- This exam is worth 100 points and has 5 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"$\times$ 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/061424 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) Let L be a linear operator and suppose y_1 and y_2 are both solutions to the differential equation $L(\vec{\mathbf{y}}) = f(t)$. Then $y_1 - y_2$ is a solution of $L(\vec{\mathbf{y}}) = 0$.

   (b) Euler's method for the initial value problem $y' = 2,\ y(0) = 0$ will yield the exact solution for all t.

   (c) Picard's Theorem guarantees that the differential equation $y' = \sqrt{t+y},\ y(0) = 0$ does NOT have a unique solution.

   (d) Every first order linear homogeneous differential equation is separable.

   (e) If using the Euler-Lagrange Two Stage method (variation of parameters) to solve the differential equation $e^{-t}y' - y = e^{2-t}$, the particular solution has the form $y_p = v(t)e^{e^t}$.

2. [2360/061424 (20 pts)] The following problems are not related.

   (a) (10 pts) You are making a secret marinade sauce for meat that involves dissolving 100 grams of Special Spice #1 in 10 gallons of vinegar in a large tank. A malefactor has decided to sabotage the mixture by creating a machine that pours 1 gallon, containing 30 grams of Special Spice #1, into the tank every minute. As soon as the machine is turned on, the malefactor also creates a hole in the tank that drains the well-mixed marinade from the tank at 2 gallons per minute. Set up, but **DO NOT SOLVE**, the initial value problem (IVP) modeling this scenario. Let $t = 0$ be the moment that the hole is created in the tank. Be sure to include the interval over which the equation is valid.

   (b) (10 pts) Use the integrating factor method to solve $(10-t)\dfrac{\mathrm{d}y}{\mathrm{d}t} + 2y = 300 - 30t,\ y(0) = 100$. Do not simply use formulas. Show all the steps needed to arrive at the solution.

3. [2360/061424 (24 pts)] After discovering the culinary sabotage noted part (a) of the previous problem, you decide to make a new batch of marinade with your trademark 100 grams of Special Spice #1. In addition to being irresistibly delicious, Special Spice #1 is also an unstable radioactive material with a half-life of 10 days. Be sure to show all your work for all parts of this problem.

   (a) (8 pts) Set up the corresponding initial value problem for this decay problem assuming that the new marinade is made at $t = 0$.

   (b) (8 pts) How much Special Spice #1 will be left after t days?

   (c) (8 pts) Your marinade will lose its flavor if under 10 grams of Special Spice #1 are left. How long after making a fresh batch of the marinade do have to use it?

MORE PROBLEMS BELOW/ON REVERSE

4. [2360/061424 (22 pts)] The following problems are not related.

   (a) (16 pts) Consider the differential equation $y' = y^3 - 3y^2 - y + 3$.

      i. (10 pts) Find all equilibrium solutions and their stability.

      ii. (6 pts) Plot the phase line for the differential equation.

   (b) (6 pts) Given the differential equation $y' + y = t^2$, draw the isoclines corresponding to slopes of $1, 0, -1$. Be sure to include the line segments showing the slope of the solutions on each isocline and label important points.

5. [2360/061424 (24 pts)] Consider the system of differential equations

$$x' = 2 - x + y$$
$$y' = y^2 - x$$

   (a) (6 pts) Find the h nullclines, if any exist.

   (b) (6 pts) Find the v nullclines, if any exist.

   (c) (6 pts) Find the equilibrium solutions, if any exist.

   (d) (6 pts) Draw a phase plane on your paper and put arrows in the phase plane showing the vector field at the points $(1, 1)$ and $(0, -2)$. Be sure to include appropriate labels.