Dynamic Models, New Gains from Trade?*

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Abstract

Yes. We state closed-form expressions for steady state gains from trade that apply in a class of dynamic trade models that includes dynamic versions of the Krugman (1980), Melitz (2003), and customer capital (e.g., Arkolakis, 2010) models. The gains are a function of the domestic trade share and the long-run elasticity of trade with respect to iceberg trade costs, similar to Arkolakis, Costinot, and Rodríguez-Clare (2012). In contrast to static settings, in a dynamic world this long-run elasticity cannot be estimated in one step by relying on tariff variation as shifters of trade costs. We show, instead, that this object can be recovered by combining two tariff elasticity estimates: the long- and the short-run. Thus, the short-run tariff elasticity indirectly enters the formula for the steady state gains from trade. Our main substantive finding is that the gains from trade are large. They depend crucially on the short-run tariff elasticity, and can be arbitrarily large even if the long-run tariff elasticity is high. Accounting for the transition path has a minor impact on the magnitude of the gains from trade, relative to simply comparing steady states.

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1. Introduction

Dynamic trade models have a tradition going back to at least the 1960s (e.g. Bardhan, 1965, 1966; Oniki and Uzawa, 1965; Inada, 1968; Stiglitz, 1970). While this line of research has been a continuous presence in the trade literature, the last decade has seen a veritable explosion of work employing dynamic quantitative trade models. While modeling and quantification have flourished, there are few analytical characterizations of the gains from trade in dynamic environments. In particular, we currently lack compact and intuitive gains from trade formulas in the spirit of Arkolakis, Costinot, and Rodríguez-Clare (2012, henceforth ACR) for dynamic economies.

This paper makes three contributions. Our theoretical contribution is to state ACR-like closed-form expressions for the gains from trade (GFT) that apply in dynamic models in steady state. Our measurement contribution is to show how empirical elasticity estimates at multiple time horizons can be used to recover the structural parameters required to calculate the GFT in a dynamic setting. The quantification contribution computes the resulting GFT, highlights the importance of the short-run trade elasticities, and compares the steady state gains implied by the formula to the gains from trade that explicitly account for the transition path between the trade regimes.

To illustrate the model features important for the results, we start with a simple dynamic Krugman (1980) model. There are multiple countries and firms. Firms face downward-sloping demand in destination markets and are monopolistically competitive. Within a period, they earn positive flow profits. In order to enter a destination market, a firm has to pay a stochastic sunk cost. A firm enters a destination market if the net present value of its expected profits from selling there cover the sunk costs of entry. This feature introduces forward-looking behavior and gradual adjustment to shocks. Following a trade cost shock, two forces will act on the welfare of the domestic agents: the gain from imported varieties, captured by the domestic trade share as in ACR; and the loss in domestic varieties. It turns out that under the inverse Pareto distributional assumption on the sunk costs, the loss of domestic varieties is a power function of the domestic trade share. Thus, the domestic trade share is a sufficient statistic for the welfare change, modulo the relevant elasticity. This elasticity is a function of the Dixit-Stiglitz substitution elasticity between firms, and the curvature of the sunk cost distribution. Intuitively, this curvature regulates how strongly domestic variety responds to foreign competition.

We then state a general set of conditions under which the closed-form expression for the gains from trade applies. The first two conditions coincide with ACR: trade is balanced; and the ratio of aggregate profits to aggregate sales is constant. The third condition puts structure on supply and demand. Total bilateral exports can without loss of generality be written as a product of sales per unit mass of firms and the mass of firms. The result requires that (i) domestic demand per unit mass of firms is CES (closely related to ACR's third assumption), and (ii) the mass of firms is a power function of sales per firm normalized by the source country wage. The assumption (ii) is an additional restriction required in a dynamic environment. While it is sensible that the mass of firms would be an increasing function of per-firm sales relative to factor cost, the power functional form of this relationship is a non-trivial restriction.

Under these conditions, we show that the ratio of steady state real consumption levels under trade relative to autarky is given by

$$\left(\lambda_{jj}\right)^{\frac{1}{(1+\chi)\varepsilon_{\kappa}^{0}}},$$

where λ_{jj} is the share of domestically-produced goods in total spending, ε_{κ}^{0} is the elasticity of the CES demand per unit mass to unit costs, and χ is the exponent governing the relationship between the mass of firms and per-firm sales. Importantly, $(1+\chi)\varepsilon_{\kappa}^{0}$ is also the *long-run* elasticity of trade with respect to the iceberg trade costs.

This formula is essentially ACR. Our theoretical contribution is to derive it in a general dynamic environment. In the process we show that in a dynamic setting the long-run trade elasticity is governed by different structural parameters than in static settings. Most importantly, the curvature of the response of entry to destination-specific sales, χ, does not appear in the ACR formulas. In the dynamic Krugman model, ε_{κ}^{0} is simply $1-\sigma$, where σ is the Dixit-Stiglitz substitution elasticity. We next show that the conditions of the proposition are satisfied by two additional important dynamic models: the customer base model à la Arkolakis (2010) with the cost of acquiring customers taking a power form; and the Melitz (2003) model with Pareto productivity and inverse Pareto sunk cost distributions.

Next, we turn to measurement. While the domestic trade shares are fairly straightforward to obtain, the long-run elasticity of trade with respect to iceberg costs is harder to pin down, because we do not normally observe iceberg trade costs. Instead, the predominant approach in the literature is to use tariff variation, as tariffs are often the only *ad valorem* component of trade costs that is relatively easily observed.[1]

The distinction between iceberg trade costs and tariffs is innocuous in static settings, as the iceberg elasticity can be easily recovered from the tariff elasticity. It is no longer innocuous in dynamic environments. To make this explicit, we state a generalization of the main proposition to an environment with both iceberg costs and tariffs. The GFT formula still requires the long-run elasticity of trade to iceberg costs. However, in a dynamic world the long-run elasticity of trade with respect to iceberg trade costs cannot be recovered from the long-run elasticity with respect to tariffs alone. In addition, the formula now features an adjustment for tariff revenue. This type of adjustment was derived in a static setting by Felbermayr, Jung, and Larch (2015). We show that in a dynamic setting, computing this adjustment requires not only the long-run trade elasticity, but also knowledge of ε_{κ}^{0} and χ individually.

To summarize, implementing the dynamic gains from trade formula faces two hurdles: (i) the long-run iceberg trade cost elasticity cannot be directly computed from the long-run tariff elasticity, and tariff elasticities are often the only reliable empirical estimates available; and (ii) implementing

[1] At least 20 papers have used tariff variation to estimate the trade elasticity in the past 25 years. See Head and Mayer (2014) and Boehm, Levchenko, and Pandalai-Nayar (2023) for bibliographies.

the tariff adjustment requires knowing not just the long-run iceberg elasticity, but ε_{κ}^{0} and χ separately. We propose a solution: these two parameters can be inferred from two tariff elasticities at different time horizons: the short- and the long-run. Intuitively, the short-run tariff elasticity is a function of ε_{κ}^{0}, while the long-run tariff elasticity is a function of both ε_{κ}^{0} and χ. Thus, with two empirical estimates – the short- and the long-run – one can recover both deep parameters.

Finally, with this approach in hand, we turn to quantification. We perform three exercises. First, we report the dynamic gains from trade for a large set of countries according to our formula and accounting for tariffs. Our preferred short– and long-run tariff elasticity estimates are taken from Boehm, Levchenko, and Pandalai-Nayar (2023). The gains from trade are large, with gains of 25-30% for even the largest countries such as the US and Brazil, and gains of over 100% for several countries. At the same time, the tariff revenue adjustment plays a small role in all but a handful of economies. Second, we highlight the role of the short-run trade elasticity by setting the long-run tariff elasticity at a conventional high value of −5, but varying the short-run elasticity. It turns out that conditional on a fixed long-run elasticity, the short-run elasticity is decisive for the overall gains from trade. In fact, in the limit as the short-run elasticity goes to -1, the gains from trade become infinite even with a high long-run elasticity. Most available estimates of the short-run elasticity are low (Fitzgerald and Haller, 2018; Boehm, Levchenko, and Pandalai-Nayar, 2023; Auer, Burstein, and Lein, 2021), suggesting that gains from trade are likely quite large, regardless of the long-run elasticity.

Third, we compare the gains implied by the formula with the gains from trade that also account for the transition path from one trade regime to another. The length of the transition path – and therefore its quantitative importance for welfare – is disciplined by the amount of time it takes the trade elasticity to converge to the long-run value. Boehm, Levchenko, and Pandalai-Nayar (2023) find that the transition takes 7-10 years. While steady state comparisons are unambiguous, the dynamic path of consumption will differ depending on whether the world is transitioning from autarky to trade, or from trade to autarky. Thus, we simulate both scenarios.

There are two main findings. First, the disparity between steady state formula-implied gains and the full welfare change over the transition path is relatively minor. Second, the steady state formula overstates the dynamic gains of moving from autarky to trade, but understates the gains from staying open to trade compared to the dynamic path of moving from trade to autarky. The intuition is as follows. When moving from autarky to trade, we start with the autarky steady state, and transition to the trade steady state slowly. Over this transition, consumption is lower than eventual steady state consumption, because agents need to invest in setting up exporting firms, and doing so requires forgoing consumption over the transition path. As a result, the dynamic gains of going from autarky to trade are below the steady state comparison. Moving from trade to autarky, countries' accumulated exporting capital has become useless, but they need to increase the mass of domestic firms to replace imports. Thus, when shocked with an unanticipated increase in trade costs, countries also decrease consumption below the eventual autarky steady state, as they accumulate domestic firms. This reduces the value of the consumption path towards autarky – effectively the denominator of the GFT – relative to steady state, and thus raises the implied GFT.

Literature. While the field of international trade has always been interested in the gains from trade, the literature on the quantification of GFT was given fresh impetus by the landmark contribution of Arkolakis, Costinot, and Rodríguez-Clare (2012), who stated closed-form expressions for the GFT in a wide class of static trade models.[2] This led to an active literature exploring various analytical and quantitative properties of the sufficient statistics formulas, such as sectoral comparative advantage (Costinot and Rodríguez-Clare, 2014; Levchenko and Zhang, 2014) or trade elasticities (Ossa, 2015; Imbs and Mejean, 2017). The formulas have also been extended in a variety of directions, such as variable markups (Arkolakis et al., 2019), non-constant trade elasticities (Melitz and Redding, 2015; Feenstra, 2018; Adão, Arkolakis, and Ganapati, 2020), gains from multinational production (Ramondo and Rodríguez-Clare, 2013), non-representative agent settings (Galle, Rodríguez-Clare, and Yi, 2023), and accounting for tariff revenue (Felbermayr, Jung, and Larch, 2015; Lashkaripour, 2021), to name a few. In static settings, Melitz and Redding (2015) and Feenstra and Weinstein (2017) highlight that allowing for changes in the mass of (potential) firms leads to welfare gains that differ from the ACR formula, implying that the GFT can then be sensitive to microfoundations. In our dynamic trade setting the mass of firms also changes, contributing to the gains from trade. Aside from the fact that ours is a dynamic setting, our contributions relative to these papers are to (i) analytically characterize the mapping between the mass of firms and the domestic trade share, yielding ACR-like GFT welfare formulas that account for endogenous mass adjustment; and (ii) establish this mapping in a class of models that covers multiple microfoundations.

The literature on analytical GFT characterizations in dynamic environments is more limited. Arkolakis, Eaton, and Kortum (2011) and Chen et al. (2024) develop results for a dynamic version of the Eaton-Kortum model, and Atkeson and Burstein (2010) and Alessandria, Choi, and Ruhl (2021) for a dynamic heterogeneous firm model. On the quantitative side, a number of papers compute gains from trade numerically in dynamic models, including accounting for the transition path (see, among others, Alvarez, 2017; Brooks and Pujolas, 2018; Mutreja, Ravikumar, and Sposi, 2018; Ravikumar, Santacreu, and Sposi, 2019, 2024; Anderson, Larch, and Yotov, 2020). We provide a more general characterization that applies to steady state comparisons in a broad class of dynamic trade models. Unlike the Eaton-Kortum setting, our analytical results cover cases in which there is net firm entry and profits. We also emphasize the importance of measurement, in particular the information contained in trade elasticities at multiple horizons in conditioning the steady state gains from trade.

The rest of the paper is organized as follows. Section 2 fully lays out the simplest dynamic model to illustrate the mechanics behind the result. Section 3 states the general proposition and establishes the mappings to other dynamic models. Section 4 quantifies the gains from trade. Section 5 concludes.

[2] Antecedents that stated similar formulas in specific settings include Eaton and Kortum (2002) for the Ricardian model, Eaton and Kortum (2005) for the Armington model, and Arkolakis et al. (2008) for the Melitz model.

2. Warmup: Welfare Gains in a Dynamic Krugman Model

This section derives the gains from trade formula in the simplest possible setup: a dynamic version of the Krugman (1980) model. It serves to introduce the notation maintained throughout the paper, and to demonstrate what features are essential for the result to go through.

2.1 Model Setup

Consider a dynamic economy with J countries indexed by i and j, and discrete time indexed by t. Each country is populated by a representative consumer who consumes C_{jt} and inelastically supplies labor L_j.

Households. Consumers in country j maximize

$$\max_{\{C_{jt}\}} \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\gamma}}{1-\gamma}$$

subject to the budget constraint

$$P_{jt}C_{jt} + \frac{B_{jt}}{1+r_{jt}^n} = w_{jt}L_j + \Pi_{jt} + R_{jt}^g + B_{jt-1}, \tag{2.1}$$

and a no-Ponzi game condition. Here, P_{jt} is the consumption price index in country j, B_{jt} are bond holdings, r_{jt}^n is the nominal interest rate, w_{jt} the nominal wage, Π_{jt} aggregate profits, and R_{jt}^g are government tariff revenues rebated to the household. The parameters β and γ denote the household's discount factor and the coefficient of relative risk aversion, respectively. We assume that firms producing in country j are exclusively owned by the consumer in j, and hence the consumer receives all profits as income.[3]

Optimal behavior implies that consumption follows the Euler equation

$$\left(1 + r_{jt}\right) \beta C_{jt+1}^{-\gamma} = C_{jt}^{-\gamma},$$

where $1 + r_{jt} = \left(1 + r_{jt}^n\right) \frac{P_{jt}}{P_{jt+1}}$ is the real interest rate in country j.

The consumption bundle C_{jt} is a CES aggregate of quantities $q_{ijt}(\omega)$ supplied by firms indexed by ω, from all countries i serving market j:

$$C_{jt} = \left(\sum_i \int_{\Omega_{ijt}} q_{ijt}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}.$$

Ω_{ijt} denotes the endogenous set of varieties produced in country i and available for purchase in

[3] All the results go through if we instead assume that the home consumers receive a constant fraction of aggregate profits.

country j and $\sigma > 1$ is the demand elasticity. Demand for each variety ω and the ideal price index satisfy:

$$q_{ijt}(\omega) = C_{jt} \left(\frac{p^c_{ijt}(\omega)}{P_{jt}} \right)^{-\sigma}, \tag{2.2}$$

$$P_{jt} = \left(\sum_i \int_{\Omega_{ijt}} \left(p^c_{ijt}(\omega) \right)^{1-\sigma} d\omega \right)^{\frac{1}{1-\sigma}},$$

where $p^c_{ijt}(\omega)$ is the price faced by the consumer in country j.

Firms. Firms are monopolistically competitive, face the downward-sloping demand curve given by (2.2), and take the ideal price index as given. The production function is linear in labor. Shipments from country i to j are subject to iceberg transport costs κ_{ijt}, so that

$$q_{ijt}(\omega) = \frac{1}{\kappa_{ijt}} l_{ijt}(\omega),$$

where $l_{ijt}(\omega)$ is the firm's labor input for producing for market j. The marginal cost of serving market j is therefore $\kappa_{ijt} w_{it}$. Profit-maximizing firms charge a constant markup over marginal cost:

$$p^x_{ijt}(\omega) = \frac{\sigma}{\sigma - 1} \kappa_{ijt} w_{it},$$

where $p^x_{ijt}(\omega)$ is the price received by the exporter. As a result, per-period profits are a constant fraction of firm revenue:

$$\pi_{ijt}(\omega) = \frac{1}{\sigma} p^x_{ijt}(\omega) q_{ijt}(\omega) = \frac{1}{\sigma} x_{ijt}(\omega). \tag{2.3}$$

Entry. Every period there is a unit mass of potential firms that can enter market j from i. Entry is subject to a stochastic sunk cost of $\xi^s_{ijt}(\omega)$ units of country i's labor. A firm ω from i that pays the sunk costs in period t sells to j from $t+1$ until it exits. Exit is random and occurs with probability δ. The value of exporting is therefore

$$v_{ijt}(\omega) = \frac{1}{1 + r^n_{it}} \left(\pi_{ijt+1}(\omega) + (1 - \delta) v_{ijt+1}(\omega) \right). \tag{2.4}$$

A potential entrant enters if the value of exporting exceeds the sunk cost of entry. The marginal firm's sunk costs $\bar{\xi}^s_{ijt}$ satisfy

$$v_{ijt}(\omega) = w_i \bar{\xi}^s_{ijt}(\omega). \tag{2.5}$$

Denote by n_{ijt} the mass of exporters from i to j. Its law of motion is

$$n_{ijt} = (1 - \delta) n_{ijt-1} + G\left(\bar{\xi}^s_{ijt-1} \right),$$

where G denotes the cumulative distribution function of ξ^s_{ijt}.

Tariffs, Aggregation, and Market Clearing. Let τ_{ijt} denote gross *ad valorem* tariffs.[4] Then the prices paid by the consumers and prices received by the exporters satisfy $p^c_{ijt}(\omega) = \tau_{ijt} p^x_{ijt}(\omega)$, and the government collects $(\tau_{ijt} - 1) p^x_{ijt}(\omega)$ revenue per unit sold.

Total exports from i to j non-inclusive of tariff payments are:

$$X_{ijt} = \int_{\Omega_{ijt}} x_{ijt}(\omega)\, d\omega = n_{ijt} x_{ijt}. \tag{2.6}$$

The tariff revenue of government j is

$$R^g_{jt} = \sum_i \left(\tau_{ijt} - 1\right) X_{ijt}.$$

Profits in country j are

$$\Pi_{jt} = \sum_i \int_{\Omega_{jit}} \pi_{jit}(\omega)\, d\omega - \sum_i \int_{\Omega^e_{jit}} w_{jt} \xi^s_{jit}(\omega)\, d\omega. \tag{2.7}$$

where $\Omega^e_{jit} = \left\{\omega \in [0,1] : \bar{\xi}^s_{jit} \geq \xi^s_{jit}(\omega)\right\}$ is the set of entrants. Trade is balanced, so that in all countries j and periods t

$$w_{jt} L_j + \Pi_{jt} + R^g_{jt} = \sum_{i=1}^{n} X_{jit}. \tag{2.8}$$

Trade balance trivially implies that all bond positions are zero: $B_{jt} = 0$. We include the bond in the households' optimization problem only to pin down the interest rates.

2.2 Steady State Welfare Gains from Trade

In this subsection, we abstract from tariff revenues: $\tau_{ijt} = 1$ for all i and j, implying that $R^g_{jt} = 0$. Since all operating firms in the model have identical quantities and prices, we will drop the firm subscript ω going forward. The steady state objects are denoted by suppressing the time subscripts. From the budget constraint (2.1), real consumption is:

$$C_j = \frac{w_j L_j + \Pi_j}{P_j}. \tag{2.9}$$

We will denote the gross proportional gains from trade as the ratio of real consumption under the current trade regime relative to autarky:

$$GFT = \frac{C_j}{C^{AUT}_j}.$$

[4] In this notation, a 5% *ad valorem* tariff implies $\tau_{ijt} = 1.05$.

In the tradition following Eaton and Kortum (2002) and Arkolakis, Costinot, and Rodríguez-Clare (2012), we seek to express (2.9) as a function of the domestic trade share and exogenous parameters. We start with the standard step that the domestic trade share is:

$$\lambda_{jj} \equiv \frac{n_{jj} x_{jj}}{Y_j} = \frac{n_{jj} \left(\frac{\sigma}{\sigma-1} w_j\right)^{1-\sigma}}{P_j^{1-\sigma}}, \tag{2.10}$$

where $Y_j \equiv P_j C_j$ is total expenditure. Solving this expression for the price index and combining the result with equation (2.9) implies that real consumption satisfies:

$$C_j \propto \frac{w_j L_j + \Pi_j}{w_j \lambda_{jj}^{-\frac{1}{1-\sigma}} n_{jj}^{\frac{1}{1-\sigma}}}. \tag{2.11}$$

From here, we proceed to show that (i) aggregate profits are a constant fraction of the labor income; and that (ii) the mass of domestic firms n_{jj} is a power function of λ_{jj}. To compute profits and the mass of entrants, we must make a distributional assumption on the sunk costs of entry. We assume that the sunk costs are drawn from an inverse Pareto distribution:

$$G\left(\xi^s\right) = \left(b\xi^s\right)^{\chi}, \tag{2.12}$$

where $\chi > 0$ is the Pareto dispersion parameter and $b > 0$ is the location parameter, that defines over the domain of this distribution: $0 < \xi^s \leq \frac{1}{b}$. We assume throughout that b is sufficiently small to ensure that not all potential entrants find it worthwhile to enter in any given period ($\bar{\xi}^s_{ijt} < \frac{1}{b}$ for all t). Under this assumption the steady state mass of firms becomes

$$n_{ji} = \frac{1}{\delta}\left(b\bar{\xi}^s_{ji}\right)^{\chi}. \tag{2.13}$$

Since $1 + r_j = 1/\beta$ in the steady state, the value of selling to i is:

$$v_{ji} = \frac{\beta}{1-\beta\left(1-\delta\right)}\pi_{ji} = \frac{\beta}{1-\beta\left(1-\delta\right)}\frac{1}{\sigma}x_{ji},$$

and the threshold sunk cost of entry is:

$$\bar{\xi}^s_{ji} = \frac{\beta}{1-\beta\left(1-\delta\right)}\frac{1}{\sigma}\frac{x_{ji}}{w_j}. \tag{2.14}$$

Equations (2.13) and (2.14) imply that the mass of firms is a power function of per-unit sales normalized by the cost of production:

$$n_{ji} \propto \left(\frac{x_{ji}}{w_j}\right)^{\chi}. \tag{2.15}$$

Combining (2.7), (2.8), (2.13), and (2.14) leads to the desired result that total profits are a constant multiple of labor income:

$$\Pi_j = \frac{\frac{1}{\sigma}\left(1 - \frac{\chi}{\chi+1}\frac{\beta}{1-\beta(1-\delta)}\delta\right)}{1-\frac{1}{\sigma}\left(1 - \frac{\chi}{\chi+1}\frac{\beta}{1-\beta(1-\delta)}\delta\right)} w_j L_j. \tag{2.16}$$

Finally, starting with the expression for steady state n_{jj} in (2.13), and combining it with (2.14), (2.16), and the expression for domestic sales x_{jj} leads to:

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}}. \tag{2.17}$$

Combining (2.11) with (2.16) and (2.17) yields the result that real consumption is proportional to the domestic trade share:

$$C_j \propto \lambda_{jj}^{\frac{1}{1-\sigma}} n_{jj}^{-\frac{1}{1-\sigma}} = \lambda_{jj}^{\frac{1}{(1-\sigma)(1+\chi)}}. \tag{2.18}$$

Since in autarky $\lambda_{jj} = 1$, (2.18) is also the gains from trade.

Note the difference with the ACR formula for the static Krugman model, $\lambda_{jj}^{\frac{1}{1-\sigma}}$, which would obtain in a setting in which n_{jj} is either exogenously fixed or constant across equilibria. Compared to the classic case and holding σ fixed, the gains from trade are moderated because international trade leads to the reduction in domestic varieties. The log change in real consumption following a change in the domestic trade share can be written as:

$$\begin{aligned} d\ln C_j &= \frac{1}{1-\sigma} d\ln\lambda_{jj} - \frac{1}{1-\sigma}\frac{d\ln n_{jj}}{d\ln\lambda_{jj}} d\ln\lambda_{jj} \\ &= \underbrace{\frac{1}{1-\sigma} d\ln\lambda_{jj}}_{\text{Gain from foreign varieties}} \quad \underbrace{-\frac{1}{1-\sigma}\frac{\chi}{1+\chi} d\ln\lambda_{jj}}_{\text{Loss of domestic varieties}}. \end{aligned} \tag{2.19}$$

The first term is the usual direct effect of the change in the interior trade share, interpreted as the utility gains from the availability of foreign goods. It increases with trade openness (recall that an increase in trade openness is a fall in λ_{jj}). The second term is the utility reduction from the loss of domestic varieties, as an increase in trade openness unambiguously lowers n_{jj}. It contributes negatively to the gains from trade. In this case, however, the net gain from openness is positive.

Two further points are worth noting. First, the loss of domestic varieties was modeled and quantified by Melitz and Redding (2015) and Feenstra and Weinstein (2017) in specific static models. We build on these contributions by deriving a parsimonious functional form (2.17) that relates domestic variety to the domestic trade share, which in turn leads to the closed-form GFT expression (2.18) requiring only data on λ_{jj}. As we show below, this property extends to several alternative microfoundations, implying these models admit the same analytical GFT formula. Second, (2.19) together with (2.15) highlight the role of the Pareto dispersion parameter χ. As evident from (2.15), χ is the elasticity of the mass of domestic varieties to the domestic profit opportunities. When χ is high, domestic variety is very sensitive to the profit opportunities, and so the fall in profits due to import competition leads to a large fall in domestic variety, and a large second term in (2.19). When χ is low, the opposite is true: import competition does not move domestic variety much, and thus the second term in (2.19) is smaller.

The long-run trade elasticity. An important reason behind the appeal of the ACR result is that the exponent on the domestic trade share is the inverse of the trade elasticity. We now show that the dynamic GFT formula shares this feature. Recall that bilateral trade flows are given by (2.6). The long-run trade elasticity with respect to iceberg trade costs therefore has the following components:

$$\frac{\partial \ln X_{ij}}{\partial \ln \kappa_{ij}} = \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} + \frac{\partial \ln n_{ij}}{\partial \ln \kappa_{ij}}$$

It is immediate that the $\frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} = 1 - \sigma$, as usual. From (2.13) and (2.14), $\frac{\partial \ln n_{ij}}{\partial \ln \kappa_{ij}} = \chi(1 - \sigma)$. Together, the long-run trade elasticity is

$$\frac{\partial \ln X_{ij}}{\partial \ln \kappa_{ij}} = (1 - \sigma)(1 + \chi),$$

and thus the gains from trade formula (2.18) features the inverse of the trade elasticity. Note that as in ACR and everywhere else in the literature, this is a partial elasticity, that ignores the general-equilibrium changes in expenditures, wages, and prices.

3. General Result

We now state the set of conditions under which the dynamic GFT formula applies.

Proposition 3.1. *Consider a class of dynamic models that satisfy the following three conditions in their steady state:*

A.1 For all countries j, trade is balanced (expenditure = revenue):

$$Y_j = w_j L_j + \Pi_j,$$

where $Y_j = C_j P_j$.

A.2 For all countries j, profits are a constant share of GDP:

$$\frac{\Pi_j}{Y_j} = const$$

A.3 For all country pairs (i, j) trade flows satisfy

$$X_{ij} = n_{ij} x_{ij} \tag{3.1}$$

where

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^{\chi} \tag{3.2}$$

and domestic per-unit-mass sales satisfy

$$x_{jj} \propto Y_j \left(\frac{w_j}{P_j}\right)^{\varepsilon_\kappa^0} \tag{3.3}$$

for some constant $\chi > 0$ *and where* $\varepsilon_\kappa^0 \equiv \partial \ln x_{ij} / \partial \ln \kappa_{ij} < 0$ *is the elasticity of exports per unit mass with respect to iceberg trade costs.*

Then

$$C_j \propto (\lambda_{jj})^{\frac{1}{\varepsilon_\kappa^0 (1+\chi)}} \tag{3.4}$$

where $\lambda_{jj} = \frac{X_{jj}}{Y_j}$, *and* $\varepsilon_\kappa^0(1+\chi)$ *is the long-run elasticity of trade flows with respect to iceberg trade costs.*

Proof. See Appendix A. □

Note that since $\lambda_{jj} = 1$ in autarky, (3.4) is also the gross proportional gains from trade in steady state. Assumptions A.1 and A.2 are identical to R1 and R2 in ACR. Assumption A.3 puts structure on supply and demand. Condition (3.1) stipulates that total exports from i to j can be written as a product of some generic mass n_{ij} and sales per unit mass of sellers x_{ij}. This in and of itself is without loss of generality, as we can in principle always express total exports as some (average) sales per firm/variety/HS code/etc. times the total number/mass of those units. The rest of A.3 puts structure x_{ij} and n_{ij}. Condition (3.3) states that domestic demand per unit mass is CES. It essentially corresponds to ACR's R3. Note that the proposition is stated in terms of the functional form for domestic sales. This is done to cover a greater range of models. In some models, such as Krugman and customer capital, international trade flows x_{ij} take the same form, modulo iceberg costs κ_{ij}. In the Melitz model, domestic sales per unit mass satisfy (3.3), while export sales contain additional terms, as will become clear below.

Finally, condition (3.2) is an additional restriction required in a dynamic environment. It puts a specific structure on how entry occurs. Qualitatively, it is intuitive: entry increases in the ratio of sales to unit costs. Section 2, in particular equations (2.3) and (2.5), illustrate how this can arise: the value of exporting scales with per period profits, which are in turn proportional to sales (numerator). Sunk costs are paid in terms of domestic labor (denominator). However, the proposition requires more than an increasing relationship: it requires that entry is a power function of this ratio. This places a restriction on the nature of the entry decision. Section 2 shows that the inverse Pareto distribution of sunk costs satisfies this restriction.

3.1 Mapping from specific models

Section 2 shows that the dynamic Krugman model satisfies the conditions of Proposition 3.1. In that model, $\varepsilon_{\kappa}^{0} = 1-\sigma$. We now go through two additional commonly used dynamic models: the customer base model and the Melitz-Pareto model.

Customer base model. In the customer base model (e.g. Arkolakis, 2010; Drozd and Nosal, 2012; Gourio and Rudanko, 2014; Fitzgerald, Haller, and Yedid-Levi, 2023), firms gradually build up the mass of customers they serve. Let there be a country i representative firm that faces downward-sloping demand (2.2) per unit mass of customers in country j. As above, its profits per unit mass of customers are given by (2.3). Let n_{ijt} be the mass of customers that the firm serves. This mass depreciates at rate δ and can be built up by investment a_{ijt}, that acts with a one-period lag. Thus, the customer mass evolves according to:

$$n_{ijt} = (1-\delta)\, n_{ijt-1} + a_{ijt-1}. \tag{3.5}$$

Investment has a cost $f(a_{ijt})$. The firm chooses the path of customer base investment to maximize the present value of profits:

$$\max_{\{a_{ijt+s}\}} \sum_{s=0}^{\infty} m_{it,t+s}^{n} \left[n_{ijt+s}\pi_{ijt+s} - w_{it} f\left(a_{ijt+s}\right) \right] \tag{3.6}$$

subject to (3.5), where $m_{it,t+s}^{n}$ is the firm's discount factor. The first-order conditions of this problem can be manipulated to yield:

$$w_{it} f'\left(a_{ijt}\right) = v_{ijt} \tag{3.7}$$

$$v_{ijt} = \frac{1}{1+r_{it}^{n}} \left(\pi_{ijt+1} + (1-\delta)\, v_{ijt+1}\right), \tag{3.8}$$

where we assumed that the discount factor of the firm coincides with that of the representative consumer. Let the cost of accessing customers be given by the following functional form:

$$f\left(a_{ijt}\right) = \frac{\chi}{(1+\chi)\,\zeta} \left(a_{ijt}\right)^{\frac{1}{\chi}+1}. \tag{3.9}$$

Then, in steady state:

$$n_{ij} = \frac{1}{\delta} a_{ij} = \left(\zeta \frac{v_{ij}}{w_i}\right)^{\chi}. \tag{3.10}$$

In turn, combining (2.3) and (3.8) yields the proportionality of v_{ij} to x_{ij}, verifying assumption A.3 in Proposition 3.1.

To see that Assumption A.2 is satisfied, note that aggregate profits can be written as:

$$\Pi_i = \sum_j \left(\frac{1}{\sigma} n_{ij} x_{ij} - w_i \frac{\chi}{(1+\chi)\,\zeta} \left(a_{ij}\right)^{\frac{1}{\chi}+1} \right). \tag{3.11}$$

Since a_{ij} is proportional to n_{ij}, and $a_{ij}^{\frac{1}{\chi}}$ is proportional to x_{ij}/w_i (see 3.10 for both), $\left(a_{ijt}\right)^{\frac{1}{\chi}+1}$ is proportional to $n_{ij}x_{ij}$, and w_i cancels out in the consumer base cost term.

The deeper microfoundation, and thus the interpretation of some equilibrium quantities (e.g., n_{ij}) or parameters (e.g. χ) are different from the Krugman model. However, this model is isomorphic to dynamic Krugman in its predictions for aggregate trade flows, and the functional forms of the trade elasticities.

Melitz-Pareto. The dynamic Melitz (2003) model differs from the Krugman model in Section 2 in two ways. First, firms are heterogeneous in productivity, denoted $\varphi(\omega)$. Continuing to assume constant Dixit-Stiglitz markups, the firm ω's price becomes:

$$p_{ijt}^{x}\left(\omega\right) = \frac{\sigma}{\sigma-1}\frac{\kappa_{ijt}w_{it}}{\varphi(\omega)}. \tag{3.12}$$

We assume that $\varphi(\omega)$ is distributed Pareto:

$$F\left(\varphi\right) = 1 - \left(\frac{\varphi_L}{\varphi}\right)^{\theta}. \tag{3.13}$$

Second, the firm in i needs to pay a per-period fixed cost ξ denominated in units of i's labor in order to serve market j.

As in Section 2, each firm must pay a stochastic sunk cost $\xi_{ijt}^{s}\left(\omega\right)$ to enter market j, drawn from an inverse Pareto distribution (2.12). Paying this sunk cost also reveals to the firm its productivity for serving market j. Thus, the entry decision is made based on expected profits. Further, due to the per-period fixed cost not all firms that pay a sunk cost will end up exporting. The marginal firm earns variable profits that just cover the per-period fixed costs: $\frac{1}{\sigma}x_{ijt}\left(\omega\right) = w_{it}\xi$. Combining (2.2) and (3.12) (and noting that without tariffs $p_{ijt}^{x}\left(\omega\right) = p_{ijt}^{c}\left(\omega\right)$) leads to the productivity cutoff for selling from i to j:

$$\varphi_{ijt}^{m} = \frac{\sigma}{\sigma-1}\kappa_{ijt}w_{it}\left(\frac{\sigma w_{it}\xi}{C_{jt}\left(P_{jt}\right)^{\sigma}}\right)^{\frac{1}{\sigma-1}}. \tag{3.14}$$

Total sales from i to j are:

$$\begin{aligned} X_{ijt} &= \int x_{ijt}(\omega)\, d\omega \\ &= n_{ijt} \int_{\varphi_{ijt}^m}^{\infty} x_{ijt}(\varphi)\, dF(\varphi) \\ &= n_{ijt} \underbrace{C_{jt}\left(P_{jt}\right)^{\sigma} \left(\left(\frac{\theta \varphi_L^{\theta}}{\theta - (\sigma - 1)} \right)^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma - 1} \frac{\kappa_{ijt} w_{it}}{\left(\varphi_{ijt}^m\right)^{\frac{\sigma-1-\theta}{\sigma-1}}} \right)^{1-\sigma}}_{x_{ijt}}, \end{aligned} \tag{3.15}$$

where the last line comes from applying the Pareto distribution. Relative to the Krugman model, there is the extra complication that the average sales per firm are affected by entry/exit of the marginal firms – movements in φ_{ijt}^m. Combining (3.14) and (3.15) leads to the following expression for φ_{ijt}^m:

$$\varphi_{ijt}^m = \left(\frac{\theta \varphi_L^{\theta} \xi \sigma}{\theta - (\sigma - 1)} \frac{w_{it}}{x_{ijt}} \right)^{\frac{1}{\theta}}. \tag{3.16}$$

In turn, combining (3.15) and (3.16) produces the following expression for x_{ijt}:

$$x_{ijt} \propto \left(\frac{Y_{jt}}{w_{it}} \right)^{\frac{\theta-(\sigma-1)}{\sigma-1}} Y_{jt} \left(\frac{\kappa_{ijt} w_{it}}{P_{jt}} \right)^{-\theta}. \tag{3.17}$$

Equation (3.17) clarifies that in the Melitz model, cross-border sales involve an additional term $\left(Y_{jt}/w_{it}\right)^{\frac{\theta-(\sigma-1)}{\sigma-1}}$ that is absent from Krugman and customer capital models. This term arises due to the extensive margin, whereby the cutoff for serving a market is a function of market size Y_{jt}, scaled by the domestic unit costs: if market size increases, more and more marginal firms will enter, increasing sales per unit mass.[5] Even though foreign sales do not follow a simple CES demand functional form, domestic sales do. If A.2 holds, then the ratio Y_{jt}/w_{jt} is constant and x_{jj} conforms to (3.3) in Proposition 3.1. We show below that A.2 holds.

In steady state, at the time sunk costs are paid, the expected profits are:

$$E\left[\pi_{ij}(\omega)\right] = \frac{1}{\sigma} x_{ij} - w_i \xi \left(\frac{\varphi_L}{\varphi_{ij}^m} \right)^{\theta}. \tag{3.18}$$

Combining with (3.16) leads to the familiar result that expected profits are a constant fraction of

[5] Recall sales per unit mass $x_{ij} = \int_{\varphi_{ijt}^m}^{\infty} x_{ijt}(\varphi)\, dF(\varphi)$ is not the same as the average sales of firms serving a market, which is $x_{ij}/(1 - F(\varphi_{ijt}^m))$. When market size increases, φ_{ijt}^m falls – less productive firms enter. This increases x_{ij} since a higher fraction of firms per unit mass sell to the market. At the same time, the average sales fall, as less productive firms can serve larger markets.

expected sales: $E\left[\pi_{ij}(\omega)\right] = \frac{\sigma-1}{\theta}\frac{x_{ij}}{\sigma}$. Since (2.13) and (2.14) hold unchanged in the Melitz model (with the qualification that here, x_{ij} is sales per unit mass of firms rather than representative firm sales), they lead to (3.2), and Assumption A.3 is satisfied.

To see that A.2 is satisfied, note that the steady state profits to country i firms from selling to j are:

$$\Pi_{ij} = \frac{\sigma-1}{\theta}\frac{1}{\sigma}n_{ij}x_{ij} - w_i\int_0^{\bar{\xi}^s{}_{ij}} \xi^s g(\xi^s)\, d\xi^s \tag{3.19}$$

$$= \frac{\sigma-1}{\theta}\frac{1}{\sigma}n_{ij}x_{ij} - \frac{\chi}{\chi+1}\beta\frac{\sigma-1}{\theta\sigma}x_{ij}n_{ij} \tag{3.20}$$

$$= \left(1-\beta\frac{\chi}{\chi+1}\right)\frac{\sigma-1}{\theta\sigma}X_{ij}, \tag{3.21}$$

where the second line uses the distributional assumption on the sunk costs ξ^s. Summing across destinations and imposing trade balance delivers Assumption A.2.

We obtain the familiar result that the elasticity of x_{ij} with respect to trade costs ε^0_κ is no longer a function of the elasticity of substitution, but of the dispersion parameter in the Pareto productivity distribution. Relative to the Krugman model, following a change in trade costs, average sales per unit mass x_{ijt} will change both because of the intensive margin (all firms' sales change) and the extensive margin (marginal firms entering/exiting). As in Arkolakis et al. (2008) and ACR, when it comes to x_{ij}, those two margins' net effect is captured by $-\theta$.

Differently from those static models, and along the lines of the Krugman model in Section 2, the gains from trade are conditioned not just by θ, but also by the curvature of the sunk costs χ, due to the adjustment of the mass of firms that pay the sunk costs to obtain productivity draws n_{jj}. Thus, the Melitz extension retains the intuitions laid out in Section 2.

3.2 Generalization to Tariffs

Often, trade elasticities are estimated using variation in tariffs. To build up towards measurement and quantification, we state a generalization of Proposition 3.1 to a case with tariffs.

Proposition 3.2. *Consider a class of dynamic models that satisfy the following three conditions in their steady state:*

A.1′ For all countries j, trade is balanced (expenditure = revenue):

$$Y_j = w_jL_j + \Pi_j + R^g_j,$$

where $Y_j = C_jP_j$ and $R^g_j = \sum_i(\tau_{ij}-1)X_{ij}$.

A.2′ For all countries j, profits are a constant share of labor income:

$$\frac{\Pi_j}{w_jL_j} = const$$

A.3′ For all country pairs (i, j) trade flows satisfy

$$X_{ij} = n_{ij} x_{ij} \tag{3.22}$$

where

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^{\chi} \tag{3.23}$$

and domestic per-unit-mass sales satisfy

$$x_{jj} \propto Y_j \left(\frac{w_j}{P_j}\right)^{\varepsilon_\kappa^0} \tag{3.24}$$

for some constant $\chi > 0$ and where $\varepsilon_\kappa^0 \equiv \partial \ln x_{ij} / \partial \ln \kappa_{ij} < 0$ is the elasticity of exports per unit mass with respect to iceberg trade costs.

Then

$$C_j \propto \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0 (1+\chi)}} \left(1 - \frac{R_j^g}{Y_j}\right)^{-\left(1 - \frac{\chi}{1+\chi} \frac{1}{\varepsilon_\kappa^0}\right)}, \tag{3.25}$$

and $\varepsilon_\kappa^0 (1+\chi)$ is the long-run elasticity of trade flows with respect to iceberg trade costs.

Proof. See Appendix A. □

Note that since $\lambda_{jj} = 1$ and $R_j^g = 0$ in autarky, (3.25) is also the gross proportional gains from trade in steady state. Because tariffs generate revenue, (3.25) differs from (3.4) by the multiplicative factor that is a function of one minus the tariff revenue share in final expenditure. This multiplicative factor is greater than 1 as long as tariff revenue is positive. Thus, it amplifies the gains from trade relative to the no-tariff formula, conditional on the same λ_{jj}. In static models, this tariff adjustment to the ACR formula was to our knowledge first stated by Felbermayr, Jung, and Larch (2015). We show that it operates in a similar way in a dynamic setting. As in Felbermayr, Jung, and Larch (2015), the exponent on the tariff adjustment term cannot be recovered from the long-run trade elasticity alone. We show below how to recover this exponent from estimates of short- and long-run trade elasticities.

The data requirements for computing (3.25) are low. In addition to the domestic trade share, all it additionally requires is the total tariff revenue as share of GDP. This information is often available from statistical authorities. For the quantification below, we will require bilateral *ad valorem* tariff rates. Thus, it will be convenient to state the following alternative functional form for this adjustment factor:

$$1 - \frac{R_j^g}{Y_j} = \sum_i \frac{1}{\tau_{ij}} \lambda_{ij},$$

where $\lambda_{ij} \equiv \frac{\tau_{ij} X_{ij}}{Y_j}$ is the tariff-inclusive expenditure shares on goods from i.

We note that the Melitz-Pareto model with tariffs is not covered by Proposition 3.2, because tariffs also affect the extensive margin conditional on drawing the sunk cost, in a way that is not captured by (3.24). Proposition A.1 in Appendix A is an extension of Proposition 3.2 that also covers the Melitz-Pareto model with tariffs. The extended proposition is identical to Proposition 3.2 except for a strictly more general functional form for average sales x_{ij}. This generalization only affects the exponent on the tariff adjustment term $\left(1 - \frac{R_j^g}{Y_j}\right)$, and leaves the component of the GFT related to λ_{jj} unaffected. As we show in the quantification below, the tariff adjustment term is not quantitatively important. In addition, the non-linearity introduced by the extensive margin in the Melitz-Pareto model vanishes as the firm size distribution approaches a power law with exponent close to -1, the empirically relevant case (Axtell, 2001; Di Giovanni, Levchenko, and Rancière, 2011; Di Giovanni and Levchenko, 2013). Appendix A contains the detailed discussion.

4. Measurement and Quantification

This section takes the dynamic trade formulas to the data. We make four main points. The first is that in a dynamic world the long-run trade elasticity with respect to iceberg costs required by the formula cannot be recovered from a single empirical estimate of the elasticity of trade with respect to tariffs. Second, we compute the gains from trade under our preferred estimates of the trade elasticities, taken from Boehm, Levchenko, and Pandalai-Nayar (2023). This exercise shows that the gains from trade are large, and that the quantitative impact of the tariff adjustment to the GFT formula in (3.25) is generally minor. Third, we highlight the point that in the dynamic world, the long-run tariff elasticity is not sufficient for computing the gains from trade, and that GFT can vary widely even conditional on the same long-run tariff elasticity. Along the way, we also compare the dynamic gains from trade to those obtained from the static ACR models. Finally, the fourth part of the section compares the GFT implied by the formula to those computed numerically taking into account the transition path.

4.1 Data

The quantification relies on several sources of data. First, computing the gains from trade using (3.4) requires the domestic absorption share λ_{jj}. Typically, domestic absorption is measured from standard datasets such as the OECD Inter-Country Input Output tables (ICIO). The ICIO contains information on all bilateral sectoral expenditures, covering manufacturing and services, and intermediate and final goods. Importantly, it also contains information on expenditure on domestic sectors. However, the ICIO does not contain information on bilateral tariff revenues, so aggregate expenditure and expenditure shares constructed from this source are not tariff-inclusive.

Computing the gains from trade when *ad valorem* tariffs are non-zero (3.25) requires the total tariff revenue as a share of total (tariff-inclusive) spending. Aggregate tariff revenues are available from the World Bank. However, the full quantitative implementation of the dynamic model additionally requires all tariff-inclusive bilateral expenditure shares λ_{ij}. Therefore, aggregate tariff revenues are not sufficient for our purposes.

To construct bilateral tariff revenue, we obtain tariff data from the TRAINS dataset. This database reports the applied tariff by country pair at the Harmonized System (HS) 6-digit level. We link these data to trade flows at the HS-6 level from the BACI version of UN-COMTRADE. To compute tariff revenue, we multiply the bilateral, product-level applied tariffs obtained from TRAINS with bilateral trade flows from BACI:

$$R_{ij}^{g} = \sum_{p} X_{ijp}^{\text{BACI}} \left(\tau_{ijp}^{\text{TRAINS}} - 1 \right),$$

where R_{ij}^{g} is bilateral tariff revenue from goods trade and p is an HS-6 product. BACI does not contain information on services trade flows. We assume that services trade flows are subject to no tariff, so the aggregate bilateral tariff rate imposed by j on i consistent with goods tariff revenues R_{ij}^{g} is:

$$\tau_{ij} - 1 = \frac{R_{ij}^{g}}{X_{ij}^{\text{ICIO}}},$$

where X_{ij}^{ICIO} is total expenditure of j on goods and services from i, sourced from the OECD ICIO database. We can then calculate all tariff-adjusted trade shares λ_{ij}:

$$\lambda_{ij} = \frac{\tau_{ij} X_{ij}^{\text{ICIO}}}{\sum_k \tau_{kj} X_{kj}^{\text{ICIO}}}.$$

Our baseline sample includes 67 countries and a rest-of-the-world in 2006.[6] We validate our tariff revenue measures by comparing $R_{j}^{g} = \sum_i R_{ij}^{g}$ with national tariff revenue obtained from the World Bank. As the World Bank tariff revenue data are provided in local currency, we convert them to US dollars using an annual exchange rate obtained from the same source. Appendix Figure A1 illustrates that our baseline tariff revenue measures are very similar to those obtained from the World Bank.

The implementation of the full dynamic path in the quantitative model in Section 4.4 additionally requires data on real GDP, which we obtain from the Penn World Tables.

4.2 Measurement: Trade Elasticities

As in ACR, the gains from trade in this class of dynamic models is a function of the domestic absorption share and exogenous parameters. Propositions 3.1-3.2 state that the domestic share is exponentiated with the inverse of the long-run iceberg trade elasticity. In dynamic models, this long-run trade elasticity is a function of different structural parameters than the "trade elasticity" in static models. We now show that this has important implications for how this long-run elasticity can be recovered

[6] Three percent of the observations show positive bilateral goods trade flows in the ICIO but have no tariffs declared in TRAINS. In these cases, we assume there is 0 tariff revenue associated with these pairs.

from the data.

The exponent in the gains from trade formula is the inverse of the long-run elasticity of trade with respect to iceberg trade costs κ_{ij}:

$$\varepsilon_\kappa \equiv \frac{d\ln X_{ij}}{d\ln \kappa_{ij}} = \frac{d\ln n_{ij}}{d\ln \kappa_{ij}} + \frac{d\ln x_{ij}}{d\ln \kappa_{ij}} = \varepsilon_\kappa^0 (1+\chi). \tag{4.1}$$

Though a few papers have used shipping cost data to compute the trade elasticity (e.g. Hummels, 2001; Shapiro, 2016; Adão, Costinot, and Donaldson, 2017), the large majority of existing trade elasticity estimates use tariffs. However, the trade elasticity with respect to tariffs differs from that with respect to iceberg costs. The tariff elasticity is:

$$\varepsilon_\tau \equiv \frac{d\ln X_{ij}}{d\ln \tau_{ij}} = \frac{d\ln n_{ij}}{d\ln \tau_{ij}} + \frac{d\ln x_{ij}}{d\ln \tau_{ij}} = (\varepsilon_\kappa^0 - 1)(1+\chi), \tag{4.2}$$

The two elasticities differ because iceberg costs are reflected in the border price, whereas tariffs are not. Most (though not all) of the literature that estimates trade elasticities in the context of static models recognizes this distinction. In static models, this distinction is fairly innocuous: to account for it, one could either add 1 to the tariff elasticity to recover the iceberg cost elasticity, or use trade flows inclusive of tariff payments in estimation. In a dynamic setting, however, neither of these simple adjustments work, requiring another strategy to recover the iceberg elasticity.[7]

Fortunately, it is possible to use tariff elasticity estimates at different horizons to reconstruct the long-run elasticity that enters the gains from trade formula. The key is to use estimates of both short- and long-run tariff elasticities to separately pin down ε_κ^0 and $1+\chi$. Knowledge of ε_κ^0 and $1+\chi$ separately is also required to compute the tariff revenue adjustment to the GFT formula as in (3.25).

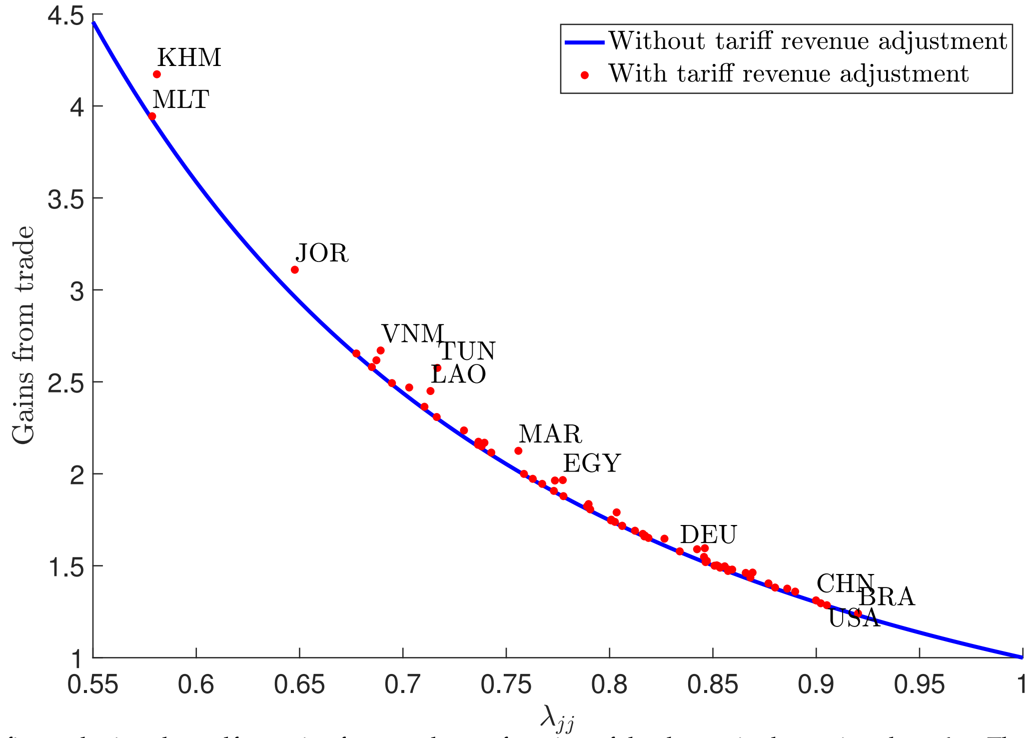
We will call the "short run" a time period over which x_{ij} can adjust but n_{ij} cannot. This is consistent with the model laid out in Section 2, in which n_{ij} only starts adjusting with a one-period lag. The short-run tariff elasticity is then:

$$\varepsilon_\tau^0 \equiv \frac{d\ln X_{ijt}}{d\ln \tau_{ijt}} = \frac{d\ln x_{ijt}}{d\ln \tau_{ijt}} = \varepsilon_\kappa^0 - 1. \tag{4.3}$$

It is immediate that with both the short- and long-run tariff elasticities (4.2)-(4.3) in hand, one can recover both ε_κ^0 and $1+\chi$, and reconstruct the object needed for the welfare gains formula, (4.1). This is the strategy we pursue in the quantification that follows.

[7] For example, in a static Armington or Krugman model the tariff elasticity is $-\sigma$, while the iceberg cost elasticity is $1-\sigma$. So simply adding 1 to the tariff elasticity would recover the iceberg elasticity. It is immediate from (4.1)-(4.2) that this won't work in the dynamic setting. It is also easy to verify that the long-run tariff elasticity of tariff-inclusive trade flows $d\ln\left(\tau_{ij}n_{ij}x_{ij}\right)/d\ln\tau_{ij}$ also does not recover the needed iceberg elasticity (4.1).

Figure 1: Steady State Gains from Trade



Notes: This figure depicts the welfare gains from trade as a function of the domestic absorption share λ_{jj}. The blue line implements the formula (3.4). The red dots implement the formula that adjusts for tariff revenue, (3.25).

4.3 Steady State Gains from Trade

Figure 1 plots the gains from trade for a sample of countries based on (3.25). It uses the long-run tariff elasticity estimate $\varepsilon_\tau = -2$, and a short-run tariff elasticity estimate $\varepsilon_\tau^0 = -1.25$ (Boehm, Levchenko, and Pandalai-Nayar, 2023). The blue line is simply the formula (3.4) that ignores tariffs and only uses information on the domestic trade share. The red dots are (3.25), and thus make the tariff adjustment using country-specific tariff revenue data. Conditional on a fixed λ_{jj}, the tariff adjustment unambiguously increases the gains from trade. However, the tariff adjustment is small quantitatively for all but a few countries.

The gains from trade are large. Even the most closed countries – the US, Brazil, China – gain on the order of 25-30% from trade. Jordan's welfare triples, and Malta's quadruples, when it goes from autarky to trade.

To highlight the role of dynamics and the short-run elasticity in conditioning the gains from trade, we now perform the following thought experiment. Suppose the value of the long-run tariff elasticity ε_τ is known. This is the object most commonly estimated (or, at least, intended to be estimated) in the literature. To make the results stark, suppose that the value of this long-run elasticity is high, for example -5 (Costinot and Rodríguez-Clare, 2014).

A researcher working on static models covered by ACR would simply add 1 to yield a long-run elasticity with respect to iceberg costs of -4, and compute the gains from trade based on this. As

ACR and Costinot and Rodríguez-Clare (2014) highlight, the basic single-sector ACR formula under this level of trade elasticity yields fairly small gains from trade. These are depicted by the red line in Figure 2.

However, even holding ε_τ fixed at -5, in a dynamic world an additional piece of information is required, that can be supplied by the short-run tariff elasticity. The elasticity required in Propositions 3.1-3.2 can be rewritten in terms of the (potentially) estimable tariff elasticities as:

$$\begin{aligned}
\varepsilon_\kappa^0(1+\chi) &= \varepsilon_\kappa^0(1+\chi)\frac{\varepsilon_\kappa^0-1}{\varepsilon_\kappa^0-1} \\
&= \underbrace{(\varepsilon_\kappa^0-1)(1+\chi)}_{\varepsilon_\tau}\frac{\varepsilon_\kappa^0}{\varepsilon_\kappa^0-1} \\
&= \varepsilon_\tau\frac{\varepsilon_\tau^0+1}{\varepsilon_\tau^0}.
\end{aligned}$$

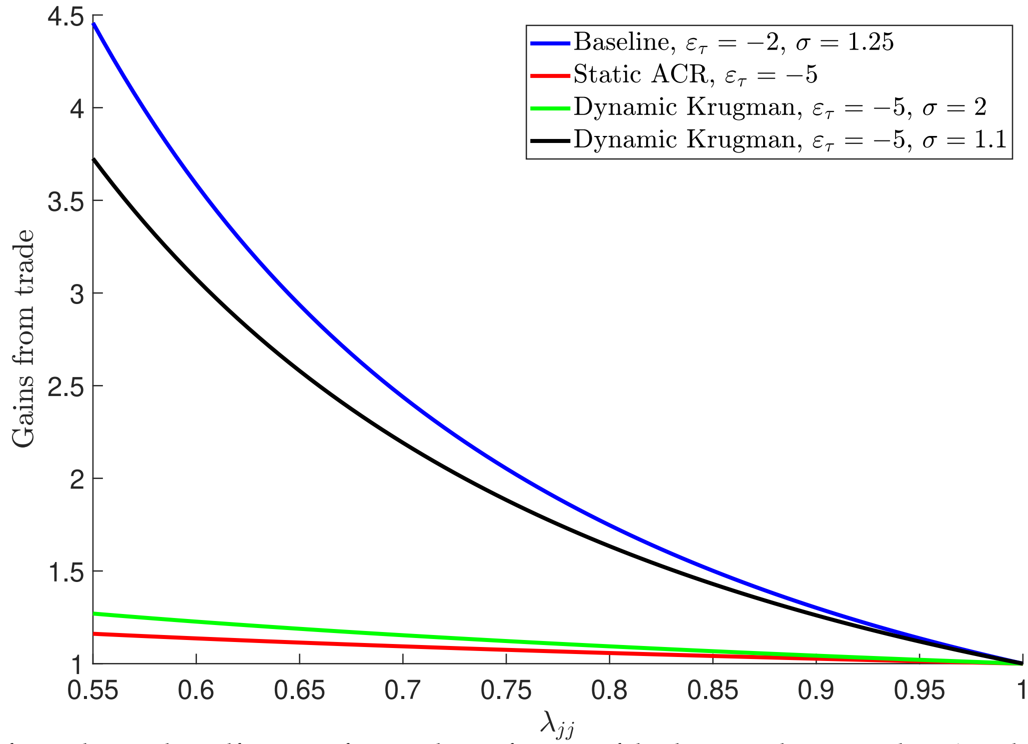
This expression makes it clear that a high long-run tariff elasticity ε_τ is consistent with very high gains from trade if the short-run tariff elasticity is low enough. Indeed, as $\varepsilon_\tau^0 \uparrow -1$, the gains from trade become infinite. The green and black lines in Figure 2 plot the gains from trade according to (3.4) under an identical long-run $\varepsilon_\tau = -5$, but for two values of ε_τ^0, -1.1 and -2. The difference in the gains from trade is drastic. Indeed, the black line is not too far from our baseline gains from trade plotted under the long-run elasticity of 2, despite a much higher long-run elasticity that it uses. However, even with an unreasonably high short-run elasticity of 2, the dynamic gains from trade are higher than in the static ACR implementation.

4.4 The Transition Path and Welfare Gains

In the final exercise, we answer the question of how costly it is that the formula compares steady states, and thus ignores the transition path. To do this, we compute welfare taking into account the transition between trade regimes. This requires calibrating the full model, and thus taking a stand on all the parameters.

We employ the Krugman model from Section 2. In addition to σ and χ, calibrated as above using short- and long-run tariff elasticity estimates, we require the depreciation rate δ, risk aversion γ, the discount factor β, and the Inverse Pareto scale parameter b. Of these, the most important one is δ, as it controls the speed of transition. The lower is δ, the slower the transition, and the greater the discrepancy between steady state and fully dynamic gains. When depreciation is full ($\delta = 1$), transition occurs in 1 period. This parameter is disciplined by the speed of convergence of the trade elasticity to the long-run. To be conservative, we assume the transition takes 15 years; this implies $\delta = 0.25$. Boehm, Levchenko, and Pandalai-Nayar (2023) reports that convergence occurs at around 10 years. We use this as robustness, as a shorter transition lowers the role of transition dynamics. The remaining parameter choices are standard. Table 1 summarizes the baseline calibration. We

Figure 2: Steady State Gains from Trade: the Role of the Short-Run Elasticity



Notes: This figure depicts the welfare gains from trade as a function of the domestic absorption share λ_{jj}. The red line depicts the ACR formula with elasticity -4, $\lambda_{jj}^{-1/4}$. The other lines implement the formula (3.4) for different values of ε_τ and ε_τ^0.

consider alternative parameter choices for robustness in Table 2. Appendix B.1 details the procedure. We solve the model for the 13 largest countries in the world by total GDP, a fourteenth country and a rest-of-the-world aggregate. We vary the fourteenth country across simulations to obtain quantitative gains from trade estimates for a larger group of countries.

While steady state comparisons are unambiguous, in a dynamic setting we have to specify whether the world is transitioning from autarky to trade, or from trade to autarky, and the welfare comparison between trade and autarky will differ in those two scenarios. Figure 3 reports three sets of gains from trade: (i) comparing autarky and trade steady states according to the formula (3.25) (blue); (ii) transitioning from autarky to the current levels of trade openness (red); and (iii) transitioning from the current levels openness to autarky (black). For scenarios (ii) and (iii) we begin in the initial steady state and then unexpectedly and permanently change trade costs κ_{ij} at time 1 to the value in the terminal steady state. When computing the welfare gains for country i, we make this adjustment to trade costs for all $j \neq i$. The GFT numbers for each country and each scenario are listed in Appendix Table A1.

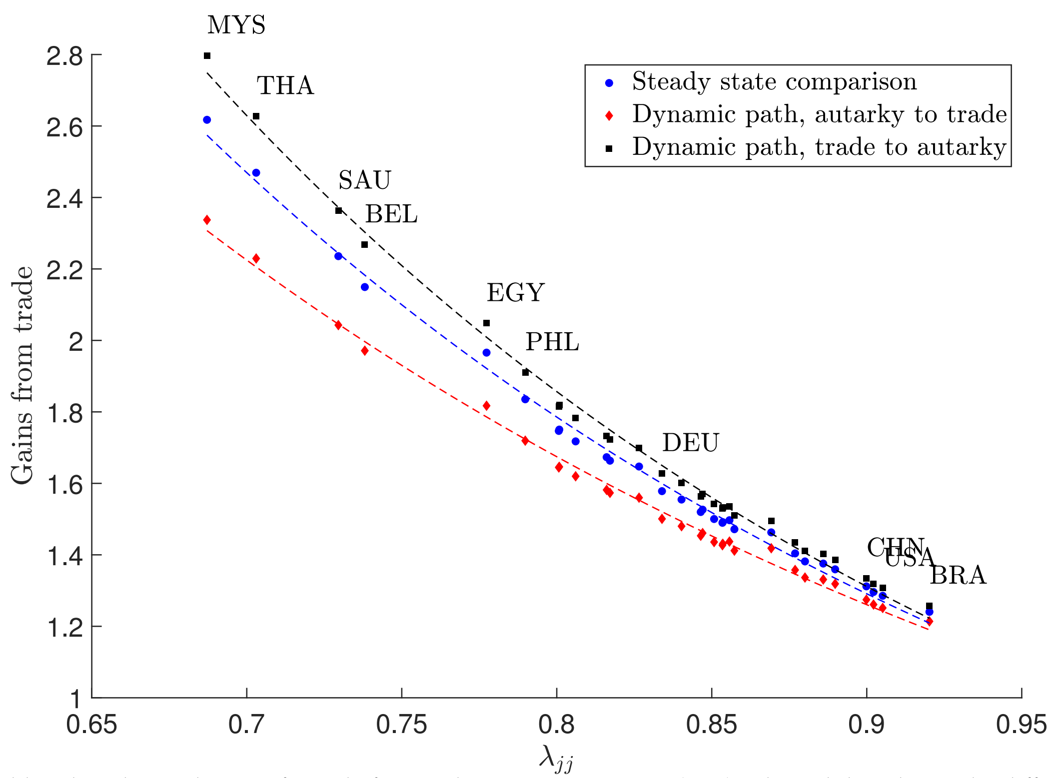
Two conclusions stand out from the figure. First, the disparity between steady state gains and the full dynamic gains is relatively minor. On average in this sample, the autarky-to-trade gains are 13.1 percent smaller, and trade-to-autarky gains are 8.5 percent larger. Second, the steady state gains are

Table 1: Baseline Calibration

Parameters	Value / Target / Source	Notes
σ	1.25	Short-run tariff elasticity
χ	0.6	Inverse Pareto shape parameter
β	0.97	Discount factor
γ	2	Relative risk aversion
δ	0.25	Exit rate
b	1	Inverse Pareto scale parameter
τ_{ij}	BACI, TRAINS	Average bilateral tariff
κ_{ij}	λ_{ij} from BACI, ICIO, TRAINS	Non-tariff trade costs
L_i	Relative real GDP from PWT	Labor endowment

Notes: The table presents the baseline calibration.

Figure 3: Steady State Gains vs. Gains over the Transition Path

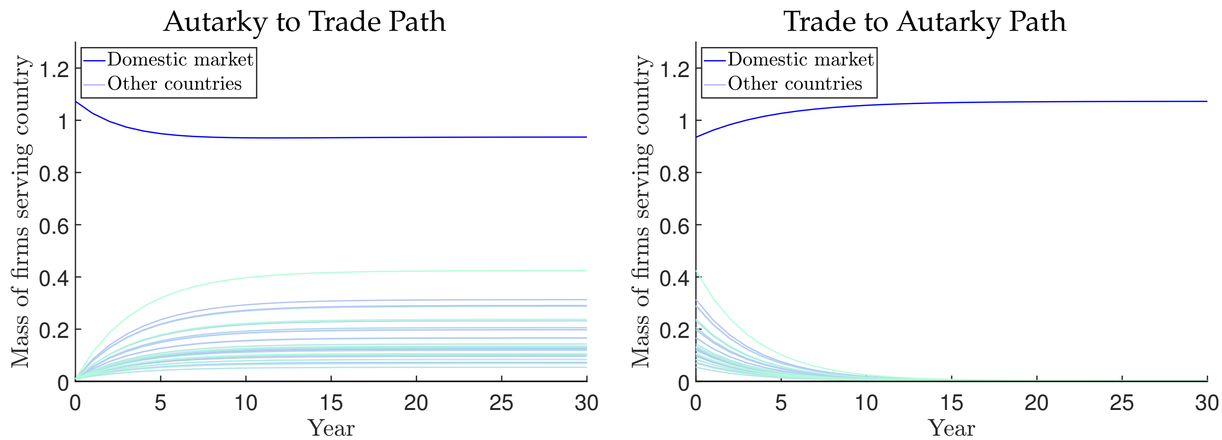


Notes: The blue dots depict the GFT formula for steady state comparisons (3.25). The red dots depict the difference in real consumption from starting in autarky and moving to the observed levels of trade,relative to remaining in autarky forever. The black dots depict the difference in real consumption between staying at the observed levels of trade forever and transitioning to autarky. Dashed lines represent an exponential fit between the gains from trade and the domestic absorption shares.

always in between those two.

To illustrate the intuition for this ranking of gains, Appendix Figure A2 plots the dynamic paths

Figure 4: Mass of Firms in the Transition Path, Malaysia



Notes: This figure shows the transition paths of the masses of Malaysian firms after a sudden change in the trade regime. The dark line denotes the mass of Malaysian firms serving the domestic market n_{jjt} and the light lines denote the masses of Malaysian firms serving the other countries n_{jit}, $j \neq i$. The left panel plots the paths following a sudden decrease in iceberg transport costs κ_{ijt} that takes the model from autarky to trade. The right panel plots the paths following a one-time increase in iceberg transport costs κ_{ijt} that takes the model from trade to autarky.

of consumption, and Figure 4 plots the evolution of the masses of firms for one country, Malaysia. When moving from autarky to trade, we start with the autarky steady state, and transition to the trade steady state slowly. Over this transition, consumption is lower than in the terminal steady state. This is because agents need to invest in "exporting capital" n_{jit}, $i \neq j$ starting from a level of 0, as illustrated in the left panel of Figure 4, and doing so requires forgoing consumption over the transition path. As a result, the dynamic gains of going from autarky to trade are below the steady state comparison.

Moving from trade to autarky, countries' accumulated exporting capital n_{jit} has become useless, because flow exports along the intensive margin x_{jit} is zero under infinite trade costs. At the same time, firms invest in their domestic operations, increasing the mass of domestic firms n_{jjt} as shown in the right panel of Figure 4. The result is an immediate drop in consumption *below* the level of the autarky steady state, and a gradual convergence of consumption to the autarky steady state level from below (Appendix Figure A2). This reduces the present value of consumption relative to the steady state – effectively the denominator of the GFT formula – and thus raises the implied GFT relative to the steady state comparison.

Robustness. We recompute the quantitative model with alternative parameter values to evaluate the role of transition dynamics in alternate settings. As discussed above, the most important parameter is δ, governing the speed of the transition. We consider alternative values of $\delta = 0.35$ and $\delta = 0.15$ corresponding to transitions from autarky to trade of around 10 years (the estimate of Boehm, Levchenko, and Pandalai-Nayar (2023)) and 30 years respectively. Additionally, we consider higher and lower demand elasticities $\sigma = 1.5$ and $\sigma = 1.15$. Finally we vary the curvature of adjustment costs χ, allowing for high curvature $\chi = 1$ and low curvature $\chi = 0.3$. The results are in Table 2.

Table 2: Robustness: Alternative parameter values

	Average Steady State Gains	Average Dynamic Gains: Autarky to Trade	Autarky to Trade: difference	Average Dynamic Gains: Trade to Autarky	Trade to Autarky: difference
baseline	0.464	0.406	-12.5%	0.502	7.58%
$\delta = 0.35$	0.464	0.420	-9.36%	0.492	5.72%
$\delta = 0.15$	0.464	0.376	-18.9%	0.523	11.3%
$\sigma = 1.5$	0.210	0.188	-11.1%	0.225	6.69%
$\sigma = 1.15$	0.893	0.780	-12.7%	0.921	3.02%
$\chi = 1$	0.359	0.313	-12.7%	0.406	11.71%
$\chi = 0.3$	0.596	0.532	-10.7%	0.621	4.05%

Notes: The table presents results from parameter robustness for the quantitative model with transition dynamics. The first column shows the average gains from trade relative to autarky in steady state, using the formula in (3.4). Columns 2 and 3 show the absolute and relative gains in the quantitative dynamic Krugman model with transition dynamics, moving from autarky to trade. Columns 4 and 5 show the absolute and relative gains in the dynamic Krugman model with transition dynamics moving from trade to autarky. The rows show alternative parameter choices. The first row shows the baseline calibration in Table 1. The next two rows change the speed of transition to a fast transition in 5 years ($\delta = 0.35$) and a slow transition ($\delta = 0.15$). The next to rows change the short-run iceberg elasticity σ. The last two rows change the convexity of adjustment χ.

Across all calibrations, the steady state gains from trade implied by the formula in (3.4) remains a good approximation of quantitative gains from trade including transition dynamics, with average differences ranging from 4.5% to −18.9%. As expected, the largest average difference is with a much slower transition of 30 years, moving from autarky to trade. Even here, the steady state gains from trade are a reasonable approximation. In all cases, the steady state gains from trade remain in between those computed in the full model going from autarky to trade and trade to autarky.

5. Conclusion

Research employing dynamic trade and spatial models has exploded in recent years. We provide closed-form gains from trade formulas that apply in a wide class of dynamic trade models. After stating the theoretical result, we emphasize measurement. We show that the short-run tariff elasticity is a crucial object even in evaluating the long-run steady state gains. In our quantification, the gains from trade are large, because the short-run elasticity is typically found to be small. Finally, we show that accounting for the transition path has a modest effect on the magnitude of the gains. Whether the steady-state formula over- or under-states the transition path gains depends on whether the transition is from autarky to trade or in the opposite direction.

References

Adão, Rodrigo, Costas Arkolakis, and Sharat Ganapati. 2020. "Aggregate Implications of Firm Heterogeneity: A Nonparametric Analysis of Monopolistic Competition Trade Models." Mimeo, Chicago Booth, Yale, and Georgetown.

Adão, Rodrigo, Arnaud Costinot, and Dave Donaldson. 2017. "Nonparametric Counterfactual Predictions in Neoclassical Models of International Trade." *American Economic Review* 107 (3):633–89.

Alessandria, George, Horag Choi, and Kim Ruhl. 2021. "Trade adjustment dynamics and the welfare gains from trade." *Journal of International Economics* 131:103458.

Alvarez, Fernando. 2017. "Capital accumulation and international trade." *Journal of Monetary Economics* 91:1 – 18.

Anderson, James E, Mario Larch, and Yoto V Yotov. 2020. "Transitional Growth and Trade with Frictions: A Structural Estimation Framework." *The Economic Journal* 130 (630):1583–1607.

Arkolakis, Costas. 2010. "Market Penetration Costs and the New Consumers Margin in International Trade." *Journal of Political Economy* 118 (6):1151–1199.

Arkolakis, Costas, Arnaud Costinot, Dave Donaldson, and Andrés Rodríguez-Clare. 2019. "The Elusive Pro-Competitive Effects of Trade." *Review of Economic Studies* 86 (1):46–80.

Arkolakis, Costas, Arnaud Costinot, and Andrés Rodríguez-Clare. 2012. "New Trade Models, Same Old Gains?" *American Economic Review* 102 (1):94–130.

Arkolakis, Costas, Svetlana Demidova, Peter J. Klenow, and Andres Rodriguez-Clare. 2008. "Endogenous Variety and the Gains from Trade." *American Economic Review* 98 (2):444–50.

Arkolakis, Costas, Jonathan Eaton, and Samuel S. Kortum. 2011. "Staggered Adjustment and Trade Dynamics." Mimeo, Yale and Penn State.

Atkeson, Andrew and Ariel Burstein. 2010. "Innovation, Firm Dynamics, and International Trade." *Journal of Political Economy* 118 (3):433–484.

Auer, Raphael, Ariel Burstein, and Sarah M. Lein. 2021. "Exchange Rates and Prices: Evidence from the 2015 Swiss Franc Appreciation." *American Economic Review* 111 (2):652–86.

Axtell, Robert L. 2001. "Zipf Distribution of U.S. Firm Sizes." *Science* 293 (5536):1818–1820.

Bardhan, P. K. 1965. "Equilibrium Growth in the International Economy." *Quarterly Journal of Economics* 79 (3):455–464.

———. 1966. "On Factor Accumulation and the Pattern of International Specialisation." *Review of Economic Studies* 33 (1):39–44.

Boehm, Christoph, Andrei A. Levchenko, and Nitya Pandalai-Nayar. 2023. "The Long and Short (Run) of Trade Elasticities." *American Economic Review* 113 (4):861–905.

Brooks, Wyatt J. and Pau S. Pujolas. 2018. "Capital accumulation and the welfare gains from trade." *Economic Theory* 66 (2):491–523.

Chen, Junyuan, Carlos Góes, Marc-Andreas Muendler, and Fabian Trottner. 2024. "Dynamic Adjustment to Trade Shocks." Mimeo, UC San Diego.

Costinot, Arnaud and Andrés Rodríguez-Clare. 2014. "Trade Theory with Numbers: Quantifying the Consequences of Globalization." *Handbook of International Economics* 4:197–261.

di Giovanni, Julian and Andrei A. Levchenko. 2013. "Firm Entry, Trade, and Welfare in Zipf's World." *Journal of International Economics* 89 (2):283–296.

di Giovanni, Julian, Andrei A. Levchenko, and Romain Rancière. 2011. "Power Laws in Firm Size and Openness to Trade: Measurement and Implications." *Journal of International Economics* 85 (1):42–52.

Drozd, Lukasz A. and Jaromir B. Nosal. 2012. "Understanding International Prices: Customers as Capital." *American Economic Review* 102 (1):364–95.

Eaton, Jonathan and Samuel S. Kortum. 2002. "Technology, Geography, and Trade." *Econometrica* 70 (5):1741–1779.

———. 2005. "Technology in the Global Economy: A Framework for Quantitative Analysis." Book manuscript.

Feenstra, Robert C. 2018. "Restoring the product variety and pro-competitive gains from trade with heterogeneous firms and bounded productivity." *Journal of International Economics* 110:16–27.

Feenstra, Robert C. and David E. Weinstein. 2017. "Globalization, Markups, and US Welfare." *Journal of Political Economy* 125 (4):1040–1074.

Felbermayr, Gabriel, Benjamin Jung, and Mario Larch. 2015. "The welfare consequences of import tariffs: A quantitative perspective." *Journal of International Economics* 97 (2):295–309.

Fitzgerald, Doireann and Stefanie Haller. 2018. "Exporters and shocks." *Journal of International Economics* 113:154–171.

Fitzgerald, Doireann, Stefanie Haller, and Yaniv Yedid-Levi. 2023. "How Exporters Grow." Forthcoming, *Review of Economic Studies*.

Galle, Simon, Andrés Rodríguez-Clare, and Moises Yi. 2023. "Slicing the Pie: Quantifying the Aggregate and Distributional Effects of Trade." *Review of Economic Studies* 90 (1):331–375.

Gourio, François and Leena Rudanko. 2014. "Customer Capital." *Review of Economic Studies* 81 (2):1102–1136.

Head, Keith and Thierry Mayer. 2014. "Gravity Equations: Workhorse, Toolkit, and Cookbook." In *Handbook of International Economics*, vol. 4, edited by Elhanan Helpman, Kenneth Rogoff, and Gita Gopinath. Elsevier B.V., 131–195.

Hummels, David. 2001. "Toward a Geography of Trade Costs." Gtap working papers, Center for Global Trade Analysis, Department of Agricultural Economics, Purdue University.

Imbs, Jean and Isabelle Mejean. 2017. "Trade Elasticities." *Review of International Economics* 25 (2):383–402.

Inada, Ken-ichi. 1968. "Free Trade, Capital Accumulation and Factor Price Equalization." *Economic Record* 44 (3):322–341.

Krugman, Paul. 1980. "Scale Economies, Product Differentiation, and the Pattern of Trade." *American Economic Review* 70 (5):950–59.

Lashkaripour, Ahmad. 2021. "The cost of a global tariff war: A sufficient statistics approach." *Journal of International Economics* 131:103419.

Levchenko, Andrei A. and Jing Zhang. 2014. "Ricardian productivity differences and the gains from trade." *European Economic Review* 65:45–65.

Melitz, Marc J. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71 (6):1695–1725.

Melitz, Marc J. and Stephen J. Redding. 2015. "New Trade Models, New Welfare Implications." *American Economic Review* 105 (3):1105–46.

Mutreja, Piyusha, B. Ravikumar, and Michael Sposi. 2018. "Capital goods trade, relative prices, and economic development." *Review of Economic Dynamics* 27:101–122.

Oniki, H. and H. Uzawa. 1965. "Patterns of Trade and Investment in a Dynamic Model of International Trade." *Review of Economic Studies* 32 (1):15–38.

Ossa, Ralph. 2015. "Why trade matters after all." *Journal of International Economics* 97 (2):266–277.

Ramondo, Natalia and Andrés Rodríguez-Clare. 2013. "Trade, Multinational Production, and the Gains from Openness." *Journal of Political Economy* 121 (2):273–322.

Ravikumar, B., Ana Maria Santacreu, and Michael Sposi. 2019. "Capital accumulation and dynamic gains from trade." *Journal of International Economics* 119:93–110.

———. 2024. "Trade liberalization versus protectionism: Dynamic welfare asymmetries." *European Economic Review* 163:104692.

Shapiro, Joseph S. 2016. "Trade Costs, CO^2, and the Environment." *American Economic Journal: Economic Policy* 8 (4):220–54.

Stiglitz, Joseph E. 1970. "Factor Price Equalization in a Dynamic Economy." *Journal of Political Economy* 78 (3):456–488.

A. Theory Appendix

Proof of Proposition 3.1. From A.1 and A.2, real consumption is proportional to the real wage:

$$C_j \propto \frac{w_j}{P_j}. \tag{A.1}$$

From A.3, the price index

$$P_j \propto w_j \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{-\frac{1}{\varepsilon_\kappa^0}}. \tag{A.2}$$

From A.3, the mass of firms

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}}, \tag{A.3}$$

where we also used A.1. Putting (A.1)-(A.3) together yields the first result.

To derive the last claim, note that:

$$\frac{\partial \ln X_{ij}}{\partial \ln \kappa_{ij}} = \frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} + \frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}}.$$

It is immediate from A.3 that $\frac{\partial \ln x_{ij}}{\partial \ln \kappa_{ij}} = -\varepsilon_\kappa^0$, and $\frac{\partial \ln n_{ij}}{\partial \ln x_{ij}} = \chi$, which gives the result.

Proof of Proposition 3.2. From A.1′,

$$Y_j = \left(1 - \frac{R_j^g}{Y_j}\right)^{-1} \left(w_j L_j + \Pi_j\right) \tag{A.4}$$

From A.2′,

$$C_j \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-1} \frac{w_j}{P_j} \tag{A.5}$$

From A.3′,

$$\frac{w_j}{P_j} = \lambda_{jj}^{-\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{\frac{1}{\varepsilon_\kappa^0}} \tag{A.6}$$

Also from A.3′,

$$n_{jj} \propto \lambda_{jj}^{\frac{\chi}{1+\chi}} \left(1 - \frac{R_j^g}{Y_j}\right)^{-\frac{\chi}{1+\chi}}, \tag{A.7}$$

Putting (A.5)-(A.7) together yields the first result. This last step also uses the fact that (A.4) and A.2′ imply that $\frac{Y_j}{w_j} \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-1}$. The proof of the claim about the trade elasticity is identical to Proposition 3.1.

Proposition A.1. *Consider a class of dynamic models that satisfy the following three conditions in their steady state:*

A.1′ For all countries j, trade is balanced (expenditure = revenue):

$$P_j C_j = w_j L_j + \Pi_j + R_j^g$$

where

$$R_j^g = \sum_i \left(\tau_{ij} - 1\right) X_{ij}$$

and trade balance holds $\sum_i X_{ij} = \sum_i X_{ji}$.

A.2′ For all countries j, profits are a constant share of labor income:

$$\frac{\Pi_j}{w_j L_j} = const$$

A.3″ For all country pairs (i, j) trade flows satisfy

$$X_{ij} = n_{ij} x_{ij}$$

where

$$n_{ij} \propto \left(\frac{x_{ij}}{w_i}\right)^{\chi}$$

and domestic per-unit-mass sales satisfy

$$x_{jj} \propto \left(\frac{Y_j}{w_j}\right)^{\varepsilon^1} Y_j \left(\frac{w_j}{P_j}\right)^{\varepsilon_\kappa^0} \tag{A.8}$$

for some constants $\varepsilon^1 > 0$ and $\chi > 0$, and where $\varepsilon_\kappa^0 \equiv \partial \ln x_{ij} / \partial \ln \kappa_{ij} < 0$ is the elasticity of exports per unit mass with respect to iceberg trade costs.

Then

$$C_j \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-\left(1 - \frac{1}{\varepsilon_\kappa^0} \frac{\chi}{1+\chi} - \frac{\varepsilon^1}{\varepsilon_\kappa^0}\right)} \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0} \frac{1}{1+\chi}} \tag{A.9}$$

where $\lambda_{jj} = \frac{X_{jj}}{Y_j}$, and $\varepsilon_\kappa^0 (1 + \chi)$ is the long-run elasticity of trade flows with respect to iceberg trade costs.

Proof. Derivations of (A.4) and (A.5) are identical to the steps in the proof of Proposition 3.2. From A.3″,

$$\frac{w_j}{P_j} = \lambda_{jj}^{\frac{1}{\varepsilon_\kappa^0}} n_{jj}^{-\frac{1}{\varepsilon_\kappa^0}} \left(\frac{Y_j}{w_j}\right)^{-\frac{\varepsilon^1}{\varepsilon_\kappa^0}}.$$

Also from A.3″,

$$n_{jj} \propto \left(\lambda_{jj} \frac{Y_j}{w_j}\right)^{\frac{\chi}{1+\chi}}.$$

Thus,

$$\frac{w_j}{P_j} \propto \left(\lambda_{jj}^{\frac{1}{1+\chi}} \left(\frac{Y_j}{w_j}\right)^{-\varepsilon^1 - \frac{\chi}{1+\chi}}\right)^{\frac{1}{\varepsilon_\kappa^0}}. \tag{A.10}$$

Putting (A.5) and (A.10) together yields the first result. This last step also uses the fact that (A.4) and A.2′ imply that $\frac{Y_j}{w_j} \propto \left(1 - \frac{R_j^g}{Y_j}\right)^{-1}$. The proof of the claim about the trade elasticity is identical to Proposition 3.1.

□

Discussion. The conditions required for Proposition A.1 are identical to the conditions in Proposition 3.2 in every way except the per-firm sales (A.8). This functional form for sales is a strict generalization of (3.24), that allows per-firm sales to depend non-linearly on home market size and bilateral tariffs (recall from (A.4), Y_j is a function of total tariff revenue). The resulting gains from trade formula (A.9) differs from (3.25) by $\left(1 - \frac{R_j^g}{Y_j}\right)^{\frac{\varepsilon^1}{\varepsilon_\kappa^0}}$. Note that the alternative formulation for per-firm sales only affects the tariff adjustment component of the GFT formula. The non-tariff component is unchanged, and λ_{jj} is still raised to the power of the trade elasticity.

Proposition A.1 covers the Melitz (2003) model with tariffs. In that case, firm ω's sales are given by

$$x_{ijt}(\omega) = \frac{1}{\tau_{ijt}} C_{jt} \left(P_{jt}\right)^{\sigma} \left(\frac{\sigma}{\sigma-1} \frac{\tau_{ijt}\kappa_{ijt}}{\varphi(\omega)} w_{it}\right)^{1-\sigma}, \tag{A.11}$$

and the cutoff firm has productivity

$$\varphi_{ijt}^{m} = \frac{\sigma}{\sigma-1} \tau_{ijt}\kappa_{ijt} w_{it} \left(\frac{\sigma \tau_{ijt} w_{it} \xi}{C_{jt}\left(P_{jt}\right)^{\sigma}}\right)^{\frac{1}{\sigma-1}}. \tag{A.12}$$

Combining these, the average firm sales are:

$$x_{ijt} \propto \left(\frac{1}{\tau_{ijt}} \frac{Y_{jt}}{w_{it}}\right)^{\frac{\theta}{\sigma-1}-1} \frac{1}{\tau_{ijt}} Y_{jt} \left(\frac{\tau_{ijt}\kappa_{ijt} w_{it}}{P_{jt}}\right)^{-\theta}. \tag{A.13}$$

Intuitively, tariffs and market size in the Melitz model affect the extensive margin, and thus appear non-linearly in the average firm sales. This property of the Melitz model with tariffs was pointed out by Felbermayr, Jung, and Larch (2015). It is easy to verify that the Melitz model with tariffs satisfies all the conditions for Proposition A.1 to hold. As equation (A.13) makes clear, the Melitz model satisfies A.3″ for $\varepsilon^1 = \frac{\theta}{\sigma-1} - 1$.

What is notable about this functional form for ε^1 is that it goes to zero as $\frac{\theta}{\sigma-1} \to 1$. Di Giovanni, Levchenko, and Rancière (2011) and di Giovanni and Levchenko (2013) show that the distribution of sales to any destination in the Melitz-Pareto model follows a power law with exponent $-\frac{\theta}{\sigma-1}$. Further, these papers document that in the data, firm sales follow a power law with exponent close to -1, known as Zipf's Law (see also Axtell, 2001). This implies that when calibrated to the observed firm size distribution, $\frac{\theta}{\sigma-1} \approx 1$ and therefore $\varepsilon^1 \approx 0$. Intuitively, ε^1 appears because tariffs affect the extensive margin of exports conditional on drawing the sunk cost. As the firm size distribution approaches Zipf's Law, the extensive margin plays no role in the aggregate outcomes (see di Giovanni and Levchenko, 2013, for a detailed treatment of this result).

B. Quantitative Appendix

B.1 Dynamic Path Simulations

This section details the procedure to compute the dynamic welfare gains presented in Figure 3 and Table A1. We use the 30-country sample listed in Table A1 and simulate two scenarios: (i) going from autarky to trade, and (ii) going from trade to autarky.

We first compute the steady state of the model under trade and under autarky. The steady state under trade matches the observed expenditure shares and tariffs for 2006. Then, we infer the change in non-tariff trade costs κ_{ij} to generate the difference between the two steady states. In both scenarios, we consider an unexpected permanent shock to the non-tariff trade costs in period 1. The direction of the shock depends on the scenario. The the non-tariff trade costs decrease in the first scenario and increase in the second scenario. We use the Newton algorithm in order to simulate the transition path of the model variables for 42 periods, where period 0 represents the initial steady state and period 41 represents the final steady state. All parameters other than non-tariff trade costs remain constant throughout the simulations.

We base the gains from trade calculations over the transition path on consumption equivalent variation. We define the present value of consumption in period 1 V_{j1} as

$$V_{j1} = \sum_{t=1}^{\infty} \beta^t \frac{\left(C_{jt}\right)^{1-\gamma}}{1-\gamma},$$

where β is the discount factor and γ is the factor of relative risk aversion.

Autarky to trade. Consider the transition path from autarky to trade. Let the superscript T denote the transition path under trade and superscript A denote the initial steady state under autarky. We then compute the present value of consumption under the transition path to trade as

$$V_{j1}^{T} = \sum_{t=1}^{\infty} \beta^t \frac{\left(C_{jt}^{T}\right)^{1-\gamma}}{1-\gamma}.$$

Now, assume a case where the household receives a constant consumption equivalent $C_j^{T,e}$ in every period, such that

$$V_{j1}^{T,e} = \sum_{t=1}^{\infty} \beta^t \frac{\left(C_{j}^{T,e}\right)^{1-\gamma}}{1-\gamma},$$

where the superscript e denotes the consumption equivalent.

Setting $V_{j1}^{T} = V_{j1}^{T,e}$ gives

$$C_j^{T,e} = \left((1-\beta) \sum_{t=1}^{\infty} \beta^t \left(C_{jt}^{T}\right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}},$$

which is our measure of welfare in the transition path to trade. The dynamic gains from trade under the first scenario are defined as

$$DGFT_j^{A\rightarrow T} = \frac{C_j^{T,e}}{C_j^{A}}.$$

Trade to autarky. In the second scenario, we analyze the transition path from trade to autarky. Now, the superscript A denotes the transition path under autarky and superscript T denotes the initial steady state under trade. We compute the present value of consumption under the autarky transition path as

$$V_{j1}^{A} = \sum_{t=1}^{\infty} \beta^t \frac{\left(C_{jt}^{A}\right)^{1-\gamma}}{1-\gamma}.$$

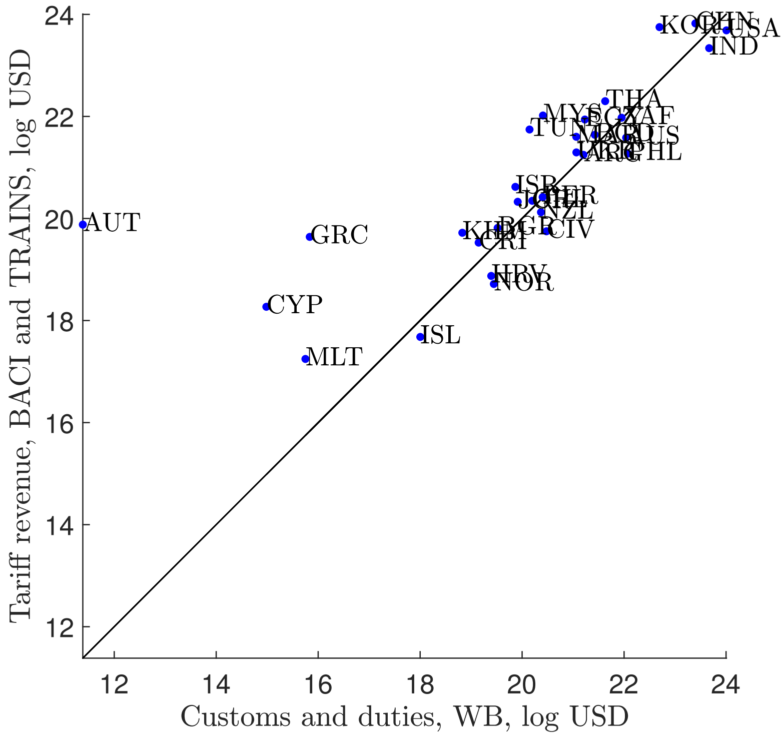
Following similar steps as in the previous case, the welfare measure in the transition path under autarky is

$$C_j^{A,e} = \left((1-\beta) \sum_{t=1}^{\infty} \beta^t \left(C_{jt}^A \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

The dynamic gains from trade in this second scenario are

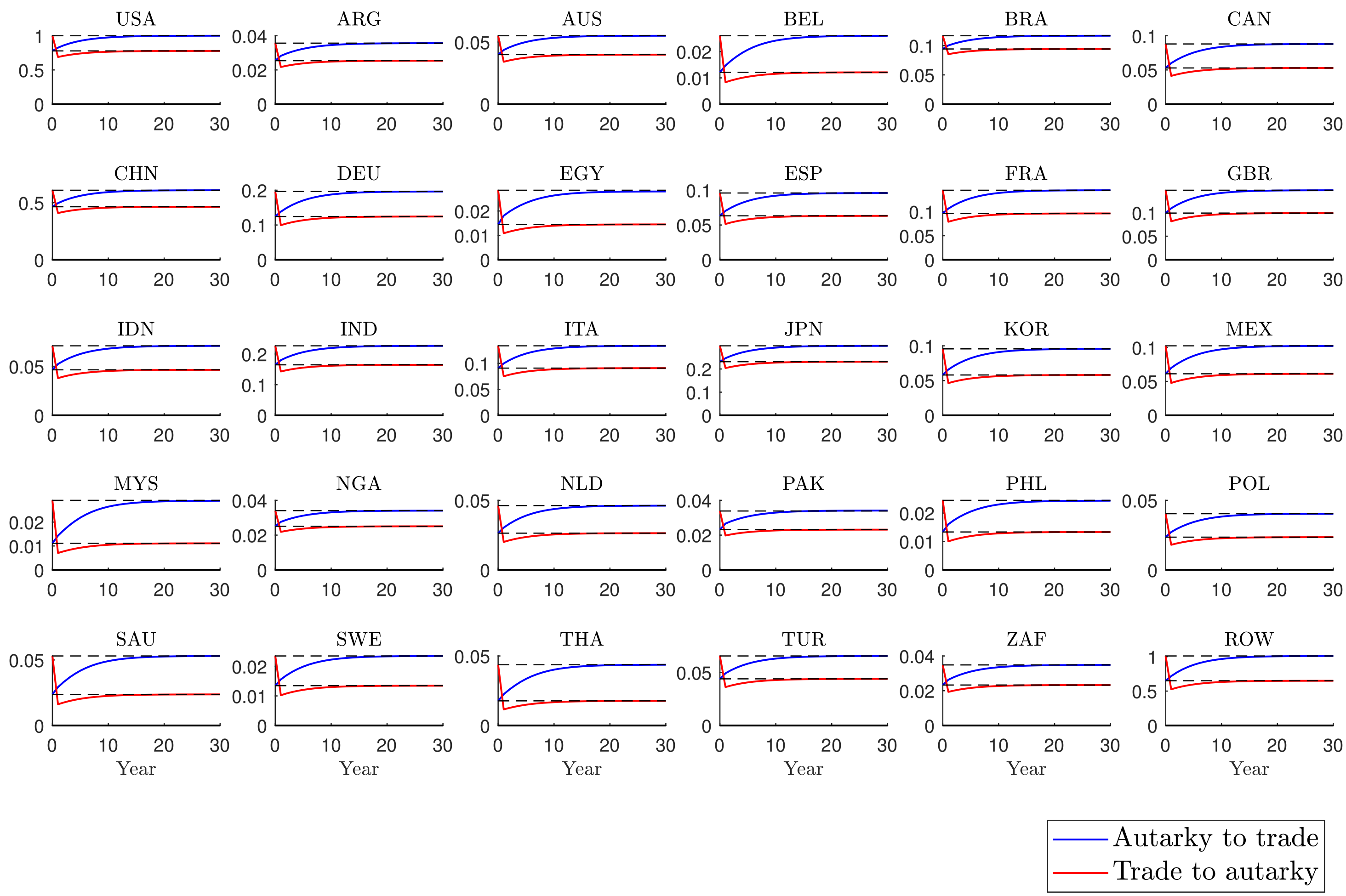
$$DGFT_j^{T\to A} = \frac{C_j^T}{C_j^{A,e}}.$$

Figure A1: Tariff Revenue Comparison



Notes: Figure shows comparison between (log) tariff revenues calculated from BACI-TRAINS and (log) customs and duties from the World Bank for the year 2006.

Figure A2: Consumption Transition Paths



Notes: This figure shows the transition paths of consumption for the countries included in our baseline calibration after a shock to iceberg transport costs κ_{ijt}. The shock is unanticipated and occurs in period 1. In the first case, we study the transition path from autarky to trade. In the second case, we study the transition path from trade to autarky. Dashed lines denote the steady state values under trade and autarky. Consumption is normalized to that of USA under the trade steady state.

Table A1: Dynamic gains from trade

Country	Steady state comp.	Dynamic path, autarky to trade	Dynamic path, trade to autarky	Country	Steady state comp.	Dynamic path, autarky to trade	Dynamic path, trade to autarky
MYS	2.617	2.337	2.797	ESP	1.520	1.453	1.564
THA	2.469	2.229	2.627	FRA	1.500	1.436	1.542
SAU	2.236	2.043	2.363	ZAF	1.497	1.437	1.536
BEL	2.149	1.971	2.268	TUR	1.491	1.431	1.532
EGY	1.966	1.817	2.048	GBR	1.490	1.427	1.530
PHL	1.835	1.719	1.910	ITA	1.472	1.412	1.511
NLD	1.750	1.646	1.819	PAK	1.463	1.418	1.495
SWE	1.747	1.645	1.816	ARG	1.404	1.358	1.435
POL	1.717	1.620	1.783	AUS	1.382	1.336	1.412
MEX	1.673	1.582	1.733	IND	1.376	1.331	1.403
CAN	1.663	1.573	1.722	NGA	1.359	1.319	1.385
KOR	1.647	1.560	1.700	CHN	1.312	1.274	1.335
DEU	1.578	1.501	1.628	JPN	1.296	1.261	1.319
ROW	1.555	1.480	1.601	USA	1.285	1.251	1.307
IDN	1.526	1.461	1.570	BRA	1.240	1.214	1.257

Notes: Table presents the numerical results for the dynamic GFT from Figure 3. The GFT formula for steady state comparisons follows (3.25), while the dynamic path calculations follow Appendix B.1.