. (18 pts) Consider the curve $y = \ln(\cos(x))$ on the interval $0 \le x \le \pi/4$.
   (a) Set up, **but do not evaluate**, an integral to find the arc length of this curve. **Fully simplify** $\mathrm{d}s$.
   (b) The curve is rotated about the x-axis to generate a surface. Set up, **but do not evaluate**, an integral to find the surface area.

3. (24 pts) Assuming a uniform density ρ, find the centroid of the region under the curve $y = \cos(x)$ and above the x-axis with $0 \le x \le \pi/2$.

4. (12 pts) A 10 m long chain with a mass of 50 kg hangs vertically from a crane. How much work is required to lift the entire chain to the top? (Use 10 m/s^2 as the acceleration due to gravity).

5. (24 pts) Determine whether the sequences below converge or diverge. For the convergent sequences, find their limit.

   (a) $a_n = \tan\left(\dfrac{\pi n^4 + 3n^2 - 1}{4n^4 - 5}\right)$ (b) $b_n = (-1)^n \dfrac{(n-1)!}{(n+1)!}$ (c) $c_n = \dfrac{(2/5)^n}{(1/4)^n}$

Trigonometric Identities

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x) \qquad \sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \sin 2x = 2\sin x \cos x \qquad \cos 2x = \cos^2 x - \sin^2 x$$

Inverse Trigonometric Integral Identities

$$\int \frac{\mathrm{d}u}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C, u^2 < a^2$$

$$\int \frac{\mathrm{d}u}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{\mathrm{d}u}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C, u^2 > a^2$$

Center of Mass Formulas

$$M_y = \rho \int_a^b x(f(x) - g(x))\,\mathrm{d}x \qquad \bar{x} = \frac{M_y}{m}$$

$$M_x = \rho \int_a^b \frac{1}{2}\left((f(x))^2 - (g(x))^2\right)\mathrm{d}x \qquad \bar{y} = \frac{M_x}{m}$$

$$m = \rho \int_a^b f(x) - g(x)\,\mathrm{d}x$$