16^{th} Annual SIAM Front Range Applied Mathematics Student Conference

March 7, 2020

Registration: 8:30 - 9:00

Morning Session I - Room 4017
9:00 - 11:35

9:00 - 9:20	Perrin Ruth *University of Colorado, Boulder*	Numerical Representation of Symbolic Datasets
9:25 - 9:45	Daniel R. Bielich *University of Colorado, Denver*	One-Synch CGS2 Algorithm in the Context of QR Factorization and Arnoldi Process
9:50 - 10:10	Lara Kassab *Colorado State University*	An Iterative Method for Structured Matrix Completion
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Kirana Bergstrom *University of Colorado, Denver*	Fast Batched Matrix Algebra with Intel® Math Kernel Library Vectorized Compact Routines
10:50 - 11:10	Stetson Zirkelbach *University of Colorado, Denver*	Traversing the Transportation Polytope
11:15 - 11:35	Jordan R. Hall *University of Colorado, Denver*	Derivative-Free Optimization and Active Subspaces in Inverse Problem Theory

Morning Session II - Room 4125
9:00 - 11:35

9:00 - 9:20	Nate Mankovich *Colorado State University*	Geometric Median of Subspaces
9:25 - 9:45	Alyssa Newman *University of Colorado, Denver*	Fractals, What Are They and How Can We Draw Them?
9:50 - 10:10	Christopher Padgett *Metropolitan State University of Denver*	Investigating poverty traps through the lens of fractal geometry and dynamical systems

Student Government Association
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We gratefully acknowledge funding support from CU Denver Student Government Association.

10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Joseph Geisz *University of Colorado, Boulder*	Persistent Homology and Extending the Sierpinski Relatives to 3D
10:50 - 11:10	Johnathan Bush *Colorado State University*	A torus model for optical flow
11:15 - 11:35	Gessner Soto	The “Brusselator”: ”Internal Entropy Production” Surrounding a Hopf Bifurcation

Morning Session III - Room 4113
9:00 - 11:35

9:00 - 9:20	Daniel Messenger *University of Colorado, Boulder*	Aggregation-Diffusion Phenomena in Domains with Boundaries
9:25 - 9:45	Lyndsey Wong *University of Colorado, Boulder*	Pattern Formation in an Amenities-Based Model of Wealth Concentration
9:50 - 10:10	Erin Ellefsen *University of Colorado, Boulder*	Finding steady states of nonlocal territorial Models in Ecology
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Daniel Jonas *Colorado State University*	In silico mouse model of infection and immunity
10:50 - 11:10	Nicholas Landry *University of Colorado, Boulder*	The effect of simplex and network degree distribution on explosive transitions in epidemic dynamics using simplicial contagion models
11:15 - 11:35	Graham Kesler O’Connor *University of Colorado, Boulder*	Ground Zero Estimation in Infection Networks

Break: 11:35 - 11:45

Lunch: 11:45 - 12:25

Plenary Address: 12:30 - 1:30, Room 2600

Dr. Jeanne Clelland
Professor
Department of Mathematics
University Colorado Boulder

Gerrymandering: What is it, how can we measure it, and what can we do about it?

Group Photographs at 1:30

Afternoon Session I - Room 4017
1:45 - 3:20

1:45 - 2:05	Lu Vy *University of Colorado, Denver*	Variance Reduction Methods Based on Multilevel Monte Carlo
2:10 - 2:30	Evan Shapiro	Propagation of Uncertainty in High-Fidelity Plasma Simulations Using Surrogate Models Generated on Sparse Grids
2:35 - 2:55	Arvind Srinivasan *University of Colorado, Denver*	Mathematical distributions for Dark Matter detection
3:00 - 3:20	Fateh Elsherif *Colorado State University*	Green Communication and Security in Wireless Networks based on Markov Process and Semivariance Optimization

Afternoon Session II - Room 4125
1:45 - 3:20

1:45 - 2:05	Keegan W$^{m.}$ Karbach *Metropolitan State University of Denver*	Analysis of fluid dynamics using the Finite Volume method as compared to other methods
2:10 - 2:30	Yifeng Mao *University of Colorado, Boulder*	Investigation of Envelope Solitary Waves in a Viscous Fluid Conduit
2:35 - 2:55	Michael Zowada *University of Colorado, Colorado Springs*	Behavior of n-lump Rational Solutions to the Kadomtsev-Petviashvili (KP) I Equation

Plenary Speaker

12:30 - 1:30, 1600

Gerrymandering: What is it, how can we measure it, and what can we do about it?

Jeanne Clelland

Professor, Department of Mathematics, CU Boulder

Gerrymandering refers to the practice of drawing legislative districts so that one political party wins a disproportionate number of seats relative to their share of the electorate. But how can we tell whether or not districts have been drawn fairly? This is a legal question and, increasingly, a mathematical one, but the mathematical tools used to measure gerrymandering are relatively new and are still evolving rapidly. One promising approach involves using computational and statistical tools to compare a specific districting plan to an "ensemble" consisting of a large number of potential districting plans. This approach, referred to as "outlier analysis," has the advantage of taking into account the inherent political geography of a region in a way that simpler measures cannot, and it has already begun to play a role in major court cases regarding redistricting in North Carolina, Pennsylvania, and elsewhere. In this talk I will describe how gerrymandering works and some of the mathematical tools that are being developed to detect it, with a focus on outlier analysis. I will also talk about an ongoing effort to collect data and perform this type of analysis for as many states as possible in advance of the next round of Congressional redistricting in 2021.

About the Speaker

Jeanne Clelland is a Professor in the Department of Mathematics at CU Boulder. She received her Ph.D. in 1996 from Duke University and works in differential geometry and the application ofgeometry to the study of partial differential equations. Professor Clelland is the author of the textbook, from Frenet to Cartan: The Method of Moving Frames. She is the 2018 winner of the Burton W. Jones Distinguished Teaching Award, from the Rocky Mountain Section of the Mathematical Association of America. Professor Clelland is also interested in applying geometry and mathematical techniques to questions of gerrymandering.

MORNING SESSION I

NUMERICAL REPRESENTATION OF SYMBOLIC DATASETS

Perrin Ruth
Advisor: Manuel Lladser
University of Colorado, Boulder

Metric dimension has been used to represent vertices of graphs as numerical vectors, which is then an acceptable input for numerical data analysis and machine learning algorithms. Hamming graphs, in particular, allow for symbolic data, e.g., genetic sequences, to be represented numerically since its nodes are strings. Unfortunately, Hamming graphs only allow for symbolic data of fixed length to be represented numerically. Here we present an extension of Hamming graphs to allow for strings of varying lengths to be studied at once. We do this by defining Levenshtein graphs from the well- known notion of edit distance. Some properties of Levenshtein graphs are revealed to assist in their future study. This research has been partially funded by the NSF grant No. 1836914.

ONE-SYNCH CGS2 ALGORITHM IN THE CONTEXT OF QR FACTORIZATION AND ARNOLDI PROCESS

Daniel R. Bielich
Advisor: Julien Langou
University of Colorado, Denver

The number of global reductions is an important metric for the parallel scalability of Krylov iterative methods. We focus on the Arnoldi-QR algorithm for nonsymmetric matrices. The underlying orthogonalization scheme is “left-looking” and “sees” columns one at a time. Thus, at least one global reduction is required per iteration. A stable method for orthogonalizing the Krylov vectors during the Arnoldi process is the classical Gram Schmidt algorithm with reorthogonalization (CGS2), requiring three reductions per step. A new variant of Arnoldi-CGS2 that requires only one reduce has been derived. Stability and strong-scaling results are presented for finding eigenvalue-pairs of a nonsymmetric matrix. A preliminary attempt to derive a similar algorithm (one reduction per Arnoldi iteration with a robust orthogonalization scheme) was presented by Hernandez et al. 2007 [1] but their method lacks numerically stability; while our new method, after extensive experiments, is much more stable and accurate. Our algorithm can also be implemented in the context of a QR factorization (as opposed to Arnoldi), and we explain the method in this context as well.

AN ITERATIVE METHOD FOR STRUCTURED MATRIX COMPLETION

Lara Kassab[1]
Advisor: Henry Adams[1]
Collaborator: Deanna Needell[2]
[1] ***Colorado State University***
[2] ***University of California, Los Angeles***

The task of filling-in or predicting missing entries of a matrix, from a subset of known entries, is known as *matrix completion*. In today’s data-driven world, data completion is essential whether it is the main goal or a pre-processing step. In recent work, a modification to the standard nuclear norm minimization for matrix completion has been made to take into account *structural differences* between observed and unobserved entries. One example of such structural difference is when the probability that an entry is observed or not depends mainly on the value of the entry. We propose adjusting an Iteratively Reweighted Least Squares (IRLS) algorithm for low-rank matrix completion to take into account *sparsity-based* structure in the missing entries. We also present an iterative gradient-projection-based implementation of the algorithm, and present numerical experiments showing that the proposed method often outperforms the IRLS algorithm in structured settings.

FAST BATCHED MATRIX ALGEBRA WITH INTEL®MATH KERNEL LIBRARY VECTORIZED COMPACT ROUTINES

Kirana Bergstrom
University of Colorado, Denver

Many HPC applications depend on matrix computations performed on large groups of very small matrices. For example, PDE-based simulations use a discretization over a large mesh, where mesh entities are represented as many small matrices. An iterative linear solver then performs a sequence of matrix operations over the grid. Intel®Math Kernel Library (Intel®MKL) 2018 provides *compact routines* that are optimized for such problems. These compact routines create true SIMD computations, in which subgroups of matrices are operated on with kernels that abstractly appear as scalar kernels while registers are filled by cross-matrix vectorization. Intel MKL compact routines provide significant performance benefits compared to standard batched techniques. We will describe the compact format, library implementation details, and performance benefits that can be achieved using these routines.

TRAVERSING THE TRANSPORTATION POLYTOPE

Stetson Zirkelbach
Advisor: Steffen Borgwardt
University of Colorado, Denver

In Linear Programming, optimal solutions occur at vertices of the polytope described by the constraints of the problem. Clustering problems can be expressed as transportation problems which have good linear programming formulations. A common occurrence is that clustering problems have multiple feasible clustering that can be expressed as separate objective functions on the same polytope. This talk explores the beginnings of traversing the simple cases of transportation polytopes between these equally optimal solutions with a separable clustering using a linear programming framework. In the talk we go through the challenges of the preserving the separable clustering for each step along the polytope in simpler cases. Then discuss the problems introduced by the construction of the polytope. Then we will show off some visualizations of the method in $\mathbb{R}^2$ along with some of the problems that can occur using the current method.

At the end, future work will be discussed focusing on how we get these problems closer to real world applications and the state of the current work, both with how the nice relaxations in the simplest case start to break down and how this works when you scale the system.

DERIVATIVE-FREE OPTIMIZATION AND ACTIVE SUBSPACES IN INVERSE PROBLEM THEORY

Jordan R. Hall
Advisor: Varis Carey
University of Colorado, Denver

We present numerical results from solving a Stochastic Inverse Problem (SIP) in a data-consistent fashion using novel optimization approaches involving Derivative-Free Optimization (DFO) and Active Subspaces. Since the objective functions used to solve SIPs typically involve a noisy model with a high-dimensional input space, gradient approximation becomes expensive; hence, we consider DFO methods. The high-dimensional input space may be reduced to resolve the model more cheaply, allowing for more efficient computations, such as optimization problems involving the model. We use a novel method to perform the optimization needed to solve a SIP which is numerically more efficient in certain cases. We perform Data-Consistent Inversion (DCI), which ensures that the solution to our SIP will only be updated in the directions uniformed by data; in the directions informed by data, the updated density (i.e., the solution to the SIP) will be a push-forward measure, in the sense that one may evaluate the updated density through the model and recover the observed density in data space. We present numerical results for a SIP posed involving a linear model with Gaussian initial and observed densities.

MORNING SESSION II

GEOMETRIC MEDIAN OF SUBSPACE

Nate Mankovich
Advisor: Michael Kirby
Collaborators: Chris Peterson & Emily King
Colorado State University

The median of a finite set of real numbers is the number which 'separates this set in half'. This concept has been generalized to a set of n-dimensional real vectors and is called the geometric median. In this talk we will explain how to calculate a geometric median of real numbers. We'll then use an analogous method to calculate the geometric median of a set of lines in three space. If there's time at the end, we'll discuss the generalization of this concept to the Grassmannian manifold.

FRACTALS, WHAT ARE THEY AND HOW CAN WE DRAW THEM?

Alyssa Newman
University of Colorado, Denver

Fractals are a Mathematical phenomenon most people have come across without realizing it. Understanding this phenomenon changes your view of the world. Fractals appear all around us all of the time, in clouds, plants, rocks, and so much more. Fractals are generated by taking a shape and transforming it in a way, you could shrink it, rotate it, squish it, move it or many other things, you do this transformation over and over again and the result is the fractal. In this talk, we will be discussing ways of creating these fractals, specifically two methods: the random dot iteration algorithm and iterated function systems. We will also look at a program I created in an independent study with Dr.Vaughn to allow anyone with any level of understanding of fractals to create their own fractals.

INVESTIGATING POVERTY TRAPS THROUGH THE LENS OF FRACTAL GEOMETRY AND DYNAMICAL SYSTEMS

Christopher Padgett
Advisor: Dr. Robert G. Niemeyer
Metropolitan State University of Denver

Wealth inequality is a major problem facing us today. The goal of this project is to identify patterns in city formation that are correlated with economic prosperity. We hypothesize that fractal dimension is an appropriate indicator of the aforementioned correlation. City formation is highly affected by policies, both past and current. We believe a change in historically discriminatory policies can impact economic conditions and fractal scaling will indicate how benefits will propagate.

PERSISTENT HOMOLOGY AND EXTENDING THE SIERPINSKI RELATIVES TO 3D

Joseph Geisz
Advisor: James Meiss
University of Colorado, Boulder

The Sierpinski Relatives are a group of 2D fractals often used as introductory examples to illustrate the construction of fractal sets and the concept of self-similarity. They are helpful for showing how to construct Iterated Function Systems which have fractal attracting sets. These and many other interesting dynamical systems have chaotic and/or fractal attractors which are difficult to analyze analytically. Persistent homology has proved to be a useful tool in exploring and describing such complex sets computationally.

In this talk we will begin by describing the Sierpinski Relatives and methods of constructing them. We will also give an introduction to persistent homology. We then define a new set of fractals that are analogous to the Sierpinski Relatives but are 3-Dimensional. We explore the properties of this new class such as their fractal dimensions, connectedness, and higher-dimensional homology.

A TORUS MODEL FOR OPTICAL FLOW

Johnathan Bush
Advisor: Henry Adams
Colorado State University

In *A naturalistic open source movie for optical flow evaluation*, Butler et al. create a database of ground-truth optical flow from the computer-generated video short *Sintel*. We study the high-contrast 3×3 patches from this video and provide evidence that this dataset is well-modeled by a torus. Our main tools are persistent homology and zigzag persistence, which are popular techniques from the field of computational topology. We show that the optical flow torus model is naturally equipped with the structure of a fiber bundle, which is furthermore related to the statistics of range images.

THE "BRUSSELATOR: "INTERNAL ENTROPY PRODUCTION" SURROUNDING A HOPF BIFURCATION

Gessner Soto

To borrow from the Wikipedia entry associated with the word "Brusselator", the word "Brusselator" is a melding of the word Brussels and oscillator. The Brussels component of the word is an homage to the Université libre de Bruxelles – or the "Free University of Brussels" as it seems to be often translated as – where both Dr. René Lefever and Dr. Ilya Prigogine were primarily based when they contributed to the general academic community this "Brusselator mechanism" in 1,967 . The oscillator component captures the particular relative curiosity that this Brusselator mechanism demonstrates is possible within physically "reasonable conditions": relative to the apparent historical predisposition to presume that "all that chemical reaction arrangements can do is converge onto a preferred distribution". This Brusselator mechanism presents a chemical reaction arrangement that can have a sustained oscillatory type of motion as its "least resistant" type of motion. The general distinction is that of between a convergence onto a stationary steady-state – associated with this presumed historical predisposition – and convergence onto a non-stationary steady-state

– of which this Brusselator arrangement one possible option. A “Hopf” type of “bifurcation” may be justifiably conceived of as the boundary between convergence onto a stationary steady-state and convergence onto a non-stationary steady-state. A limit-cycle may be justifiably presumed to be the most basic type of non-stationary steady-state. The base mathematical component of this Brusselator mechanism is specifically a collection of two coupled differential-equations that capture the presumed evolution of the presence of two transient atoms or molecules that actualize as the product of meditative steps in the contrived transformation of two chemical species from one arrangement of matter to another arrangement of matter. The Brusselator mechanism itself exists within the backdrop of an “open system”: a context that exchanges both “energy” and matter with its immediate external environment. The complete Brusselator mechanism is able to be reduced to two differential-equations with the presumption that the concentrations of the four chemical species that are not transient – the two that “go into” the system and the two that “exit” the system – be maintained essentially constant. The entropy component is one whose conception is grounded in “the physical” – in a system-scale type of analysis. It may justifiably be stated that the change in entropy is greater than zero when the “preferred distribution” of some arrangement has not actualized: the “separation” from the “equilibrium state” presumably both initiates and sustains internal motion within the context of this arbitrary arrangement “in an effort” to reconcile “an imbalance” relative to whatever it is that optimality references. Arrangements that exists within an “open” context are able to indefinitely be maintained away from their “preferred distribution”. It is presumably the case that multiple open systems can “come together” to mutually aid the other in their respective drive back to their respective “preferred distribution” within some macro-context that all of these open systems exist within. The “big” question underlying this project was – and is – why are limit-cycles common constituents of arrangements at all-scales in the “out-there”? .

MORNING SESSION III

AGGREGATION-DIFFUSION PHENOMENA IN DOMAINS WITH BOUNDARIES

Daniel Messenger[1]
Advisors: Ralf Wittenberg[2] **& Razvan C. Fetecau**[2]
[1] ***University of Colorado, Boulder***
[2] ***Simon Fraser University***

This talk relates to a class of mathematical models for the collective behaviour of autonomous agents, or particles, in general spatial domains, where particles exhibit pair-wise interactions and may be subject to environmental forces. Such models have been shown to exhibit non-trivial behaviour due to interactions with the boundary of the domain. More specifically, when there is a boundary, it has been observed that the swarm of particles readily evolves into unstable states. Given this behaviour, we investigate the regularizing effect of adding noise to the system in the form of Brownian motion at the particle level, which produces linear diffusion in the continuum limit. To investigate the effect of linear diffusion and interactions with the boundary on swarm equilibria, we analyze critical points of the associated energy functional, establishing conditions under which global minimizers exist. Through this process we uncover a new metastability phenomenon which necessitates the use of external forces to confine the swarm. We then introduce numerical methods for computing critical points of the energy, along with examples to motivate further research. Finally, we consider the short-time dynamics of the stochastic particle system as the noise strength approaches zero.

PATTERN FORMATION IN AN AMENITIES-BASED MODEL OF WEALTH CONCENTRATION

Lyndsey Wong
Advisor: Nancy Rodriguez
Collaborator: Ali Hassan
University of Colorado, Boulder

Gentrification refers to the influx of income into a community leading to the improvement of an area through renovation or the introduction of local amenities. This is usually accompanied by an increase in the cost of living, which displaces lower income

populations. Gentrification is a core issue that affects many urban areas, and the dynamics of such are not fully understood. To better understand this problem, we will introduce a model for the dynamics of wealth and amenities. In order to find when we have inhomogeneous solutions to this model, we present two approaches. The first is to perform a linear stability analysis in order to find when small perturbations of constant equilibrium solutions become unstable. The second is to prove the existence of a global bifurcation of these solutions from the constant equilibrium solution.

FINDING STEADY STATES OF NONLOCAL TERRITORIAL MODELS IN ECOLOGY

Erin Ellefsen
Advisor: Nancy Rodriguez-Bunn
University of Colorado, Boulder

There are certain populations of animals that tend to move in social groups. We investigate territory development of these social groups by studying a system of non-local continuum equations. However, these equations pose both analytical and computational problems. Therefore, we perform a long-wave approximation to investigate a local approximation to this system. We take advantage of the structure of these local and non-local systems in order to find steady state solutions. We compare the steady states of the local and non-local model to see if the local model is a good enough approximation as we move towards future projects.

IN SILICO MOUSE MODEL OF INFECTION AND IMMUNITY

Daniel Jonas
Collaborators: Michael Kirby, Alan Schenkel
Colorado State University

An organism's immune system tries to protect it by identifying the presence of pathogens and attempting to eliminate them. The defense is twofold: innate immune cells mobilize rapidly, while acquired immune cells slowly develop into pathogen-killing specialists. These responses incur collateral tissue damage, which anti-inflammatory mediators seek to control. This system of checks and balances is responsible for host survival. Experimental research has demonstrated how vastly complex these interactions are, indicating a place for theoretical and computational study. In this work we develop a comprehensive differential equation model of the immune system by considering interactions between immune system components in the presence of pathogen or tissue trauma. Through this step-by-step construction we explore the dependence of the anti-inflammatory mediators on pathogen levels, and also how they temper the immune response at the end of infection. We then challenge the "virtual mouse" with typical pathogens of varying virulence and observe the outcomes via model simulation. We find that anti-inflammation can downregulate the activation and proliferation of immune cells or promote apoptosis as cessation mechanisms, suggesting the need for in vivo experiments. Bifurcation theory describes how the outcomes of infection depend on model parameters, from which we conclude that initial insult and pathogen growth rate allow us to predict whether or not the in silico mouse overcomes the disease in a deterministic framework.

THE EFFECT OF SIMPLEX AND NETWORK DEGREE DISTRIBUTION ON EXPLOSIVE TRANSITIONS IN EPIDEMIC DYNAMICS USING SIMPLICIAL CONTAGION MODELS

Nicholas Landry
Advisor: Juan G. Restrepo
University of Colorado, Boulder

Incorporating higher order connections, i.e., simplices, into contagion and opinion modeling is valuable because the resulting complex network is closer to mimicking the true structure of real-world networks. We consider the SIS epidemic model on a network where contagion is mediated by both 1-simplices (pairwise interactions) and 2-simplices (three-way interactions). We examine the role that the correlation between the 1-simplex and the 2-simplex network has on the dynamics of contagion models. In particular, we generalize recent results that used an Erdös-Renyí network with simplices distributed independent of degree and found that the 2-simplex network can lead to hysteretic behavior. We find that increasing heterogeneity in the degree distribution of the 1-simplex layer suppresses explosive transitions. In addition, connecting the 2-simplex network with uniform probability and independent of the pairwise degree instead of correlated to the pairwise degree also suppresses these transitions. We verify these results with microscopic simulations using the configuration model with a power-law and uniform degree distribution. In addition, we compare these results with predictions made using a mean-field model.

GROUND ZERO ESTIMATION IN INFECTION NETWORKS

G. Kesler O'Connor
Advisor: Manuel E. Lladser
University of Colorado, Boulder

In this talk, we explore the problem of estimating "ground zero" on infection networks using a static set of observer nodes. Our model assumes infections spread over finite, undirected trees, with random propagation delays along edges. We present a concise expression for the joint characteristic function of the observed delays, that depends only on the structure of the graph, and the relative location of the observers with respect to the alleged source, and which may enable a maximum likelihood estimation approach for detecting ground zero. This research has been partially funded by the NSF grant No. 1836914.

AFTERNOON SESSION I

VARIANCE REDUCTION METHODS BASED ON MULTILEVEL MONTE CARLO

Lu Vy
Advisor: Yaning Liu
University of Colorado, Denver

If we could see into the future, then finance would be a lot easier. Unfortunately, we can't, so stock traders work with mathematicians. When a time machine isn't available, the next best option is a good mathematical model. While many excellent models exist to predict stock prices, their complexity often evades an analytic solution. When this is the case, simulation becomes the best alternative. What began in the Los Alamos Laboratories as Monte Carlo estimation evolved over the next 80 years to become something ubiquitous in financial mathematics. Today, Monte Carlo computational methods are so heavily used that pseudo-random numbers alone hardly suffice. Predicting the modern market requires efficiency, and to this end, a number of variance reduction techniques emerged. In this paper, I juxtapose two, and find that their combined effects are synergistic.

In 2004, Okten introduced a method of generating high quality random numbers. Somewhat paradoxically, he proposed re-randomizing the un-randomized Halton sequence. Doing so, he argued, allows us to keep the space-filling property of quasi-random sequences without the troublesome correlation between terms. In 2018, Giles published a paper that introduced the multi-level Monte Carlo (MLMC) algorithm. Instead of high-quality numbers, he sought efficiency in the structure of the algorithm. Monte Carlo algorithms naturally require averaging many estimates, but due to computational expenses, one must choose between averaging many poor estimates and averaging a few good estimates. Giles found a way to incorporate both, thus producing an even better estimate. Both methods work well by themselves, but no one has yet tried to combine them. That is the primary motivation behind my research.

PROPAGATION OF UNCERTAINTY IN HIGH-FIDELITY PLASMA SIMULATIONS USING SURROGATE MODELS GENERATED ON SPARSE GRIDS

Evan Shapiro

Predicting turbulence driven transport in future scale fusion reactors is critical to ensuring reactor stability during operation. We are interested in making predictions on the scaling of ion temperature gradient induced heat transport in an experimental tokamak fusion reactor using XGC1 5-D gyrokinetic simulation software. Specifically, we will scale the dimensionless radius and the ap- plied heating power in the simulation. Parameter drift during simulation, and uncertainty in the plasma heat deposition profile, require that the uncertainty be propogated through the simulation to fully characterize the quantity of interest, the net heat flux, in response to an uncertain parameter space. The XGC1 simulation suite that enables these predictions are computationally expensive, and the parameter space under consideration is of high enough dimension to make a blind uncertainty quantification computational experiment infeasible. Thus, we consider constructing a heat transport model-surrogate using sparse grid polynomial interpolation with data from a smaller set of simulation runs. This polynomial interpolant will allow to perform sensitivity analysis on a set of identifiable parameters.

MATHEMATICAL DISTRIBUTIONS FOR DARK MATTER DETECTION

Arvind Srinivasan
Advisors: Amy Roberts & Anthony Villano
University of Colorado, Denver

In the analysis of Cryogenic Dark Matter Search (CDMS) data, a quantity, yield, can be calculated from each collision based on measured quantities. These measured quantities are phonon energy and charge energy of each collision. Each have some variance due to sensor accuracy. The yield quantity is a ratio of a simple linear combinations of charge energy and phonon energy. This talk is about calculating and analyzing the distribution of the yield quantity given a model of the system in which both random variables are independent. We demonstrate that the distribution of the yield quantity is pathological under some circumstances – having undefined nth-degree moments for n>=1. Based on this analysis, I recommend that the dark matter research community move away from reporting yield and instead consider reporting their directly measured quantities.

GREEN COMMUNICATION AND SECURITY IN WIRELESS NETWORKS BASED ON MARKOV DECISION PROCESS AND SEMIVARIANCE OPTIMIZATION

Fateh Elsherif
University of Colorado, Denver

In this work, we use two optimization frameworks and use them to formulate three problems in wireless communication and networking, and develop computationally efficient algorithms to solve them. These two optimization frameworks are: 1) We use Markov decision process (MDP)—a framework to trade off immediate cost for expected long-term cost. 2) Semivariance optimization—a framework to trade off potential gains for possible losses. First, we study the problem of base station (BS) dynamic switching for energy efficient design of fifth generation (5G) cellular networks and beyond. We formulate this problem as a Markov decision process (MDP) and use an approximation method known as policy rollout to solve it. This method employs Monte Carlo sampling to approximate the Q-value. Simulation results show the potential of our novel approach of exploiting user mobility information within the MDP framework to achieve significant energy savings while providing quality-of-service guarantees. Second, we study the problem of wireless jamming-mitigation multipath routing. To address this problem, we propose a new framework for mitigating jamming risk based on semivariance optimization. Semivariance is a mathematical quantity used originally in finance and economics to measure the dispersion of a portfolio return below a risk-aversion benchmark. We map the problem of jamming-mitigation multipath routing to that of portfolio selection within the semivariance risk framework. Then we use this framework to design a new, and computationally feasible, RF-jamming mitigation algorithm. We use simulation to study the properties of our method and demonstrate its efficacy relative to a competing scheme that optimizes the jamming risk in terms of variance instead of semivariance. Directly optimizing objective functions that involve exact semivariance introduces certain computational issues. However, there are approximations to the semivariance that overcome these issues. Based on one of these solutions, we develop an efficient algorithm for solving semivariance optimization problems. Our algorithm provides a general approach to solving semivariance optimization problems, and can be used in other applications. Last, we consider the problem of multiple–radio-access technology (multi-RAT) connectivity in heterogeneous networks (HetNets). Multi-RAT is a potential method to increase throughput, to enhance communication reliability, and to minimize communication latency. We propose a new risk-averse multi-RAT connectivity (RAM) algorithm. Our RAM algorithm allows trading off expected throughput for risk measured in throughput semivariance. We formulate the multi-RAT connectivity problem as a semivariance-optimization problem. However, the objective function of the optimization problem considered here is different from the objective function of the optimization problem above that also uses semivariance to quantify risk (because the underlying standard form of portfolio selection is different). In addition, the set of constraints is different in this optimization problem: We introduce new capacity constraints to account for the stochastic capacity of the involved wireless links. We also introduce a new performance metric, the risk-adjusted throughput; risk-adjusted throughput is the ratio between the expected throughput and the throughput semideviation. We evaluate the performance of our algorithm through simulation of a system with three radio-access technologies: 4G LTE, 5G NR, and WiFi.

AFTERNOON SESSION II

ANALYSIS OF FLUID DYNAMICS USING THE FINITE VOLUME METHOD AS COMPARED TO OTHER METHODS

Keegan W$^{m.}$ Karbach
Metropolitan State University of Denver

The Finite Volume Method (FVM) is one of the most widely used and versatile numerical techniques used in computational fluid dynamics. In this method, a computational domain is discretized into a number of control volumes over whose faces the flux of the variable of interest can be calculated. The integral forms of these flux equations are converted to differential forms using Green's Theorem and these differential equations are numerically solved using difference methods. The power of this method lies in the fact that this domain discretization can take on any internal topology - coarse or fine, structured or unstructured, internal or external. In addition to this flexibility, any resulting solutions satisfy conservation laws for transport of mass, energy, and momentum due to nature of using flux between control volumes for computation.
In this talk, the mathematical basis for FVM will be briefly reviewed, the discretization of both the computational domain and the Navier-Stokes equations will be investigated, and the entire process will be illustrated with a simple example of a real-world system. Additionally, the implementation of FVM will be compared with the Finite Element Method and Spectral Methods.

INVESTIGATION OF ENVELOPE SOLITARY WAVES IN A VISCOUS FLUID CONDUIT

Yifeng Mao
Advisor: Mark Hoefer
University of Colorado, Boulder

Localized solitary waves arise in a wide range of nonlinear physical systems including shallow water waves, magnetic materials, and optical telecommunications in nonlinear fibers. An envelope solitary wave is a localized, coherent disturbance riding on top of a plane periodic wave. Recent theoretical work suggests their existence in the strongly nonlinear regime of a viscous fluid conduit. The conduit equation is an accurate model of viscous fluid conduit interfacial waves and has been studied since the 1980s. Fluid flow through a pipe-like conduit in viscously deformable media applies in many physical environments including the dynamics of magmatic and glacial systems. The conduit equation is non-integrable and envelope solitary wave solutions do not satisfy a traveling wave ordinary differential equation, leading to challenges in their study. This work presents approximate envelope solitary wave solutions by analyzing a higher order nonlinear Schödinger equation (HNLS) approximation of the conduit equation. The HNLS equation admits a one-parameter family of wave solutions. Additional solutions are sought using a numerical fixed point scheme.

BEHAVIOR OF N-LUMP RATIONAL SOLUTIONS TO THE KADOMTSEV-PETVIASHVILI (KP) I EQUATION

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A class of real, non-singular and rationally decaying solutions to the KP I equation will be considered in this talk. The dynamics of these so-called "lump" solutions is atypical of solitons and will be discussed in detail. Asymptotic methods for estimating the height and location of these peaks is given.