1. [2360/050724 (16 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given. Please write your answers in a single column separate from any work you do to arrive at the answer.

   (a) The eigenvalues of $\begin{bmatrix} 1 & a & 0 \\ 0 & 2 & b \\ 0 & 0 & 3 \end{bmatrix}$, $a, b \in \mathbb{R}$, are $1, 2, 3$, regardless of the values of a and b.

   (b) If $\mathbf{A}$ and $\mathbf{B}$ are nonsingular square matrices of the same order, then $\left(\mathbf{A}^{-1}\mathbf{B}\mathbf{A}^{\mathrm{T}}\right)^{\mathrm{T}} = \left(\mathbf{A}^{\mathrm{T}}\right)^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{A}$.

   (c) Let $\mathbb{V}$ be a vector space. Any set of vectors in $\mathbb{V}$ that contains the zero vector must be linearly dependent.

   (d) For $n \times n$ matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$, $\operatorname{Tr}\left(\mathbf{A} + \mathbf{B}^{\mathrm{T}} + \mathbf{C}\right) = \operatorname{Tr}\mathbf{A}^{\mathrm{T}} + \operatorname{Tr}\mathbf{B} + \operatorname{Tr}\mathbf{C}^{\mathrm{T}}$.

   (e) If $\mathbf{A}$ is an $m \times n$ matrix where $m \neq n$, then $\left|\mathbf{A}^{\mathrm{T}}\mathbf{A}\right|$ is not defined.

   (f) The equation $e^{w+z}\csc z \dfrac{\mathrm{d}w}{\mathrm{d}z} - wz\sin w = 0$ is separable.

   (g) The equation $y' = y^4 + 4y^2$ has an unstable equilibrium solution.

   (h) There are no values of t where the trajectories of the system $\begin{aligned} x' &= x^2 + y^4 + 4 \\ y' &= e^{x-y} - 1 \end{aligned}$ have horizontal tangents.

SOLUTION:

(a) **TRUE** The matrix is upper triangular so the eigenvalues are the diagonal elements.

(b) **FALSE** $\left(\mathbf{A}^{-1}\mathbf{B}\mathbf{A}^{\mathrm{T}}\right)^{\mathrm{T}} = \left(\mathbf{A}^{\mathrm{T}}\right)^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\left(\mathbf{A}^{-1}\right)^{\mathrm{T}} = \mathbf{A}\mathbf{B}^{\mathrm{T}}\left(\mathbf{A}^{\mathrm{T}}\right)^{-1}$

(c) **TRUE** Let $\left\{\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2, \vec{\mathbf{0}}, \vec{\mathbf{v}}_3, \ldots, \vec{\mathbf{v}}_n\right\}$ be the set of vectors. Then $0\vec{\mathbf{v}}_1 + 0\vec{\mathbf{v}}_2 + c\vec{\mathbf{0}} + 0\vec{\mathbf{v}}_3 + \cdots 0\vec{\mathbf{v}}_n = \vec{\mathbf{0}}$ is satisfied for all values of c, including $c \neq 0$.

(d) **TRUE** For example, let $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$. Then

$$\begin{aligned}
\operatorname{Tr}\left(\mathbf{A} + \mathbf{B}^{\mathrm{T}} + \mathbf{C}\right) &= \operatorname{Tr}\begin{bmatrix} a_{11} + b_{11} + c_{11} & a_{12} + b_{21} + c_{11} \\ a_{21} + b_{12} + c_{21} & a_{22} + b_{22} + c_{22} \end{bmatrix} \\
&= a_{11} + b_{11} + c_{11} + a_{22} + b_{22} + c_{22} \\
&= a_{11} + a_{22} + b_{11} + b_{22} + c_{11} + c_{22} \\
&= \operatorname{Tr}\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} + \operatorname{Tr}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \operatorname{Tr}\begin{bmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{bmatrix} \\
&= \operatorname{Tr}\mathbf{A}^{\mathrm{T}} + \operatorname{Tr}\mathbf{B} + \operatorname{Tr}\mathbf{C}^{\mathrm{T}}
\end{aligned}$$

(e) **FALSE** If $\mathbf{A}$ is $m \times n$, then $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ is $n \times n$, a square matrix for which a determinant can be computed.

(f) **TRUE** Separating variables gives $e^w w^{-1}\csc w\,\mathrm{d}w = ze^{-z}\sin z\,\mathrm{d}z$

(g) **FALSE** $y' = y^2\left(y^2 + 4\right)$ showing that $y = 0$ is an equilibrium solution. However, $y' > 0$ for $y > 0$ and $y < 0$ meaning that $y = 0$ is semistable.

(h) **FALSE** The system does have an h nullcline ($y = x$ where $y' = 0$). Trajectories have horizontal tangents there.

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2. [2360/050724 (14 pts)] Let $\vec{\mathbf{p}} = 4t^2 + 8t + 5$ and let $\vec{\mathbf{p}}_1 = t^2 + t + 1$, $\vec{\mathbf{p}}_2 = 3t^2 + 3t + 4$ and $\vec{\mathbf{p}}_3 = 2t^2 + 3t + 2$. Is $\vec{\mathbf{p}} \in \operatorname{span}\left\{\vec{\mathbf{p}}_1, \vec{\mathbf{p}}_2, \vec{\mathbf{p}}_3\right\}$? If so, write it as a linear combination of $\vec{\mathbf{p}}_1, \vec{\mathbf{p}}_2, \vec{\mathbf{p}}_3$. If not, explain why not. Justify your answer.

SOLUTION:

We need to see if constants c_1, c_2, c_3 exist such that $\vec{\mathbf{p}} = c_1\vec{\mathbf{p}}_1 + c_2\vec{\mathbf{p}}_2 + c_3\vec{\mathbf{p}}_3$. That is,

$$c_1\left(t^2+t+1\right) + c_2\left(3t^2+3t+4\right) + c_3\left(2t^2+3t+2\right) = 4t^2+8t+5$$

$$\left(c_1+3c_2+2c_3\right)t^2 + \left(c_1+3c_2+3c_3\right)t + \left(c_1+4c_2+2c_3\right) = 4t^2+8t+5 \quad \text{equate coefficients}$$

$$\left.\begin{aligned} c_1+3c_2+2c_3 &= 4 \\ c_1+3c_2+3c_3 &= 8 \\ c_1+4c_2+2c_3 &= 5 \end{aligned}\right\} \implies \begin{bmatrix} 1 & 3 & 2 \\ 1 & 3 & 3 \\ 1 & 4 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 1 & 3 & 3 & 8 \\ 1 & 4 & 2 & 5 \end{array}\right] \xrightarrow[R_3^*=-1R_1+R_3]{R_2^*=-1R_1+R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 1 \end{array}\right] \xrightarrow[R_2\leftrightarrow R_3]{R_1^*=-3R_3+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow{R_1^*=-2R_3+R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$\implies \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -7 \\ 1 \\ 4 \end{bmatrix} \implies \vec{\mathbf{p}} = -7\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + 4\vec{\mathbf{p}}_3 \qquad \text{Yes, } \vec{\mathbf{p}} \in \text{span}\left\{\vec{\mathbf{p}}_1, \vec{\mathbf{p}}_2, \vec{\mathbf{p}}_3\right\}$$

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3. [2360/050724 (18 pts)] Consider the function $f(t) = \begin{cases} 0 & t < 0 \\ 2t & 0 \le t < 2 \\ 4-(t-2)^2 & t \ge 2 \end{cases}$

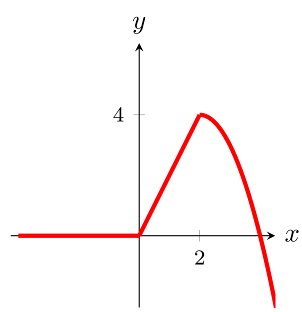
(a) (6 pts) Draw a well labeled graph of $f(t)$.

(b) (6 pts) Write $f(t)$ using step functions.

(c) (6 pts) Find the Laplace transform of $f(t)$.

SOLUTION:

(a) Graph of $f(t)$.



(b) $f(t) = 2t\,\text{step}(t) - 2t\,\text{step}(t-2) + \left[4-(t-2)^2\right]\text{step}(t-2) = 2t\,\text{step}(t) - 2(t-2)\,\text{step}(t-2) - (t-2)^2\text{step}(t-2)$

(c) Using the latter expression from part (b) gives

$$\begin{aligned} \mathscr{L}\left\{f(t)\right\} &= 2e^{-0s}\mathscr{L}\left\{t\right\} - 2e^{-2s}\mathscr{L}\left\{t\right\} - e^{-2s}\mathscr{L}\left\{t^2\right\} \\ &= \frac{2}{s^2} - \frac{2e^{-2s}}{s^2} - \frac{2e^{-2s}}{s^3} \end{aligned}$$

Alternatively, using the first expression from part (b) gives

$$\begin{aligned} \mathscr{L}\left\{f(t)\right\} &= 2e^{-0s}\mathscr{L}\left\{t\right\} - 2e^{-2s}\mathscr{L}\left\{t+2\right\} + 4e^{-2s}\mathscr{L}\left\{1\right\} - \mathscr{L}\left\{(t-2)^2\,\text{step}(t-2)\right\} \\ &= \frac{2}{s^2} - 2e^{-2s}\left(\frac{1}{s^2}+\frac{2}{s}\right) + \frac{4e^{-2s}}{s} - e^{-2s}\mathscr{L}\left\{t^2\right\} \\ &= \frac{2}{s^2} - 2e^{-2s}\left(\frac{1}{s^2}+\frac{2}{s}\right) + \frac{4e^{-2s}}{s} - e^{-2s}\left(\frac{2}{s^3}\right) \\ &= \frac{2}{s^2} - \frac{2e^{-2s}}{s^2} - \frac{4e^{-2s}}{s} + \frac{4e^{-2s}}{s} - \frac{2e^{-2s}}{s^3} \\ &= \frac{2}{s^2} - \frac{2e^{-2s}}{s^2} - \frac{2e^{-2s}}{s^3} \end{aligned}$$

4. [2360/050724 (14 pts)] You and your roommate are concocting an elixir that will help you pass your differential equations exam. To make it, you have two 10-gallon tanks. Initially, Tank 1 is filled with fresh water and Tank 2 is half filled with well-mixed solution in which 7 grams of the miracle spice *diffyQ* is dissolved. For $t > 0$, solution containing 2 grams of *diffyQ* per gallon enters Tank 1 at 3 gallons per hour and this well-mixed solution flows from Tank 1 into Tank 2 at the rate of 4 gallons per hour. Well-mixed solution from Tank 2 flows into Tank 1 at 1 gallon per hour and exits Tank 2 at 2 gallons per hour. The elixir will be perfect when Tank 2 is full. Write a system of differential equations using matrices and vectors that models this initial value problem, providing an appropriate t interval over which the solution is valid.

SOLUTION:

Begin by noting that the the volume in Tank 2, $V_2(t)$, is not constant since the flow in and flow out differ. Indeed,

$$\frac{\mathrm{d}V_2}{\mathrm{d}t} = \text{flow in} - \text{flow out} = 4 - 1 - 2 = 1,\ V_2(0) = 5 \implies V_2(t) = t + 5$$

We use the standard continuity equation stating that the rate of change of the mass of spice in each tank equals the difference between the incoming and outgoing mass rates of the spice. Let $x_1(t), x_2(t)$ be the amount of *diffyQ* in Tank 1 and Tank 2, respectively.

$$x_1'(t) = \left(2\,\frac{\text{gram}}{\text{gallon}}\right)\left(3\,\frac{\text{gallon}}{\text{hour}}\right) + \left(\frac{x_2}{t+5}\,\frac{\text{gram}}{\text{gallon}}\right)\left(1\,\frac{\text{gallon}}{\text{hour}}\right) - \left(\frac{x_1}{10}\,\frac{\text{gram}}{\text{gallon}}\right)\left(4\,\frac{\text{gallon}}{\text{hour}}\right) = -\frac{2x_1}{5} + \frac{x_2}{t+5} + 6$$

$$x_2'(t) = \left(\frac{x_1}{10}\,\frac{\text{gram}}{\text{gallon}}\right)\left(4\,\frac{\text{gallon}}{\text{hour}}\right) - \left(\frac{x_2}{t+5}\,\frac{\text{gram}}{\text{gallon}}\right)\left(3\,\frac{\text{gallon}}{\text{hour}}\right) = \frac{2x_1}{5} - \frac{3x_2}{t+5}$$

With $\vec{\mathbf{x}}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ we have

$$\vec{\mathbf{x}}' = \begin{bmatrix} -\frac{2}{5} & \frac{1}{t+5} \\ \frac{2}{5} & -\frac{3}{t+5} \end{bmatrix} \vec{\mathbf{x}} + \begin{bmatrix} 6 \\ 0 \end{bmatrix},\ \vec{\mathbf{x}}(0) = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$$

The solution is valid for $t \in [0, 5]$ or $0 \le t \le 5$.

5. [2360/050724 (20 pts) Use Laplace Transforms to solve $\dfrac{\mathrm{d}y}{\mathrm{d}t} + 4y = 10te^{-4t} + \delta(t-2),\ y(0) = 0.$

SOLUTION:

$$sY(s) + 4Y(s) = 10\mathscr{L}\{t\}\Big|_{s\to s+4} + e^{-2s}$$

$$(s+4)Y(s) = \frac{10}{(s+4)^2} + e^{-2s}$$

$$Y(s) = \frac{10}{(s+4)^3} + \frac{e^{-2s}}{s+4}$$

$$y(t) = \mathscr{L}^{-1}\left\{\frac{10}{(s+4)^3} + \frac{e^{-2s}}{s+4}\right\}$$

$$= 5e^{-4t}\mathscr{L}^{-1}\left\{\frac{2}{s^3}\right\} + \text{step}(t-2)\mathscr{L}^{-1}\left\{\frac{1}{s+4}\right\}\Bigg|_{t\to t-2}$$

$$= 5t^2e^{-4t} + e^{-4(t-2)}\,\text{step}(t-2)$$

6. [2360/050724 (20 pts)] Solve the initial value problem $t\vec{\mathbf{x}}' = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}\vec{\mathbf{x}},\ \vec{\mathbf{x}}(1) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$, $t > 0$. Assume solutions are of the form $t^\lambda \vec{\mathbf{v}}$ as was done in the homework and write your final answer as a single vector.

Solution:

$$\begin{vmatrix} 1-\lambda & 2 \\ -1 & 4-\lambda \end{vmatrix} = (1-\lambda)(4-\lambda)+2 = \lambda^2-5\lambda+6 = (\lambda-2)(\lambda-3) = 0 \implies \lambda = 2,3$$

$$\lambda = 2: \left[\begin{array}{cc|c} -1 & 2 & 0 \\ -1 & 2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right] \implies \vec{\mathbf{v}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\lambda = 3: \left[\begin{array}{cc|c} -2 & 2 & 0 \\ -1 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \implies \vec{\mathbf{v}} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

General solution is $\vec{\mathbf{x}}(t) = c_1 t^2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 t^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ to which we apply the initial condition, yielding

$$c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \implies \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$c_1 = \frac{\begin{vmatrix} 2 & 1 \\ 4 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{-2}{1} = -2 \qquad c_2 = \frac{\begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{6}{1} = 6$$

$$\vec{\mathbf{x}}(t) = -2t^2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 6t^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6t^3 - 4t^2 \\ 6t^3 - 2t^2 \end{bmatrix}$$

7. [2360/050724 (16 pts)] Consider an harmonic oscillator having a mass attached to a spring. For the following parts, write, but **DO NOT SOLVE**, the appropriate differential equation that governs the displacement, $x(t)$, with the given description.

   (a) (4 pts) The oscillator is unforced, experiences a damping force proportional to twice the instantaneous velocity, has circular (angular) frequency of 3 s^{-1} and a mass of 3 kg.

   (b) (4 pts) The oscillator is forced by $10\cos 2t$, has a restoring (spring) constant of 4 N/m and is in resonance.

   (c) (4 pts) The oscillator is unforced, critically damped, has mass of 2 kg and a restoring (spring) constant of 2 N/m.

   (d) (4 pts) The oscillator is undamped, has a spring (restoring) constant of 3 N/m and circular (angular) frequency of 2 s^{-1} and is forced by $\sin t$ for $\pi \le t \le 2\pi$ and unforced otherwise. Write the forcing function as a single function, not as a piecewise defined function.

Solution:

(a) $\omega_0 = 3 = \sqrt{k/3} \implies k = 27; \qquad 3\ddot{x} + 2\dot{x} + 27x = 0$

(b) For resonance, need $\omega_0 = 2 = \sqrt{4/m} \implies m = 1; \qquad \ddot{x} + 4x = 10\cos 2t$

(c) Critically damped means $b^2 - 4(2)(2) = 0 \implies b = 4; \qquad 2\ddot{x} + 4\ddot{x} + 2x = 0$

(d) $\omega_0 = 2 = \sqrt{3/m} \implies m = \frac{3}{4}; \qquad \frac{3}{4}\ddot{x} + 3x = \sin t\,[\text{step}(t-\pi) - \text{step}(t-2\pi)]$

8. [2360/050724 (22 pts)] Consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 4 \\ -2 & -4 & k \\ 3 & 6 & 12 \end{bmatrix}$ and the vector $\vec{\mathbf{b}} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$.

   (a) (6 pts) There is one real value of k that makes the matrix $\mathbf{A}$ have a single pivot column. Find it, making sure you are correct because the remaining parts depend on it.

   (b) (12 pts) Using the value of k you found in part (a):

      i. (4 pts) Find a particular solution of $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$.

      ii. (4 pts) Find a basis for the solution space of $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{0}}$ and state its dimension.

      iii. (4 pts) Is $\vec{\mathbf{b}} \in$ Col $\mathbf{A}$? Explain briefly.

   (c) (4 pts) Is $\mathbf{A}$ invertible, regardless of the value of k? Explain briefly.

Solution:

(a)

$$\left[\begin{array}{rrr|r} 1 & 2 & 4 & 2 \\ -2 & -4 & k & -4 \\ 3 & 6 & 12 & 6 \end{array}\right] \xrightarrow[R_3^*=-3R_1+R_3]{R_2^*=2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 0 & 0 & k+8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \implies k=-8 \quad \text{for system to have a single pivot column}$$

(b) If $k=-8$ we have the following augmented matrix in RREF

$$\left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \implies \vec{\mathbf{x}} = \begin{bmatrix} 2-2r-4s \\ r \\ s \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + r\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix},\ r,s \in \mathbb{R}$$

i. $\vec{\mathbf{x}}_p = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

ii. $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix} \right\}$, dimension is 2

iii. Yes. Since the system is consistent, $\vec{\mathbf{b}} \in \operatorname{Col} \mathbf{A}$.

(c) No. The RREF has a row of zeros in a row not containing k. Also, $|\mathbf{A}| = 0$.

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9. [2360/050724 (10 pts)] Consider the linear system of differential equations given by $\vec{\mathbf{x}}' = \mathbf{A}\vec{\mathbf{x}}$ where $\mathbf{A} = \begin{bmatrix} 2 & 4 \\ 1 & k \end{bmatrix}$ and $k \in \mathbb{R}$. For each of the following, find all values of k such that the system exhibits the given behavior. If there are no values of k satisfying the requirements, write NONE. Hint: The fact that $x^2 - 4x + 20 = (x-2)^2 + 16$ may be of help.

(a) The equilibrium point at $(0,0)$ is a degenerate node.

(b) The system has nonisolated equilibrium points.

(c) The equilibrium point at $(0,0)$ is a center.

(d) The equilibrium point at $(0,0)$ is stable.

(e) The equilibrium point at $(0,0)$ is an unstable node.

SOLUTION:

Note that $\operatorname{Tr}\mathbf{A} = k+2,\ |\mathbf{A}| = 2k-4,\ (\operatorname{Tr}\mathbf{A})^2 - 4|\mathbf{A}| = (k+2)^2 - 4(2k-4) = k^2 - 4k + 20 = (k-2)^2 + 16$

(a) NONE Need $(k-2)^2 + 16 = 0$ but $(k-2)^2 + 16 > 0$ for all k.

(b) $k = 2$ Need $2k - 4 = 0 \implies k = 2$

(c) NONE Need $k + 2 = 0 \implies k = -2 \implies |\mathbf{A}| < 0 \implies$ saddle

(d) NONE Need $k + 2 \leq 0 \implies k \leq -2 \implies |\mathbf{A}| < 0 \implies$ unstable

(e) $k > 2$ Need $k + 2 > 0 \implies k > -2$ and need $2k - 4 > 0 \implies k > 2$ [and we have $(k-2)^2 + 16 > 0$ for all k]

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