Work out the following problems and simplify your answers.

1. (30 pts) Determine if the following series converge or diverge. Fully justify your answer and state which test you used.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(n)}{n^2+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$ (c) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n}$

2. (15 pts) Suppose we have the series

   $$s = \ln\left(\frac{2}{3}\right) + \ln\left(\frac{3^2}{2\cdot 4}\right) + \ln\left(\frac{4^2}{3\cdot 5}\right) + \ln\left(\frac{5^2}{4\cdot 6}\right) + \cdots$$

   (a) Find a simple expression for the partial sums s_n of the series s.
   (b) Does the series converge or diverge? Fully justify your answer. If the series converges, find its sum.

3. (15 pts) Consider the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{4}{(n!)^2}$.

   (a) Show that the series converges.
   (b) Estimate the error in using the partial sum s_3 to approximate s.

4. (25 pts) Consider the power series

   $$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{4n!}(n-1)!$$

   (a) Find the radius and interval of convergence for the power series $f(x)$
   (b) Using interval notation, for what values of x is $f(x)$ absolutely convergent, conditionally convergent, and divergent?

5. (15 pts) Starting with the Maclaurin series for $\frac{1}{1-x}$, write out a power series for the function below and determine its radius of convergence without the use of the Ratio or Root Tests.

   $$f(x) = \frac{5}{1-4x^2}.$$

Common Maclaurin Series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots \qquad R = 1$$

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