- This exam is worth 100 points and has 6 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/062824 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) If $\mathbf{A}, \mathbf{B}$ are square matrices of the same order, then it is always the case that $\mathbf{AB} = \mathbf{BA}$

   (b) If $\mathbf{A}$ is a square matrix with $|\mathbf{A}| = 0$, then $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{0}}$ is inconsistent.

   (c) If $\mathbb{U}$ is the set of all points in the xy-plane except the origin, then $\mathbb{U}$ is not a vector space.

   (d) If $\mathbf{A}$ is an $n \times n$ matrix, then $\operatorname{Tr}\mathbf{A} = \operatorname{Tr}\left(\mathbf{A}^{\mathrm{T}}\right)$.

   (e) If $|\mathbf{AB}| = 0$, and $\mathbf{A}$ is nonsingular, then 0 is an eigenvalue of $\mathbf{B}$.

2. [2360/062824 (21 pts)] Let $\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 \\ -2 & 1 & 0 \\ 6 & -3 & 0 \end{bmatrix}$.

   (a) (8 pts) Find the eigenvalues of $\mathbf{A}$ and give their algebraic multiplicities.

   (b) (13 pts) For each eigenvalue, find a basis for the eigenspace and give the geometric multiplicity of the eigenvalue.

3. [2360/062824 (15 pts)] Which of the following subsets of $\mathbb{R}^3$ [parts (a) and (b)] and $\mathbb{M}_{22}$ [part (c)] are subspaces. Fully justify your answers.

   (a) (5 pts) The solution set of $x_1 + x_2 + x_3 = 0$.

   (b) (5 pts) Vectors in $\mathbb{R}^3$ having the form $\begin{bmatrix} a \\ a \\ 1 \end{bmatrix}$ where $a \in \mathbb{R}$.

   (c) (5 pts) Matrices in $\mathbb{M}_{22}$ of the form $\begin{bmatrix} a & b \\ -b & c \end{bmatrix}$ where $b \geq 0, a, c \in \mathbb{R}$.

4. [2360/062824 (20 pts)] The following problems are not related. Provide full justification for your answers.

   (a) (10 pts) Can the vectors $\left\{ \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -12 \\ 6 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix} \right\}$ be a basis for $\mathbb{R}^3$?

   (b) (10 pts) Let $\vec{\mathbf{p}}_1 = t^2 + 5$, $\vec{\mathbf{p}}_2 = 5t^2 + t$, $\vec{\mathbf{p}}_3 = 5t + 5$. Do constants c_1, c_2, c_3, not all zero, exist such that $c_1\vec{\mathbf{p}}_1 + c_2\vec{\mathbf{p}}_2 + c_3\vec{\mathbf{p}}_3 = 0$ for all real t?

MORE PROBLEMS BELOW/ON REVERSE

5. [2360/062824 (16 pts)] The following problems are not related.

(a) (8 pts) Find all values of k such that $\mathbf{G}\vec{\mathbf{x}} = \vec{\mathbf{0}}$ has nontrivial solutions, where $\mathbf{G} = \begin{bmatrix} 1 & 0 & k \\ 0 & k & 0 \\ k & 0 & 9 \end{bmatrix}$.

(b) (8 pts) If $\mathbf{B}$ is an invertible matrix with $\mathbf{B}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 3 & 2 & -1 \\ 1 & 2 & 3 \end{bmatrix}$, find the solution of $\mathbf{B}^{\mathrm{T}}\vec{\mathbf{x}} = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}$

6. [2360/062824 (18 pts)] Some friends of yours need the solution to a linear system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$. They have already performed a number of elementary row operations on the augmented matrix, and have given you the following:

$$\left[\begin{array}{cccc|c} 0 & -2 & 0 & -6 & -10 \\ 0 & 1 & 2 & 7 & 11 \\ 0 & 1 & 0 & 3 & 5 \end{array}\right]$$

Your friends (and grader) have several requests:

(a) (6 pts) Find a particular solution of the system.

(b) (8 pts) Find a basis for the solution space of the associated homogeneous system. What is its dimension?

(c) (4 pts) Find the (general) solution, $\vec{\mathbf{x}}$, to the problem.