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Abstract

A generalized spatial autoregressive model that bridges the complementarity and the conformity peer effect is proposed. A weight matrix is defined as an adjacency matrix minus a diagonal degree matrix multiplied by a conformity parameter between zero and one. This conformity parameter identifies the relative magnitude of one's complementary benefit and deviation cost from his or her own peer group. The social multiplier effect arises only when the complementary benefit overwhelms the deviation cost, and the threshold for the positive multiplier is lower for more centralized networks. The model is applied to the microfinance data from Karnataka, India. In contrast to the common belief, evidence of strong conformity is found by utilizing multiple dimensions of the social network. Furthermore, it is shown that different dimensions of one's social network are driven by distinct types of peer effects.

1 Introduction

Since the introduction of the spatial autoregressive model (SAR) for estimating geographical relationships, the model's usage has been constantly expanded to various fields, including analysis of social networks and accompanying peer effects. In earlier studies, simpler specifications such as linear-in-means or Euclidean distance models were used to represent underlying networks. Due to the increased interest in the peer effect in empirical studies and recent progression in data collection techniques, more complicated specifications of networks have become available for econometricians, and estimation strategies utilizing them have also been considerably improved.

In the context of social networks, a peer effect is interpreted as an interdependency between individuals in their behaviors. Unlike geographical objects and their relationship within geographical spaces, people influence each other for more sophisticated motives. Broadly speaking, the peer effect can be categorized into complementarity and conformity peer effects based on their underlying motives. Under complementarity,

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people enjoy extra utility by engaging in the same action with their peer groups. On the other hand, under conformity, people get disutility by deviating from their peers. Although these motives' consequences appear similar, their policy implications are vastly different. For instance, suppose that an organizer of a microfinance program wants to introduce it to a specific village. The program is expected to be more beneficial when people are tied together and encourage each other to set up a plan for the responsible use of money. Then, the organizer may successfully spread the program if she can encourage a core group of people to join it first. As they keep participating in the program, their friends or relatives will find it even more beneficial due to the complementary benefit arising from the already established basis and soon want to join the program altogether. As a result, the organizer can achieve her goal more effectively at a smaller cost. This scenario is an example of what is called the "social multiplier" effect, and this multiplier effect is working in a positive direction. The organizer's job is to carefully choose the target group based on criteria such as network centralities. A pitfall in this picture is that the peer effect is implicitly assumed to be complementarity. If the initially targeted group is under the conformity motive rather than complementarity, the target group's participation will soon dwindle because they would not want to behave differently from the other villagers. Their peers might also be willing to join in conforming with them, but the overall impact will be much lower than in the former case. Under this circumstance, the social multiplier effect is working in a negative direction, and the organizer should significantly increase the size of the target group or give them stronger incentives to compensate for this.

Due to this policy implication, several attempts have been made to distinguish and estimate these motives within the SAR model. Unfortunately, as pointed out by Boucher and Fortin (2016), the two motives are not identified with the ordinary SAR model that uses an adjacency matrix with zero diagonal elements as a weight matrix. This negative identification result is attributed to the fact that the underlying microeconomic foundations for the two motives are observationally equivalent. To address this, the "local-average" model is suggested by (Ushchev and Zenou 2020; Patacchini and Zenou 2012). In these papers, the authors model conformity as an additional utility from the average effort of one's peer group. Since the weight matrix is row-sum normalized, the conformity peer effect can be distinguished from the complementarity in which the additional utility comes from the aggregated effort ("the local-aggregate"). This approach is further sophisticated by Liu, Patacchini, and Zenou (2014). They present a microfoundation for the local-average model and perform the J-test to choose a suitable peer effect for a given dataset. This branch of literature, however, has a limitation that the social multiplier arises even under the conformity motive. A noticeably different approach is taken by Boucher (2016) by using the graph Laplacian as a weight matrix for the pure conformity peer effect. In his model, conformity is described as a disutility from the difference between one's own and peers' actions. This specification is a distinguished feature from the previous literature, where there is no such disutility. As a result, there is no multiplier effect as expected.

This paper departs from the existing literature by considering the two pure types of peer effects as two extrema of a single peer effect model that differ by the relative magnitude of the complementary benefit and the deviation cost. In the generalized spatial autoregressive model (GSAR), a *conformity parameter* is introduced to measure such magnitude. The most straightforward advantage of this approach is that it is a more realistic characterization of the peer effect. Frequently, one's true motive of peer effect is more complicated. Back to the previous example, the villagers may join the program partly because of the benefit of cooperating in the microfinance groups and, at the same time, partly because of peer pressure. The GSAR model can capture such situations properly with the flexible conformity parameter. Another benefit of this flexibility is that no model selection is required. A common approach for researchers who want to study peer effect has been arbitrarily assuming the motive they believe to be true or relying upon model selection for choosing the correct one. There are two issues with these strategies. First, there is no guarantee that one of the pure models will be selected. In other words, when each specification is tested against the other, both or none of the nulls may be rejected. In this case, the researchers are forced to draw another arbitrary conclusion to interpret the result. Even after that, the direction of the social multiplier effect is still unclear because both positive and negative multipliers cannot coexist. Second, the pure conformity model requires considerably larger samples due to the smaller variance caused by using the difference of outcome variables as an explanatory variable. The insignificant estimates from small samples may prohibit a researcher from performing model selection reliably.

To support the GSAR model, a microeconomic foundation is built upon the network utility and the private utility, where the network utility comprises the complementary benefit and the deviation cost. Furthermore, it is shown that the parameters for each component are identified by the peer effect and the conformity parameters in the econometrics model. This flexibility introduces a question on the nature of the social multiplier effect. Because the two motives are now on contiguity, the multiplier must also be a continuous function. To answer this, the concept of the social multiplier is extended to positive and negative multipliers. Under the positive multiplier effect, the complementarity motive dominates, and the aggregated output of individuals will be greater than the sum of isolated ones. On the other hand, with the negative multiplier effect, the deviation cost will be overwhelming, and individuals will tend to move toward their social norm. As a next step, a threshold for the positive multiplier is obtained as a function of both peer effect and conformity parameters. To have a positive multiplier, the peer effect must be sufficiently higher than the conformity. This threshold is characterized by a decreasing function of the sum of squared degrees divided by the sum of degrees, which is parallel to the "tendency to make hubs" centrality of Saberi et al. (2021). The intuition is that it is easier to have a positive multiplier if people are clustered around a small number of individuals.

The GSAR model is extended to the rational expectation model of Lee, Li, and Lin (2014) and applied to the case of a microfinance program in Indian villages collected and studied by Banerjee et al. (2013). Implementing the GSAR model with binary outcome variables and multiple networks, several results distinguished from the original study are drawn. Under the assumption of the simultaneous-move game, the estimation shows that there is a significant peer effect on the equilibrium, and its dominating motive is conformity. Moreover, few dimensions of the social network actually transmit the peer effect. By estimating the conformity parameter separately for each relevant network, it is found that different dimensions of an individual's social network may have different underlying motives, meaning that people have distinctive tendencies to conform depending on their reference groups.

The rest of the paper is organized as follows. Section 2 reviews related literature. Section 3 presents the GSAR model with a microeconomic foundation with the extended social multiplier effect, and the baseline model is extended to the binary outcome model with rational expectation. Section 4 applies the model to the microfinance data of Banerjee et al. (2013), and empirical results are shown. Lastly, section 5 provides concluding remarks.

2 Related Literature

The inflection problem of Manski (1993) draws attention to the identification of the "endogenous" and the "contextual" effect because only the former generates a positive social multiplier. He shows that those two effects are not identified with the linear-in-means model under complete networks. As a solution, utilizing more detailed specification of networks is suggested (Kelejian and Prucha 1998, 2010; Lee 2003), and the existence of the multiplier effect is also identified as a result. Nevertheless, the endogenous effect of the standard SAR model is still based on the complementarity peer effect, and the multiplier effects under different motives are not fully explained.

The existence of the social multiplier effect based on the linear-in-means model is empirically studied by Glaeser, Sacerdote, and Scheinkman (2003). In their work, the authors find evidence of the multiplier in the form of $1/(1-x)$ from multiple datasets. The limitation of this conventional form is that, as in Manski (1993), a network is assumed to be complete. Its result is a significantly higher multiplier for most cases, considering social networks are generally sparse. Indeed, (Giorgi, Frederiksen, and Pistaferri 2019) shows that such a measure can be misleading under more realistic network structures. The definition of the social multiplier effect proposed in this paper is rooted in the marginal effect analysis employed by many social network studies (Liu, Patacchini, and Rainone 2017; Chomsisengphet, Kiefer, and Liu 2018). This common notion of the multiplier under general networks is extended to incorporate the distinct impacts of the complementarity and conformity motives with a single metric.

This paper connects to the SAR model with binary outcomes of Lee, Li, and Lin (2014). It is assumed that the players have complete information on network structures and attributes of their peers. Because the GSAR model does not depend on the assumption or the form of the outcome variable, it can be applied to the case of incomplete information (Yang and Lee 2017) or the Tobit model (Yang, Lee, and Qu 2018).

For the application to the microfinance program, the GSAR model is also extended for the higher-order networks. A particular line of literature on geographical relationships employs a convex combination to utilize multiple networks. Debarsy and J. LeSage (2018) concerns the convex combination model to address the scaling issue between multiple networks derived from continuous measures. While it has the advantage that it is easier to include networks with different distance measures, more complicated estimation techniques are required (Debarsy and J. P. LeSage 2022). In contrast, networks are defined as binary relationships in many social networks, making distance measures less problematic. Therefore, the higher-order specification of this paper follows the simpler form of Blommestein (1983) and Huang (1984). It is worth noting that the method used in Hsieh and Lin (2017) can estimate the peer effects across networks. For the case of this study, however, the peer effect between villages is believed to be minimal.

3 Model

Consider a set of samples with N individuals who are linked to each other by a non-stochastic, exogenous social network. The network is defined by a graph on a set of vertices, V, and a nonempty set of edges, E, which is denoted as $G_N = G_N(V, E)$, where the edges are unweighted and undirected. Each vertex corresponds to an individual in a network. Self-loops or multiple edges between the same pair of vertices are prohibited, but vertices may be isolated. A set of vertices that are connected, or adjacent, to vertex i is called a neighborhood, and its size is referred to as a degree of i, d_i. The adjacency between the vertices is denoted as a binary variable, $a_{ij} \in \{0, 1\}$, which is zero if there is no edge between i and j and unity if there exists one. This relationship is summarized into a single $N \times N$ adjacency matrix, A. Note that, by construction, its diagonal is all zero. The sum of a_{ij} over all individuals other than i is equal to i's own degree, d_i, and an $N \times N$ degree matrix is constructed as D. In short, a graph derived from a network, G_N, is summarized into an adjacency matrix and a degree matrix defined as follows:

Definition 1.

$$A = \begin{cases} [a_{ij}] \text{ if } i \neq j \\ 0 \text{ otherwise} \end{cases} \qquad \textit{(Adjacency matrix)}$$

$$D = \begin{cases} [d_i] \text{ if } j = i, \\ 0 \text{ otherwise} \end{cases} \qquad \textit{(Degree matrix)}$$

In the literature of the spatial autoregressive model (SAR), the connectivity of individuals is represented by a weight matrix. Throughout this paper, it is defined as a combination of the adjacency and the degree matrices multiplied by *a conformity parameter*, $\gamma \in [0, 1] \subset \mathbb{R}$.

Definition 2. *A generalized weight matrix derived from the network structure, G_N, is an $N \times N$ square matrix defined as follows:*

$$W(\gamma) = A - \gamma D,$$

This definition encompasses both the ordinary SAR model ($\gamma = 0$) and the graph Laplacian variant of Boucher (2016) ($\gamma = 1$) as special cases. The proposed weight matrix is a type of the negative generalized graph Laplacian, where its diagonal may be any nonnegative numbers.

The generalized spatial autoregressive model (GSAR) is proposed as follows:

$$y = \rho W(\gamma) y + X\beta + \varepsilon, \tag{1}$$

where $\rho > 0$ and X is an $N \times K$ covariate matrix. The outcome variable, y, is an $N \times 1$ vector of real numbers. The vector of coefficients, β, is a $K \times 1$ vector and measures the direct effect of one's attributes. The idiosyncratic error, ε, is a random variable independent of the individual characteristics and the network structure, and no particular distribution is imposed. The parameter of interest is $\theta = (\rho, \gamma, \beta')'$.

Assumption 1. *(Exogenous network) The network structure, G_N, is independent of the idiosyncratic error, ε.*

Assumption 2. *(Model stability) The domain of the peer effect parameter is bounded by the spectral radius of $W(\gamma)$. That is,*

$$0 < \rho < 1/|\xi|,$$

where ξ is an eigenvalue of $W(\gamma)$ with the largest absolute value.

This is a common assumption for the SAR literature and also applies to the generalized weight matrix. If Assumption 2 is satisfied, then the inverse of $I - \rho W(\gamma)$ exists, and the model outcome is derived as a reduced form:

$$y = (I - \rho W(\gamma))^{-1}(X\beta + \varepsilon). \tag{2}$$

Identification

The identification result is derived for the IV estimator based on the linearly independent set matrices, $\{I, W(\gamma), W^2(\gamma)\}$.(Kelejian and Prucha 1998; Bramoullé, Djebbari, and Fortin 2009) The set of parameter, θ, is identified by non-isolated vertices in G_N. As the set of edges is nonempty, the largest subgraph with connected vertices can be found. Denote it as G_s, and the associated adjacency, degree and weight matrices as A_s, D_s, and W_s. Accordingly, the outcome vector of G_s is y_s, and the covariate vector is X_s. The peer effect and the conformity parameters are identified by utilizing this subgraph.

Assumption 3. *Let* $T = (I - \rho_0 W_s(\gamma_0))$ *and the true set of parameters* $\theta_0 = (\rho_0, \gamma_0, \beta_0')'$. *(i)* $T^{-1}X_s\beta_0 \neq 0$, *(ii)* $A_s T^{-1} X_s \beta_0 \neq 0$ *and (iii) For any nonzero constant* $c \in \mathbb{R}$, $cD^{-1}A_s T^{-1}X_s\beta_0 \neq T^{-1}X_s\beta_0$.

Assumption 3 excludes some cases where the parameters cannot be identified. Note that $T^{-1}X_s\beta_0$ is the outcome under the DGP. Some intuitions can be provided. First, if the outcome vector from the DGP without the idiosyncratic error is a zero vector, which is trivial. Second, the sum of the outcomes of one's peers is equal to zero. Third, the average of the outcomes of one's peers is not equal to a multiple of the outcome under the DGP.

Assumption 4. $plim_{N\to\infty} N^{-1}[Q_A X\beta_0, -Q_D X\beta_0, X]$ *exists and is a finite positive definite matrix of full rank.*

Proposition 1. *The parameters of the GSAR model,* $\theta = (\rho, \gamma, \beta')'$ *are identified with an IV estimator.*

Proof. See Appendix A. □

Microfoundation

Now, a microfoundation of the econometric model (1) is provided and analyzed. The utility function of any given individual i is represented as a quadratic utility function that consists of linear benefits and quadratic costs.

$$u_i(y_i, y_{-i}, x_i, \varepsilon_i) = \underbrace{\underbrace{\lambda\Big(\sum_{j\neq i} a_{ij} y_j y_i\Big)}_{\text{Complementary benefit}} \underbrace{- \frac{\delta}{2}\Big(\sum_{j\neq i} a_{ij}(y_j - y_i)^2\Big)}_{\text{Deviation cost}}}_{\text{Network net benefit}} + \underbrace{\Big((x_i\beta + \varepsilon_i)y_i - \frac{1}{2}y_i^2\Big)}_{\text{Private net benefit}}, \tag{3}$$

where $\lambda > 0$, $\delta \geq 0$ and a_{ij} is the (i, j) element of A. Given this, player i chooses an action, y_i, to maximize his or her utility, u_i. The network net benefit of i is maximized for the set of actions of the other players, y_{-i}. The private net benefit is determined by his or her own attributes, x_i, and an idiosyncratic shock, ε_i. The benefit arises from complementarity, which has been focused by many studies (Ballester, Calvó-Armengol, and Zenou 2006; Bramoullé, Kranton, and D'Amours 2014). The players benefit from each others' engaging in the same activity if they are linked by an edge, a_{ij}, in a network. On the other hand, the deviation cost measures the loss of utility from taking different actions from one's peers with a quadratic function (Boucher 2016).

The first-order condition of (3) induces each player's best-response function.

$$\begin{aligned} y_i^* &= \Big(\sum_{j\neq i} a_{ij}\big((\lambda+\delta)y_j - \delta y_i\big)\Big) + x_i\beta + \varepsilon_i \\ &= (\lambda+\delta)\Big(\sum_{j\neq i} a_{ij}\big(y_j - \frac{\delta}{\lambda+\delta}y_i\big)\Big) + x_i\beta + \varepsilon_i \\ &= \rho\Big(\sum_{j\neq i} a_{ij}\big(y_j - \gamma y_i\big)\Big) + x_i\beta - \varepsilon_i, \end{aligned}$$

where $\rho = \lambda + \delta$ and $\gamma = \delta/(\lambda + \delta)$. The pair of parameters from this function, (λ, δ) is identified by those from the estimation model, (ρ, γ).

Proposition 2. *The peer effect parameter, ρ, and the conformity parameter, γ, from the estimation model (1) are uniquely determined by the complementary benefit, λ, and the deviation cost, δ, and vice versa.*

Proof. Given (ρ, γ), the complementary benefit and the deviation cost are obtained by solving the following system of equations:

$$\begin{cases} \lambda = \rho - \rho\gamma \\ \delta = \rho\gamma. \end{cases} \tag{4}$$

Suppose that there exists another pair, $(\rho', \gamma') \neq (\rho, \gamma)$, which satisfies (4). Then, $\delta = \rho\gamma = \rho'\gamma'$ and $\lambda = \rho - \rho\gamma = \rho' - \rho'\gamma'$. Using the first equation, the latter is rewritten as $\rho'\gamma' + \rho = \rho'\gamma' + \rho'$. Then, it is obtained that $\rho = \rho'$, which is a contradiction and proves the uniqueness of ρ. Trivially, γ is also uniquely determined. Similarly, the opposite direction also holds. $\square$

Then, if the conformity parameter, γ, is equal to zero, it corresponds to the case where the deviation cost parameter, δ, is zero. In other words, the standard SAR model estimates the pure complementarity peer effect. In contrast, if γ is equal to one, the entire portion of the peer effect, ρ, is equal to the deviation cost, and the model connects to the pure conformity of Boucher (2016).

This identification result is derived from the distinguished structure of the complementary benefit and the deviation cost. In Boucher and Fortin (2016), the deviation cost is modeled as a quadratic function, $-\frac{\delta}{2}(y_i - \sum_{j\neq i} a_{ij}y_j)^2$. Note that the square of the difference is taken with y_i and the aggregated outcome of the peers. Therefore, the marginal deviation cost does not increase with respect to d_i, which is the same structure with the complementary benefit. Then, this pure conformity model leads to a SAR model, which is observationally equivalent to the pure complementarity model. As noted in Boucher (2016), this issue is shared by the local-average model as well. Due to its distinguished structure of the benefit and the cost, the GSAR model can incorporate the two components together and identify them at the same time.

An advantage of this approach is that the net benefit from the peer effect can be described as a tension between the complementary benefit and the deviation cost. By allowing the relative magnitudes of these components to be flexible, the two pure models become to be two extrema of the proposed generalized SAR model, and the characteristic of a peer effect is determined by the dynamics between the network benefit and cost from one's peers. Thus, a determinant of the characteristic can be investigated. The other advantage is that the GSAR model does not require model selection. The model selection procedure assumes that either of the models must be true and would not provide any useful information if both models are rejected. Allowing the mixture of the two models can prevent the complexity accompanying model selection.

Figure 1: A star network with $N = 3$

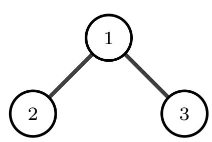


Figure 2: The pure complementarity peer effect

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}; \quad X\beta = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \quad (I - 0.15A)^{-1}X\beta = \begin{bmatrix} 1.05 \\ 0.16 \\ 0.16 \end{bmatrix}$$

Figure 3: The pure conformity peer effect

$$A - D = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}; \quad X\beta = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \quad (I - 0.15(A - D))^{-1} X\beta = \begin{bmatrix} 0.80 \\ 0.10 \\ 0.10 \end{bmatrix}$$

A simple star network with three vertices is focused to elucidate further the difference between the existing models (Figure 1). From this, an adjacency matrix and a graph Laplacian can be induced, with associated reduced forms (Figure 2 and 3). Suppose everyone's ex-ante outcome without peer effect is $(1, 0, 0)$. The first vertex may be considered as an "injection" point, which means that he or she is a target individual of a policymaker. Then, the ex-post outcomes are obtained by multiplying the inverse resolvent matrix respectively, as $(1.05, 0.16, 0.16)$ for the complementarity and $(0.80, 0.10, 0.10)$ for the conformity model. The evidence of a positive social multiplier is found from the increased outcome of the first vertex in the former. Meanwhile, in the latter, everyone has moved toward each other, and the outcome of the first vertex has decreased, which suggests a negative social multiplier.

Next, consider the local-average model with a row-normalized adjacency matrix (Figure 4). This simple example demonstrates that the local-average model does not precisely describe the conformity peer effect. The aggregated outcome of the vertices is still higher than the ex-ante outcome despite the conformity motive it attempts to describe.[1]

Figure 4: The local-average model

$$A_{norm} = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}; \quad X\beta = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \quad (I - 0.15A_{norm})^{-1} X\beta = \begin{bmatrix} 1.02 \\ 0.15 \\ 0.15 \end{bmatrix}$$

3.1 Social Multiplier Effect

As previously illustrated, the social multiplier effect appears differently depending on the relative magnitude of the complementarity benefit and the deviation cost. Thus, there arises a need for a rigorous definition of it. Early literature has adopted the common idea of a multiplier and suggested a simple metric, $1/(1 - \rho)$. This is a "naive" social multiplier rooted in the linear-in-means model with complementarity peer effect. Its limitation is on the complete network assumption, which means that everyone in a network is connected and under the influence of each other. Even when the complementarity assumption is accepted, this is far from

1. This, however, does not suggest that the local-average model is not useful in any case. If one is interested in the complementarity benefit from the average behavior of a group, it shows distinguished patterns compared to the local-aggregate model.

the truth in the context of social networks, as it is well-known that social networks are characterized by their sparsity. Consequently, the real picture of the multiplier should be more complex than the naive multiplier.

The nature of the multiplier effect can be further investigated by focusing on the marginal effect of covariates on outcome variables.[2] Consider the reduced form of the model (2),

$$y = (I - \rho W(\gamma))^{-1}(X\beta + \varepsilon).$$

For simplicity, assume that X is an $N \times 1$ vector and β is a scalar. Then, the marginal effect of i's covariate on all of the players is,

$$\frac{\partial y}{\partial x_i} = (I - \rho W(\gamma))^{-1}\beta. \tag{5}$$

Intuitively, the marginal effect from β is multiplied by the inverse matrix of $I - \rho W(\gamma)$, which means that the inverse matrix dictates the shape of the overall multiplier effect. By definition of a matrix inverse, it can be decomposed into

$$(I - \rho W(\gamma))^{-1} = \frac{1}{\det(I - \rho W(\gamma))} W(\gamma)_{adj},$$

where $W(\gamma)_{adj}$ is an adjugate matrix of $W(\gamma)$. Since the ij-th element of an adjugate matrix represents the number of paths between i and j, the overall multiplier effect will be rooted on the magnitude of the determinant. From this, a definition of the social multiplier is proposed as follows:

Definition 3. *Given a nonstochastic, exogenous G_N, the social multiplier is defined as a function of ρ and γ, which is*

$$\mu(\rho, \gamma) = 1/|(I - \rho W(\gamma))|.$$

If $\mu > 1$, the social multiplier effect is positive. If $0 < \mu < 1$, the social dispersion effect is negative.

The boundaries for the pure models are derived as follows:

Corollary 1. $\mu(\rho, 0) > 1$ *and* $0 < \mu(\rho, 1) < 1$

Proof. For $\gamma = 1$, it is suffice to show that $|I - \rho W(1)| > 1$. For any arbitrary matrix, M, and its eigenvalues, $\lambda_i(M)$, a determinant of M is equal to $\Pi_{i=1}\xi_i$. For any eigenvalues of M, $1 + \xi_i(M) = \xi_i(I + M)$ and $c\xi_i(M) = \xi_i(cM)$ hold. Since a graph Laplacian matrix is positive semidefinite, all of its eigenvalues are nonnegative. Then, $|\rho M| = \rho(\Pi_{i=1}\xi_i) > 0$ and $|I - \rho W(1)| > 1$.

2. The idea of the marginal effect multiplied by $(I - \rho W)^{-1}$ has been widely used in the literature of the SAR model (Chomsisengphet, Kiefer, and Liu 2018) (add more citations)

For $\gamma = 0$, $|I - \rho W(0)|$ is equal to the difference between 1 and the sum of paths of all lengths discounted by ρ. Therefore, $0<|I - \rho W(0)|<1$ by Assumption 2. □

As the conformity parameter is flexible in the GSAR model, the social multiplier is hinged upon the relative magnitudes of the two motives. Accordingly, a threshold where the social multiplier is exactly equal to 1 can be found.

Proposition 3. *The threshold for the positive social multiplier is a solution of $\mu(\rho, \gamma) = 1$, which is*

$$\bar{\rho}(\gamma) = \frac{2\gamma}{1 + \gamma^2 \sum_{i=1}^N d_i^2 / \sum_{i=1}^N d_i}.$$

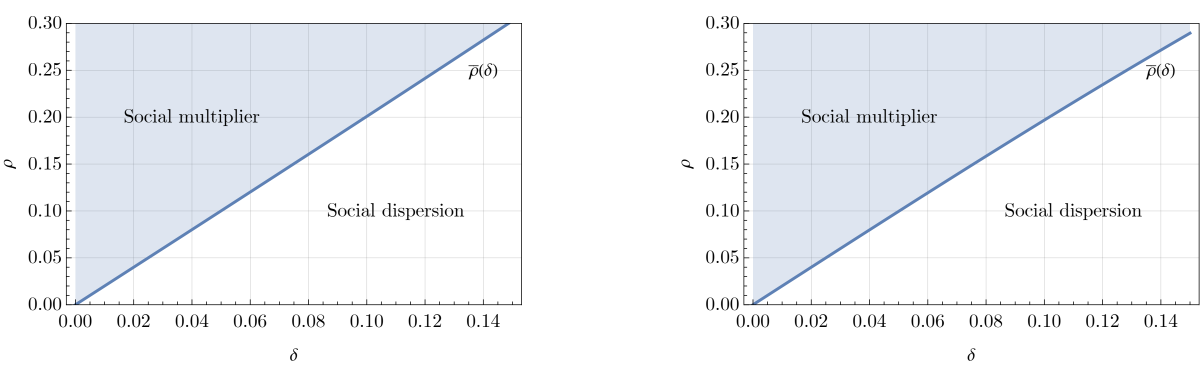
If $\rho > \bar{\rho}$, there exists a positive social multiplier effect, and $\rho < \bar{\rho}$, there exists a negative social multiplier effect.

Proof. See Appendix A. □

This result suggests that, for the positive social multiplier effect, the complementary benefit must be sufficiently higher than the conformity motive of individuals. Note that the coefficient, $\sum_{i=1}^N d_i^2 / \sum_{i=1}^N d_i$, is parallel to the "tendency to make hubs centrality" proposed by Saberi et al. (2021). The implication is that the more vertices are linked with a specific "hub" vertex, the easier for the social multiplier effect to arise.

The threshold is based on the sparse approximation of matrix determinants proposed by Ipsen and Lee (2011). The performance of approximation depends on the sparsity of non-diagonal elements of a matrix, and, indeed, most of social networks are qualified as sparse. A graphical presentation of a numerical and an approximated solution for a random graph with $N = 10$ and a density of 10% is shown in Figure 5. The

Figure 5: The numerical (left) and the approximated solution (right)



Note: The graph is generated randomly with $N = 10$ and density of 10%.

shaded areas represent where the multiplier effect is positive. The example shows that the approximated solution is tracking the actual solution relatively well, even for the higher density than the graphs in the real world.

3.2 Extension: Binary Choice Model with Rational Expectation

In this section, the GSAR model is extended to the binary outcome variables with rational expectation. Individuals are considered as players who maximizes their utility without directly observing the others' outcomes. Samples are considered as equilibrium outcomes of a simultaneous-move game (Lee, Li, and Lin 2014; Yang and Lee 2017). The individuals make decision simultaneously, based on the rational expectation formed on the others' unobservable outcomes. The outcome variable of i, y_i, is related with his or her own latent variable, y_i^*, by $y_i = I(y_i^* > 0)$.

With the latent variable, the GSAR model is rewritten as below:

$$y_i^* = \rho w_i(\gamma) E[y|G_N, X] + x_i\beta - \varepsilon_i,$$

where $y = (y_1, \ldots, y_N)'$ and w_i is the i-th row of W. Individual characteristics are presented as a $1 \times K$ vector, x_i, which is i-th row of $N \times K$ vector X. The idiosyncratic shock, ε_i, follows a symmetric distribution independent to X and G_N, of which CDF is $F_i(\cdot)$ and PDF is $f_i(\cdot)$.

The rational expectation on the unobserved outcomes is formed upon complete information on the network structure and the individual characteristics.

Assumption 5. *(Rational expectation) The entire non-stochastic network structure, G_N, and the population characteristics, X, are public information.*

Assumption 6. *(The best-response) The latent outcome variable, y_i^*, is a maximizer of the expected utility of each individual.*

Denote the choice probability, $P[y_i = 1|G_N, X]$, as p_i. Then the expected outcome, $\mathbb{E}[y_i|G, X]$, is equal to p_i and computed as follows:

$$\begin{aligned}
p_i &= P_i\big[\rho w_i(\gamma)\mathbb{E}(\mathbf{y}|G, X) + x_i\beta - \varepsilon_i > 0\big] \\
&= P_i\big[\rho w_i(\gamma)\mathbf{p} + x_i\beta - \varepsilon_i > 0\big] \\
&= 1 - F_i(\rho w_i(\gamma)\mathbf{p} + x_i\beta) \\
&= F_i(\rho w_i(\gamma)\mathbf{p} + x_i\beta),
\end{aligned}$$

where $\mathbf{p} = (p_1, \ldots, p_N)'$. For simplicity, denote $\eta_i(\theta, \mathbf{p}) = \rho w_i(\gamma)\mathbf{p} + x_i\beta$.

The unique equilibrium of the game is a solution of the following nonlinear system of N equations, which is a function of the parameters, θ. Denote such solution as $\mathbf{p}^*$. Therefore, the solution for each i, p_i^*, is a function of the parameter vector, θ.

$$p_i^*(\theta) = F_i(\eta_i(\theta, \mathbf{p}^*(\theta))). \tag{6}$$

This is a well-defined nonlinear operator defined on a Banach space (Yang and Lee 2017). Although existence of equilibrium is guaranteed by the Brouwer fixed-point theorem, there may be multiple equilibria. To focus only on the case of unique equilibrium, the stability assumption which corresponds to Assumption 2 is required.

Assumption 7. *(Uniqueness)*

$$|\rho|||W|| \max_i f_i(\eta_i) < 1$$

Then, by the contraction mapping theorem, there exists a unique equilibrium. On this equilibrium, the social multiplier is defined in the similar way to the continuous outcome model.

Definition 4. *The social multiplier of the binary outcome model is,*

$$\mu(\rho, \gamma) = 1/\det\left(I - f_{diag}(\rho w_i \mathbf{p} + x_i\beta)\rho W\right),$$

where $f_{diag}(\rho w_i \mathbf{p} + x_i\beta)$ is an $N \times N$ diagonal matrix of which i-th diagonal element is $f_i(\rho w_i \mathbf{p} + x_i\beta)$.

Identification of the parameters is shown by the similar way to the continuous outcome model. The following assumptions are parallel to those given in the previous chapter. Notations for the connected subgraph, G_s, from G_N follows those of the continuous model.

Assumption 8. *For any nonzero constant $c \in \mathbb{R}$,*

$$p_{i,s} \neq c \sum_{j \neq i} \frac{1}{d_{i,s}} a_{ij,s} p_{j,s},$$

where $p_{i,s}$ is the probability vector of the corresponding individuals in G_s.

Assumption 9. *(i) $F_i(\cdot)$ is strictly increasing distribution function with unity variance that is known to the econometrician. (ii) $plim_{N\to\infty} N^{-1}[A\mathbf{p}, -D\mathbf{p}, X]'[A\mathbf{p}, -D\mathbf{p}, X]$ exists and is a finite positive definite matrix of full rank.*

Proposition 4. *Under Assumption 8 and 9, the set of parameter, $\theta = (\rho, \gamma, \beta')'$, is identified.*

Proof. See Appendix A. □

4 Application: The Peer Effects in the Microfinance Program

4.1 Data Description

This study utilizes the data set collected by Banerjee et al. (2013) in collaboration with Bharatha Swamukti Samsthe (BSS), a non-profit organization that conducted the microfinance program in Karnataka, India. The authors had conducted a household-level survey prior to the entry of BSS into 43 rural villages in the target districts including household characteristics such as roof materials, ownership of houses, access to electricity, religion, and castes. An individual-level survey was conducted for the members of households that are randomly selected among eligible ones to collect more detailed information. As a result, additional data were obtained for about 46% of the sample households. BSS had reported the take-up rate of the program periodically.

The main interest of the individual survey was the social network information. Respondents were asked to provide the names of their peers who belong to a total of twelve dimensions of their peer network respectively as follows: (1) from whom they would borrow money, (2) to whom they would lend money, (3) to whom they give advice, (4) from whom they find help for important decision making, (5) from whom one would borrow kerosine and rice, (6) to whom one would lend kerosine and rice, (7) to whom they visit for free time, (8) who visits them for free time, (9) from whom they seek medical advice, (10) relatives in the village, (11) non-relative people with whom they socialize and (12) people with whom they go to temples together. Note that some networks, such as (1) and (2), are examined bidirectionally. The average overlapping rate of the twelve networks is roughly 50%, which satisfies the linear independence assumption. These network data were collected for each individual and then collapsed into household-level in a way that a pair of households is considered connected if any of the household members are connected. Due to the random sampling, it is possible that a household designated as a neighborhood may not have been sampled, and there is no way to verify if the relationship is reciprocal. For this reason, the surveyed networks are symmetrized. The dimensions surveyed by a set of questions with "directions" such as "from whom they would borrow money" and "to whom they would lend money," are also flattened to a single matrix throughout this paper.

Some villagers were designated as "leaders," who mostly consist of self-help group leaders, teachers, and shop owners. They were initially informed about the microfinance program in private meetings held by the institution and encouraged to invite their neighbors to the following sessions. Households with the leaders are considered leader households.

The entries without additional details such as caste or religion are removed to control the household characteristics.[3] As a result, a total of 7,919 households are used. The descriptive statistics can be found in Figure 1. There is no noticeable difference in the characteristics depending on the sample sizes. The additional characteristics are crucial due to the potential endogeneity between the socioeconomic status variables and the peer networks. In the original paper, the individual-level demographics are not used for the analysis. In this paper caste and religion are controlled by imposing the attributes of the heads of households on each sample. This is not a significant issue, as the other member of the households are likely to be identical with their heads'.

As pointed out by many studies, random removal of nodes on a network does not guarantee unbiasedness and consistency. If such an issue cannot be avoided, it is advisable to clarify its expected impacts. Comparing the density and the spectral radius of the networks under different sample sizes, it appears that the density increases while the radius decreases. It may be interpreted as a result of more nodes with smaller degrees and some bridging the others being removed. In this case, a downward bias is expected on the estimates of the peer effects. Consequently, if any estimates of peer effects are found significant under this smaller sample, it can be believed as a lower bound. Also, all 1,672 of the excluded samples are only from the first two villages, so its impact will be contained in those two.[4]

The dummy variables for the income proxies are constructed to present lower qualities, considering that the most demands are from underprivileged households.

Network Characteristics

Definitions and graph statistics of the surveyed networks are summarized in Table 2. For the individual networks, the average degree is 3.284, and the average density is 0.023, which is common sparsity for social networks.[5][6] The union network is constructed by the villagers who are connected through any of the individual dimensions.

A well-known issue in social network studies is the endogeneity of networks, which is the case where the networks are correlated with unobserved attributes of individuals. Although this cannot be completely eliminated, it can be mitigated by investigating the source of correlation and finding proper control variables. In Table 3, the relationships between the chosen attributes of the respondents and those of those who were

3. Christianity is excluded due to its extremely small sample size

4. See Table A1 for full descriptive statistics.

5. The respondents were asked to report up to four persons for each network. It can potentially cause a mismeasurement problem suggested by Griffith (2022). Its potential impact, however, is only an underestimation of peer effect, not an overestimation. If a peer effect turns out to be positive, it is not an issue anymore, which is the case in this study.

6. The degrees over four is due to the directionally surveyed networks merged into a single one.

Table 1: Descriptive statistics

Variables	Description	Mean	S.D.
Outcome variable			
Microfinance take-up	Yes = 1, No = 0	0.178	0.382
Income proxies			
Roof material: tile	Yes = 1, No = 0	0.37	0.48
Number of rooms		2.41	1.3
Number of beds		0.82	1.23
No latrine in house	Yes = 1, No = 0	0.73	0.45
House not owned	0, if the house is privately owned, 1, otherwise	0.1	0.3
No access to electricity	Yes = 1, No = 0	0.38	0.48
Social Status			
Leader group	1, if the household belongs to the leader group, 0, otherwise	0.13	0.33
Other backward castes	Base caste variable	0.53	0.5
General caste	Yes = 1, No = 0	0.12	0.33
Minority castes	Yes = 1, No = 0	0.2	0.13
Scheduled caste	Yes = 1, No = 0	0.28	0.45
Scheduled tribe	Yes = 1, No = 0	0.05	0.22
Religion			
Hinduism	Base religion variable	0.95	0.21
Islam	Yes = 1, No = 0	0.05	0.21
Village			
Village Population		184.16	86.08

N = 7,919
Number of villages = 43

chosen as their peers are shown. The two attributes are the number of beds per capita and rooms per capita, which are the proxies for income. According to the numbers shown, they tend to report those who are wealthier as peers.[7] Since the income proxies are included in the regression, it can be expected that the network endogeneity is controlled by the explanatory variables.

The other potential source of endogeneity is networks related to outcome variables. However, this possibility can be precluded for this study, as it is not conceivable for the villagers to adjust their social network according to the microfinance program that was not introduced at the time of the survey.

4.2 Methodology

By adopting the SAR model, this study departs from Banerjee et al. (2013) of which the main interest is the diffusion process and the rate of transmission of the microfinance program. Exploiting the panel data provided by BSS, they simulate the process over the surveyed network and find that non-participants play a

7. It is believed that the correlation with the income proxies is caused by the survey questions focused on the purpose of spreading microfinance. If a researcher wants to survey the networks free from the endogeneity problem, it is desirable to devise questions carefully.

Table 2: Networks description

Network	Description	Degree	Density	Spectral radius
Money	From/to whom they may borrow/lend money	4.244	.029	7.493
		(.804)	(.016)	(1.793)
Advice	From/to whom they may find/give advice	3.263	.024	6.598
		(.627)	(.015)	(1.250)
Kerosine	From/to whom they may borrow/lend kerosine or rice	3.740	.025	6.771
		(.901)	(.012)	(2.072)
Medical	From whom they seek medical advice	2.947	.020	5.354
		(.735)	(.012)	(1.752)
Non-relatives	Non-relative people with whom they socialize	4.360	.031	7.645
		(.649)	(.018)	(1.364)
Relatives	Relatives in the village	2.172	.015	4.607
		(.479)	(.009)	(.875)
Temple	With whom they go to temples together	.351	.003	1.287
		(.172)	(.002)	(1.884)
Visit	Who they visit/Who visit them for free time	5.278	.035	8.570
		(1.247)	(.016)	(2.340)
	Average	3.284	0.023	6.040
Union	Union of all networks	9.22	.06	14.07
		(1.69)	(.04)	(3.2)

Standard errors in parentheses.

Table 3: Respondents-peers differences in characteristics

	Beds per capita	Rooms per capita
Borrow money	-0.46	-0.64
Lend money	0.07	-0.02
Give advice	-0.06	-0.15
Help decision	-0.38	-0.61
Borrow kerosine and rice come	0.11	0.08
Borrow kerosine and rice go	-0.18	-0.27
Medical advice	-0.12	-0.23
Non relatives household	-0.21	-0.38
Relatives household	-0.08	-0.14
Temple company	0.01	0.00
Visit come	-0.08	-0.22
Visit go	-0.22	-0.37

Note: The numbers are the characteristics of the respondents minus those of those who were revealed as peers. The negative signs mean that the characteristics of the peers are higher. For example, the respondents reported that they borrow money from those who have 0.46 more beds on average.

crucial role in the diffusion of the program. While those who adopt the program are more likely to pass the information, the larger number of non-participants is more pivotal, even with the lower transmission rate. In terms of the peer effect, however, they find that the number of participants in one's neighborhood does not significantly affect the diffusion rate. Moreover, depending on the value of their parameters, it even turns out to be negative. While this is not impossible in theory, no plausible explanation of what causes the negative peer effect is provided in the original research.[8] As a related study, an attempt is made by Chandrasekhar and Lewis (2011), where the issue is attributed to a sampled network and consequent missing network links.

In this paper, however, a distinct approach is taken with the specification of a network. As mentioned in the previous section, they surveyed a total of twelve dimensions of social networks and took a union of them for their study. By doing so, it is ignored that an individual's social network has multiple dimensions, and one communicates through a specific dimension with only specific topics. If this is the case, the misspecification problem of the original paper, in fact, is an overly dense network, not the opposite. In Table 2, it can be confirmed that the union network is significantly different from its individual dimensions in terms of graph statistics. The spectral radius (an eigenvalue of the largest absolute magnitude) of the union network is 15.08, which is significantly higher than 5.76 on average over the twelve networks. Also, density is four times higher than the individual average as 4% to 1%. To address this, the higher-order SAR model is used, and the exact dimensions that channel the peer effect on microfinance are identified.

Another departing point is the underlying assumption on the observed samples. The SAR model mainly differs from the diffusion model by assuming that the observed state is a stable equilibrium. Under this assumption, an equilibrium is an outcome of rational expectation formed by players based on complete information on the networks and the other players.[9] To obtain a stable equilibrium, the magnitude of the peer effect must not dominate the marginal effect of each player's characteristic. On the other hand, the original paper views the diffusion process mainly as a function of graph statistics and individual characteristics as secondary. Regardless of individual characteristics, if one is not reached by an informer, they do not have any chance to make a decision. Consequently, the central role is played by the position of the leaders, who are the initial injection points on a graph. In addition to this, in the SIR (Susceptible-Infected-Recovered) diffusion model, one has only one opportunity to make a decision at the time of receiving information, and there is no further chance to change one's mind in the future. In contrast, in the SAR model, it is assumed that the information is already common knowledge. Each player's concern is to expect their neighbors' possibilities of participation and decide theirs accordingly.

Estimation is performed by the higher-order binary GSAR introduced in the previous section. To control unobserved differences in each village, group fixed effect dummy variables, τ_g, are included. For each village, the following regression is used:

$$\sum_{q=1}^{Q} \rho_q W_{q,g}(\gamma) E[y_g | G_N, X_g] + x_{i,g} + \tau_g - \varepsilon_{i,g}, \quad \varepsilon_i \sim \text{Logistic}(0,1),$$

8. Complementarity does not imply that the extra utility must come from using the same goods and services together at the same time. It may come from mitigating any uncertainties of unknown products. (Bursztyn et al. 2014)

9. If one accepts the reduced SAR model as a linear/nonlinear operator equation, the stable equilibrium can be viewed as a result of an infinite number of repeated interactions between individuals.

where $Q = 8$, which is the number of networks. Each network is required to be linearly independent from each other for identification of ρ_q. As mentioned, since the average overlapping rate of each network is 50%, this condition is satisfied.

An estimate of the parameters of interest, $\hat{\theta} = (\hat{\rho}', \hat{\gamma}, \hat{\beta}')'$, is an $(Q + 1 + K) \times 1$ vector and obtained by maximizing the following log-likelihood function, $\mathcal{L}$, with respect to θ:

$$\mathcal{L}(\theta) = \sum_{g=1}^{G} \sum_{i=1}^{N_g} \big[y_{ig} \log F(\eta^*_{ig}) + (1 - y_{ig}) \log(1 - F(\eta^*_{ig})) \big]$$

where $\eta^*_{ig} = \eta^*_{ig}(\theta, \mathbf{p}^*_g(\theta)) = \sum_{q=1}^{Q} \rho_q w_{iq,g}(\gamma)\mathbf{p}^*_g(\theta) + x_{ig}\beta$ and the equilibrium choice probability vector, $\mathbf{p}^*_g(\theta)$. is based on equation (6). The number of groups, G, is equal to 43.

4.3 Results

Parameter Estimation

As a benchmark, estimation without the peer effect is conducted with the ordinary logit model (Model 1).[10] In Table 4, it appears that the leader groups have a higher tendency to join the program. It is not unexpected, considering that they were those who were initially encouraged to participate.

The additional characteristics that are not included in the original research (castes and religion) show significant correlations with the take-up rate. Because the other backward castes are the major group in most of the villages, they are set as the base for the caste variables. The underprivileged castes, such as scheduled caste and scheduled tribe, appear to be more likely to participate in the program. In contrast, the general caste, which is a relatively more favored group, shows a negative estimate. Also, Islam, a minor religion, has a significantly higher effect compared to the base religion variable, Hinduism. Among the income proxies, only having no in-house latrine and no access to electricity show a significantly positive correlation.

The standard SAR estimation is carried out using the union network to verify existence of the peer effect (Model 2). Note that using the standard model implies the pure complementarity peer effect because the conformity parameter is fixed as zero. The result shows evidence of a significant peer effect.[11] Under the peer effect, it appears that the coefficient of the "leader group" variable is reduced by the largest margin. This suggests that the leaders were also affected by the complementary benefit from their peers. Therefore without the peer effect, the direct effect from being designated as leaders is overestimated.

Table 5 reports a set of estimates with the higher-order SAR model: the pure complementarity model

10. The estimates from this model are used as the initial point for the estimation of the rest of the models.
11. This result highlights the methodological difference between this paper and Banerjee et al. (2013). For a further discussion, see Appendix C.

Table 4: Ordinary logit and SAR with the union network

	Model 1		Model 2	
	Mean	SD	Mean	SD
ρ				
Union			0.125***	0.018
β				
Constant	-1.376***	0.289	-1.864***	0.223
Roof material: tile	0.105	0.076	0.099	0.076
No. of rooms	-0.026	0.032	-0.053*	0.032
No. of beds	-0.018	0.034	-0.025	0.034
No latrine in house	0.353***	0.085	0.401***	0.085
House not owned	0.020	0.103	0.031	0.101
No access to electricity	0.190***	0.073	0.212***	0.072
Leader group	1.098***	0.167	0.572***	0.087
General caste	-0.312**	0.134	-0.254***	0.125
Minority castes	-0.045	0.235	0.228	0.239
Scheduled caste	0.493***	0.083	0.395***	0.076
Scheduled tribe	0.406***	0.146	0.372***	0.138
Islam	0.627***	0.087	0.935***	0.147
Multiplier			1.40	
Likelihood	3451.06		3431.58	
AIC	6928.12		6891.16	
BIC	7018.82		6988.84	

Significant at *10%, **5%, ***1%.

with $\gamma = 0$ (Model 3), the pure conformity model with $\gamma = 1$ (Model 4), and the GSAR model with $\gamma \in [0, 1]$ (Model 5). Model 3 reveals that only three of the networks, Relatives, Temple, and Visit, transmit significant peer effects. Note that the first four networks highly correlated with the income proxies turn out to be insignificant, which suggests that they are properly controlled by the covariates. In Model 4, no significant peer effect is found, and the coefficients of the household characteristics are similar to Model 1.

The main result of this paper is Model 5. In addition to the peer effects, the conformity parameter, γ, is estimated as 0.284 and significant at 95% confidence level. Although the number is closer to zero than one, this does not imply that complementarity is the dominant motive. The social multiplier computed from the estimates is 0.009, which means that the complementary benefit is not high enough to overwhelm the deviation cost for the villagers. This becomes more apparent with the marginal effects that will be presented later.

Despite the existence of a significant conformity motive, the pure conformity model does not capture such peer effect from the data. One of the possible explanations is the higher requirement of the pure complementarity model. Due to the smaller variance in the difference of choice probabilities, $p_j - p_i$, the pure conformity model needs more samples to achieve the same level of performance as the pure complementary model. The other reason is attributed to the misspecified model. It can be the case that not only the villagers care more about conformity but also benefit from complementarity for some degree. For such instances, using the two pure models and conducting model selection will not provide an accurate picture of the peer effect.

Table 5: Higher-order SAR models

	Model 3: $\gamma = 0$			Model 4: $\gamma = 1$			Model 5: $\gamma \in [0, 1]$		
	Mean		SD	Mean		SD	Mean		SD
ρ									
Money	-0.034		0.079	0.034		0.347	-0.058		0.115
Advice	-0.065		0.058	0.050		0.343	-0.109		0.080
Kerosine	0.042		0.068	0.047		0.345	0.047		0.097
Medical	0.001		0.079	0.035		0.394	0.101		0.113
Non-relatives	-0.006		0.063	0.000		0.254	-0.044		0.090
Relatives	0.141	**	0.071	0.056		0.389	0.188	*	0.103
Temple	0.371	***	0.125	0.059		0.977	0.539	***	0.198
Visit	0.234	***	0.061	0.042		0.292	0.390	***	0.108
γ							0.284	**	0.123
β									
Constant	-1.783	***	0.223	-1.381	***	0.303	-1.671	***	0.223
Roof material: tile	0.126	*	0.074	0.114		0.088	0.147	*	0.080
No. of rooms	-0.065	**	0.031	-0.031		0.036	-0.086	**	0.034
No. of beds	-0.019		0.033	-0.021		0.039	-0.019		0.036
No latrine in house	0.402	***	0.083	0.374	***	0.106	0.420	***	0.092
House not owned	0.051		0.099	0.021		0.115	0.054		0.106
No access to electricity	0.216	***	0.070	0.193	**	0.086	0.226	***	0.077
Leader group	0.768	***	0.134	1.125	***	0.192	0.696	***	0.137
General caste	-0.217	*	0.117	-0.328	**	0.145	-0.226	*	0.120
Minority castes	0.369		0.234	-0.070		0.269	0.458	*	0.249
Scheduled caste	0.328	***	0.070	0.495	***	0.090	0.305	***	0.071
Scheduled tribe	0.325	**	0.128	0.413	**	0.164	0.346	**	0.135
Islam	0.606	***	0.086	0.690	***	0.139	0.657	***	0.101
Multiplier	3.53			0.00			0.009		
Likelihood	-3414.81			-3446.18			-3409.37		
AIC	6871.62			6934.36			6862.74		
BIC	7018.14			7080.88			7016.23		

Significant at *10%, **5%, ***1%.

As an extension of Model 5, the conformity parameters are separately estimated for each network dimension of interest (Model 6). Then, the weight matrix for q-th dimension is constructed as $W_{q,g}(\gamma_q) = A_{q,g} - \gamma_q D_{q,g}$. Because of the higher computational burden from estimating Q additional parameters, only the two networks that appear to be relevant from Model 5 are chosen for this extension, which are "Relatives" and "Visit."[12] The estimation result is shown in Table 6. The conformity parameter for the " Relatives "

12. Temple company is excluded due to the insufficient number of samples for the increased estimation complexity.

Table 6: Network specific conformity parameters

	Model 6		
	Mean		SD
ρ			
Relatives	0.410	**	0.179
Visit	0.250	**	0.121
γ			
Relatives	0.644	**	0.256
Visit	0.091		0.367
β			
Constant	-1.769	***	0.229
Roof material: tile	0.137	*	0.082
No. of rooms	-0.079	**	0.034
No. of beds	-0.027		0.037
No latrine in house	0.413	***	0.094
House not owned	0.058		0.107
No access to electricity	0.230	***	0.078
Leader group	0.836	***	0.143
General caste	-0.234	*	0.121
Minority castes	0.308		0.257
Scheduled caste	0.335	***	0.074
Scheduled tribe	0.340	**	0.135
Islam	0.667	***	0.108
Multiplier	0.007		
Likelihood	-3411.4		
AIC	6856.80		
BIC	6975.41		

Significant at *10%, **5%, ***1%.

network appears to be 0.644, which is even higher than that of Model 5. On the other hand, the "Visit" network shows 0.091, which is almost zero. This result confirms the well-known fact that diverse aspects of one's social network convey different types of peer effects through them.

From the perspective of model selection, Model 6 appears to be the best model according to both AIC and BIC.

Marginal Effects

The impact of the peer effect on the outcome variables is measured by the overall change caused by manipulating each explanatory variable. For the SAR models, it can be computed in two ways: the "naive" and the "sophisticated" approach. The naive marginal effect is obtained from $f(\eta^*_{ig})\hat{\theta}$ for continuous variables("No. of rooms" and "No. of beds") and $P[y_{ig} = 1|x_{ik,g} = 1] - P[y_{ig} = 1|x_{ik,g} = 0]$ for dummy variables. This approach ignores the multiplier effect by ignoring the change in the probability caused by the change in explanatory variables. That is, it can be obtained by taking a derivative on the choice probability, $p_{ig}(\hat{\theta}) = F(\sum_{q=1}^{Q} W_q(\hat{\gamma})\mathbf{p}_g^*(\hat{\theta}) + x_{ig}\hat{\beta})$.

The sophisticated approach computes the marginal effect of an explanatory variable considering its effect on the equilibrium together to incorporate the multiplier effect. In this process, a change in i's characteristic changes not only i's own choice probability but the others' as well. Denote the aggregated weight matrix of g-th village as $W_g^{(A)} = \sum_{q=1}^{Q} \hat{\rho}_q W_q(\hat{\gamma})$ and its i-th row as $w_{i,g}^{(A)}$. The direct and indirect effects for the continuous variables ("No. of rooms" and "No. of beds" variables) are

$$\begin{aligned}\frac{\partial p_{i,g}^*}{\partial x_{ik,g}} &= f(w_{i,g}^{(A)}\mathbf{p}_g^* + x_{i,g}\hat{\beta})\left(\hat{\beta}_k + w_{i,g}^{(A)}\frac{\partial M_g}{\partial x_{ik,g}}\right)\\ \frac{\partial p_{j,g}^*}{\partial x_{ik,g}} &= f(w_{i,g}^{(A)}\mathbf{p}_g^* + x_{i,g}\hat{\beta})\left(w_{j,g}^{(A)}\frac{\partial M_g}{\partial x_{ik,g}}\right),\end{aligned}$$

where

$$\frac{\partial M_g}{\partial x_{ik,g}} = \left(I_{N_g} - f_{diag}(W_g^{(A)}\mathbf{p}_g^* + X_g\hat{\beta})W_g^{(A)}\right)^{-1} f_{diag}(W_g^{(A)}\mathbf{p}_g^* + X_g\hat{\beta})\left(\frac{\partial X_g}{\partial x_{ik,g}}\right),$$

and $\frac{\partial X_g}{\partial x_{ik,g}}$ is the $N \times K$ matrix of which (i,k) entry is 1 and 0 elsewhere. For the binary variables, the marginal effect of x_k is computed as $P[y_i = 1|x_{ik} = 1, M_g(x_{ik=1})] - P[y_i = 1|x_{ik} = 0, M_g(x_{ik=0})]$. These two effects are reported as direct and indirect marginal effects. The marginal effects are averaged over individuals.[13]

The results for all of the specifications are shown in Table 7 and Table 8, in percentage points. In Model 1, the magnitudes are smaller than in any other model, due to the lack of the multiplier effect. The main focus for the rest of the models is the difference between the naive and the sophisticated effects. In Model 2 and 3, the direct effects are larger than the naive effects in general due to the complementarity peer effect. This result confirms the positive multiplier effect computed from the parameter estimates. Policymakers might be interested in the marginal effect of the leaders. Under the complementarity, their direct effect tends to be higher than the naive effect, which suggests the positive feedback of benefit under the positive multiplier. Meanwhile, in the rest of the models where conformity appears to be dominant, the direct effects are lower than the naive effects. It is as expected, because the individuals will move toward the social norm, and such a motive will attenuate the consequences of any increase in their attributes. Note that the same pattern is observed on Model 5 as well, where the conformity parameter is estimated much lower than one. This implies that the pure conformity model is not necessary for the existence of the conformity motive. Instead, the relative magnitude of the peer effect and the conformity parameter is a more relevant indicator.

13. For the treatment of the marginal effects for the SAR model, see Chomsisengphet, Kiefer, and Liu (2018).

An interesting observation is that the indirect effects of Model 5 and 6 are higher than that of Model 3. Again, consider the leader group as an example. Even though those who are appointed as leaders might be less willing to join the program compared to those under complementarity, the other people's take-up rate can be increased even more by the same conformity motive. Considering the social status of the leaders, this interpretation is not inconceivable.

Table 7: Marginal effects: Model 1-3

	Model 1	Model 2			Model 3		
		Naive	Direct	Indirect	Naive	Direct	Indirect
Roof material: tile	1.43	1.34	1.34	0.002	1.69	1.72	0.004
No. of rooms	-0.35	-0.71	-0.71	-0.001	-0.86	-0.88	-0.002
No. of beds	-0.14	-0.35	-0.35	-0.001	-0.26	-0.27	-0.001
No latrine in houses	2.72	5.13	5.16	0.008	5.09	5.17	0.012
House not owned	0.16	0.42	0.42	0.001	0.69	0.70	0.002
No access to electricity	1.49	2.91	2.92	0.005	2.94	2.99	0.007
Leader group	4.83	8.68	8.73	0.013	9.14	9.31	0.021
General caste	-2.40	-3.24	-3.26	-0.005	-2.77	-2.81	-0.007
Minority castes	-0.35	3.27	3.29	0.005	5.44	5.54	0.013
Scheduled caste	3.89	5.59	5.61	0.009	4.55	4.63	0.011
Scheduled tribe	3.13	5.50	5.52	0.009	4.70	4.79	0.011
Islam	7.89	15.64	15.73	0.024	12.30	12.54	0.028
Union		1.69					
Money					-0.45		
Advice					-0.87		
Kerosine					0.56		
Medical					0.01		
Non-relatives					-0.09		
Relatives					1.89		
Temple					4.96		
Visit					3.13		

The numbers are in percentage points.

Effective Networks

The effective networks are constructed using only the ones found to convey significant peer effects. As a result, a total of three networks, Relatives household, Temple company, and Visit, are selected, and an aggregated effective network is constructed as a union of them. Its graph statistics are presented along with the previously shown statistics table.

The result shows that the union network is noticeably overspecified compared to the aggregated network. The outcome density is 4%, which is lower than that of the union network, 6%. More importantly, the spectral radius, which is a more accurate measure of a network's connectivity, is 9.78. Compared to 14.07

Table 8: Marginal effects: Model 4-6

	Model 4			Model 5			Model 6		
	Naive	Direct	Indirect	Naive	Direct	Indirect	Naive	Direct	Indirect
Roof material: tile	1.83	1.55	0.002	1.99	1.85	0.007	1.86	1.71	0.005
No. of rooms	-0.65	-0.55	-0.001	-1.14	-1.06	-0.004	-1.07	-0.99	-0.003
No. of beds	-0.28	-0.23	0.000	-0.25	-0.23	-0.001	-0.36	-0.33	-0.001
No latrine in houses	5.52	4.72	0.005	5.28	4.93	0.019	5.30	4.89	0.016
House not owned	0.28	0.24	0.000	0.73	0.67	0.002	0.80	0.74	0.002
No access to electricity	2.66	2.24	0.002	3.08	2.85	0.011	3.17	2.90	0.009
Leader group	14.31	11.66	0.012	9.96	9.14	0.033	10.30	9.34	0.029
General caste	-2.73	-4.09	-0.004	-2.97	-2.77	-0.010	-3.01	-2.79	-0.009
Minority castes	-2.01	-1.72	-0.002	6.80	6.24	0.023	4.53	4.12	0.013
Scheduled caste	7.11	5.94	0.006	4.22	3.91	0.014	4.72	4.32	0.014
Scheduled tribe	6.55	5.39	0.006	5.12	4.72	0.017	5.00	4.55	0.014
Islam	21.49	17.05	0.018	10.92	9.97	0.036	13.73	12.35	0.038
Money	0.50			-0.79					
Advice	1.62			-1.47					
Kerosine	1.19			0.65					
Medical	0.96			1.33					
Non-relatives	-1.26			-0.56					
Relatives	2.46			2.43			5.56		
Temple	3.43			7.06					
Visit	0.41			5.15			3.39		

The numbers are in percentage points.

Table 9: Comparison with the relevant networks

	Aggregated		Union		Selected		Individual	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Degree	6.36	1.27	9.22	1.69	2.36	0.16	3.00	0.24
Density	0.04	0.02	0.06	0.04	0.02	0.01	0.03	0.01
Spectral Radius	9.78	2.35	14.07	3.20	4.12	0.54	5.22	0.66

Note: the selected networks are Relatives, Temple and Visit. The statistics are averaged over the three networks.

of the union network, this is a significantly lower number. Network researchers are often concerned about network misspecification, especially with missing links. However, this can be considered as evidence that suggests that misspecification in the other direction, or overspecification, could also be a problem.

5 Conclusion

In this study, a generalized SAR model that unifies and estimates the complementarity and the conformity peer effects is proposed. The distinguished feature of the model is its weight matrix that is a generalized graph Laplacian with its diagonal elements multiplied by the conformity parameter. With the microeconomic foundation, it is shown that the peer effect from a network consists of the complementary benefit and the deviation cost, which are identified in the estimation model. A definition of the social multiplier is drawn upon the marginal effect analysis, and it is suggested that the positive multiplier effect arises only when the complementary benefit is sufficiently higher than the deviation cost. The threshold for the positive multiplier is derived as a function of the ratio between the sum of squared degrees and the sum of degrees.

The proposed model is also extended to incorporate the rational expectation model of Lee, Li, and Lin (2014) and multiple networks. As a result, the GSAR model can be implemented for discrete outcome variables. This extension is demonstrated with the data from the microfinance program in Indian villages collected by Banerjee et al. (2013). In the original research, the authors found no sign of peer effects. In contrast, the multiple surveyed networks and the GSAR model captures a significant peer effect with conformity motive. Moreover, it turns out that only a small number of dimensions of the social network transmit the peer effect. By estimating the conformity parameter separately for the different dimensions, it is presented that various dimensions of one's social network may be driven by distinct motives.

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A Proofs

The following Lemma will be used for the identification result for both continuous and binary outcome models.

Lemma 1. *Let $Q_A = A_s T^{-1}$ and $Q_D = D_s T^{-1}$, where $T = I - \rho_0 W(\gamma_0)$. Then, $(\rho - \rho_0)Q_A X_s \beta_0 - (\rho\gamma - \rho_0\gamma_0)Q_D X_s \beta_0 = 0$ if and only if $\rho = \rho_0$ and $\gamma = \gamma_0$.*

Proof. Suppose that $\rho \neq \rho_0$. Then, $(\rho - \rho_0)A_s T^{-1} X\beta_0 = (\rho\gamma - \rho_0\gamma_0)D_s T^{-1} X\beta_0$. First, if $\gamma = \gamma_0$, $(\rho - \rho_0)A_s T^{-1} X\beta_0 = (\rho - \rho_0)\gamma_0 D_s T^{-1} X\beta_0$. If $\gamma_0 = 0$, $A_s T^{-1} X\beta_0 = 0$, and this violates Assumption 4 (ii). If $\gamma_0 \neq 0$, $\frac{1}{\gamma_0} D_s^{-1} A_s T^{-1} X\beta_0 = 0$, which is also a violation of Assumption 4 (iii). Therefore, $\rho = \rho_0$.

Next, suppose that $\gamma \neq \gamma_0$. If $\rho = \rho_0$, $\rho_0(\gamma - \gamma_0)D_s T^{-1} X\beta_0 = 0$, since D_s is an invertible diagonal matrix, $T^{-1} X\beta_0 = 0$, and it is a violation of Assumption 4 (i). Next, suppose that $\rho \neq \rho_0$. If $\rho\gamma - \rho_0\gamma_0 = 0$, it violates Assumption 4 (ii), and, if $\rho\gamma - \rho_0\gamma_0 \neq 0$, it violates Assumption 4 (iii).

Therefore, $(\rho - \rho_0)Q_A X_s \beta_0 - (\rho\gamma - \rho_0\gamma_0)Q_D X_s \beta_0 = 0$ implies $\rho = \rho_0$ and $\gamma = \gamma_0$. It is trivially satisfied if $\rho = \rho_0$ and $\gamma = \gamma_0$. □

Proof of Proposition 1. Consider an $N \times (K + 2)$ instrument vector Z.

$$
\begin{aligned}
\alpha(\theta) &= \text{plim}_{N\to\infty} N^{-1} Z'(y - \rho W_s(\gamma)y - X\beta) \\
&= \text{plim}_{N\to\infty} N^{-1} Z'[(I - \rho W_s(\gamma))(T^{-1} X_s \beta_0 + T^{-1}\varepsilon) - X\beta] \\
&= \text{plim}_{N\to\infty} N^{-1} Z'[(I - \rho W_s(\gamma) T^{-1} X\beta_0 - X\beta] + \text{plim}_{N\to\infty} N^{-1} Z'(I - \rho W_s(\gamma))T^{-1}\varepsilon \\
&= \text{plim}_{N\to\infty} N^{-1} Z'[(I - \rho W_s(\gamma))T^{-1} X_s \beta_0 - X_s \beta]
\end{aligned}
$$

As $(I - \rho W(\gamma))T^{-1} = [T + (\rho_0 + \rho)A_s - (\rho_0\gamma_0 - \rho\gamma)D_s]T^{-1} = I + (\rho_0 - \rho)A_s T^{-1} - (\rho_0\gamma_0 - \rho\gamma)D_s T^{-1}$,

$$
\begin{aligned}
\alpha(\theta) &= \text{plim}_{N\to\infty} N^{-1} Z'[(\rho_0 - \rho)A_s T^{-1} X\beta_0 - (\rho_0\gamma_0 - \rho\gamma)D_s T^{-1} X_s \beta_0 + X_s\beta_0 - X_s\beta] \\
&= \text{plim}_{N\to\infty} N^{-1} Z'[Q_A X\beta_0, -Q_D X\beta_0, X] \begin{bmatrix} \rho_0 - \rho \\ \rho_0\gamma_0 - \rho\gamma \\ \beta_0 - \beta \end{bmatrix}.
\end{aligned}
$$

By Lemma 1, the parameters are identified under Assumption 4. □

The following proof follows Liu and Zhou (2017).

Proof of Proposition 4. Consider two observationally equivalent sets of parameters, $\theta = (\rho, \gamma, \beta)$ and $\tilde{\theta} =$

$(\tilde{\rho}, \tilde{\gamma}, \tilde{\beta})$. Then, for each set, there exists an equilibrium that satisfies

$$p_i^*(\theta) = F_i(\rho a_i \mathbf{p}^* - \rho\gamma d_i \mathbf{p}^* + x_i\beta)$$
$$\tilde{p_i}^*(\tilde{\theta}) = F_i(\tilde{\rho} a_i \mathbf{p}^* - \rho\tilde{\gamma} d_i \mathbf{p}^* + x_i\tilde{\beta}),$$

where a_i and d_i are i-th row of A and D. Since $p_i^*(\theta)$ and $\tilde{p_i}^*(\tilde{\theta})$ are observationally equivalent, $p_i^*(\theta) = \tilde{p_i}^*(\tilde{\theta})$, and $F_i(\rho a_i \mathbf{p}^* - \rho\gamma d_i \mathbf{p}^* + x_i\beta) = F_i(\tilde{\rho} a_i \mathbf{p}^* - \rho\tilde{\gamma} d_i \mathbf{p}^* + x_i\tilde{\beta})$. Under Assumption 4. (i), the model parameters are identified if

$$\begin{bmatrix} A\mathbf{p} \\ -D\mathbf{p} \\ X \end{bmatrix}' \begin{bmatrix} \rho - \tilde{\rho} \\ \rho\gamma - \tilde{\rho}\tilde{\gamma} \\ \beta - \tilde{\beta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{7}$$

In this model, vertices may be isolated, but by Assumption (), the largest subgraph with connected vertices can be found. Denote it as G_s and of which adjacency and degree matrices as A_s and D_s. The peer effect and the conformity parameters are identified by utilizing G_s. Then, under Assumption 3, the following statement holds:

$$\rho A_s\mathbf{p} - \rho\gamma D_s\mathbf{p} = \tilde{\rho} A_s\mathbf{p} - \tilde{\rho}\tilde{\gamma} D_s\mathbf{p}, \text{ if and only if } \rho = \tilde{\rho} \text{ and } \gamma = \tilde{\gamma}.$$

Then, Assumption 4 (ii) implies that θ is identified from (7) by including the isolated individuals. □

Proof of Proposition 3. The threshold for the social multiplier effect is obtained by solving the following equation with respect to ρ:

$$\det(I - \rho W) = \det(I - \rho(A - \gamma D)) = 1.$$

Denote the solution as $\bar{\rho}(\gamma)$. Then, apply the sparse matrix approximation (Higham 2008; Ipsen and Lee 2011) and obtain

$$\det(I - \rho(A - \gamma D)) \approx \exp(\mathrm{tr}(\log(I - \rho(A - \gamma D)))) = 1.$$

By taking a logarithm on both-hand sides, the problem reduces to the following equation.

$$\mathrm{tr}(\log(I - \rho(A - \gamma D))) = 0$$

For the logarithm in the trace, use the power series expansion of matrix logarithm, which is available due to Assumption 2.

$$\log(I-\rho W) = \frac{(-\rho(A-\gamma D))^1}{1} - \frac{(-\rho(A-\gamma D))^2}{2} + \frac{(-\rho(A-\gamma D))^3}{3} + \cdots \tag{8a}$$

$$= -\rho(A-\gamma D) - \frac{\rho^2(A-\gamma D)^2}{2} - \frac{\rho^3(A-\gamma D)^3}{3} - \cdots \tag{8b}$$

The expansion (1b) is truncated at the second term and each term is separated by the property of trace.

$$\mathrm{tr}\left(-\rho(A-\gamma D) - \frac{\rho^2(A-\gamma D)^2}{2}\right) = 0 \tag{9a}$$

The first term of (2a) is represented as a sum of degrees:

$$\mathrm{tr}(-\rho(A-\gamma D)) = -\rho(\mathrm{tr}(A) - \gamma \mathrm{tr}(D)) = \rho\gamma \sum_i d_i,$$

and the second term is:

$$\begin{aligned}\mathrm{tr}\left(-\frac{\rho^2(A-\gamma D)^2}{2}\right) &= \mathrm{tr}(-\frac{\rho^2}{2}(A^2 - \gamma AD - \gamma DA + \gamma^2 D^2)) \\ &= -\frac{\rho^2}{2}\sum_{i=1}^{N}\sum_{j\neq i} a_{ij}a_{ji} - \frac{\rho^2\gamma^2}{2}\sum_i d_i^2 \\ &= -\frac{\rho^2}{2}\sum_{i=1}^{N} d_i - \frac{\rho^2\gamma^2}{2}\sum_i d_i^2. \quad \text{(by the symmetry assumption)}\end{aligned}$$

Then, the equation (2a) is rewritten by adding the two terms:

$$\rho\gamma\sum_{i=1}^{N} d_i - \frac{\rho^2}{2}\sum_{i=1}^{N} d_i - \frac{\rho^2\gamma^2}{2}\sum_i d_i^2 = 0$$

$$\gamma\sum_{i=1}^{N} d_i - \rho\frac{1}{2}\sum_{i=1}^{N} d_i - \rho\frac{\gamma^2}{2}\sum_i d_i^2 = 0$$

The equation is a polynomial equation of degree 1, and its solution is

$$\bar{\rho}(\gamma) = \frac{2\gamma\sum_{i=1}^{N} d_i}{\sum_{i=1}^{N} d_i + \gamma^2\sum_{i=1}^{N} d_i^2} = \frac{2\gamma}{1+\gamma^2\sum_{i=1}^{N} d_i^2/\sum_{i=1}^{N} d_i}.$$

This completes the proof. □

B Tables

Table A1: Full descriptive statistics

			Whole samples		Participating samples		Participating samples w/ caste data	
			N=14890		N=9591		N=7919	
			Number of villages = 75		Number of villages = 43		Number of villages = 43	
	Min	Max	Mean	SD	Mean	SD	Mean	SD
Outcome variable								
Microfinance take-up	0	1			0.182	0.386	0.178	0.382
Income proxies								
Roof material: tile	0	1	0.32	0.47	0.34	0.47	0.37	0.48
Number of rooms	0	20	2.36	1.32	2.34	1.31	2.41	1.3
Number of beds	0	50	0.85	1.33	0.86	1.42	0.82	1.23
Without Latrine	0	1	0.73	0.44	0.73	0.45	0.73	0.45
House not owned	0	1	0.1	0.3	0.1	0.3	0.1	0.3
Without electricity	0	1	0.38	0.49	0.38	0.48	0.38	0.48
Social Status								
Leader group	0	1	0.12	0.33	0.12	0.33	0.13	0.33
Other backward castes							0.53	0.5
General caste							0.12	0.33
Minority castes							0.2	0.13
Scheduled caste							0.28	0.45
Scheduled tribe							0.05	0.22
Religion								
Hinduism	0	1	0.96	0.2	0.95	0.21	0.95	0.21
Islam	0	1	0.04	0.2	0.05	0.21	0.05	0.21
Village								
Village Population					223.21	56.17	184.16	86.08
Graph statistics								
Union								
Average degree					9.2	1.65	9.22	1.69
Density					0.04	0.01	0.06	0.04
Spectral radius					15.08	2.56	14.07	3.2
Individual networks								
Average degree					2.73	0.3	3	0.24
Density					0.01	0.001	0.03	0.01
Spectral radius					5.76	0.39	5.22	0.66

C Discussion of Banerjee et al. (2013)

This section discusses the methodological differences between the GSAR model and the diffusion model of Banerjee et al. (2013), BCDJ henceforth. The main finding of BCDJ is the role of non-participants in the information diffusion process. Although the non-participants are less likely to pass information to others, they play a larger role in the overall take-up rate due to their larger number. Meanwhile, the authors do not find any evidence of the "endorsement effect," which corresponds to the peer effect of this paper, in their study. In other words, not only the probability of participation does not increase with the number of already participating peers, but even slightly decreases. Their paper does not fully explain this negative result, but some hints are suggested by Chandrasekhar and Lewis (2011). In that paper, the insignificant peer effect is attributed to the missing links in sampled networks. As mentioned in Chapter 4, the organizers surveyed only 50% of the villagers, so the issue of misspecification is not inconceivable.

Conversely, the peer effect is estimated as significantly positive in this paper with the same surveyed networks. This result may be viewed from a different angle because the missing links can only decrease the estimate of peer effect, not increase. For the estimation, BCDJ uses a two-step procedure where the coefficients for the independent variables are estimated without networks on the leader group first, and those first-step estimates are used to estimate the probability of passing information and the endorsement effect. Their methodology is based on the diffusion process, where the individuals are not aware of the microfinance program until they are reached by those who have already acquired that information. Once they learn about the program, a decision must be made at the moment of learning, and there is no second chance thereafter (the Susceptible, Infectious, Recovered (SIR) model). At each stage, the decision is made by the following logistic function:

$$\log \frac{p_{it}}{1-p_{it}} = X_i\hat{\beta} + \lambda F_{it},$$

where F_{it} is the number of participants in one's peer group divided by the number of the peers informed about the program. The subscript t denotes the choice probability obtained for each time period. Note that $\hat{\beta}$ is not estimated along with λ. It is a first-step estimate from the leader group, and the other parameters, including λ, is estimated later based on $\hat{\beta}$. Other than this, the specification is very similar to the local-average model, and a guess can be made that the idea of the endorsement effect is based on the positive social multiplier. From this, a hypothetical SAR estimation can be conducted to simulate their methodology. The peer effect is separately estimated with the coefficients for the household characteristics estimated from the leader groups. The two-step procedure is: 1) estimate the coefficients only with the leader groups, and

2) estimate the peer effect with the entire sample using the estimated coefficients. The main assumption implicitly made here is that the coefficients do not vary across the rest of the individuals outside of the leader group. As the leaders are chosen by the organization based on their characteristics, it is reasonable to assume that there is no significant social link between them. Therefore, the estimated coefficients will be asymptotically unbiased if there are no unobservable differences. Unfortunately, a regression including the leader dummy as a covariate shows that the leaders have a significantly higher tendency to participate.[14] Therefore, one can expect that the coefficients for the non-leader individuals, which should be lower than those of the leaders, will be overestimated and compensated by the negative peer effect. This conjecture is supported by the estimation result shown in A2. The explanatory variables are based on the dataset and

Table A2: The two-step and the SAR estimation

	Two-step estimation	SAR
	N=995	N=7919
Constant	-.871***	-1.668***
	(.239)	(.107)
Number of rooms	-.155**	-.086***
	(.066)	(.029)
Number of beds	-.089	-.056*
	(.075)	(.030)
No private latrine	.260	.129*
	(.177)	(.076)
House not owned	.127	.335***
	(.178)	(.062)
	N=7919	N=7919
Peer effect	-.122***	.082***
	(.015)	(.012)

codes published by the authors. It shows that the estimated peer effect is significantly negative, which is parallel to the estimate reported by BCDJ.[15] As expected, the standard SAR regression on the entire sample, including the leader group, shows a lower constant term and a positive peer effect. The lower estimate for the constant suggests that the entire group has a much lower baseline preference for microfinance.

Another possible explanation is the difference in the timeline. First, the SIR diffusion model does not allow late decisions. The spread pattern over time assumes that the newcomers are only those who are newly informed about the program. However, it is also possible that they initially declined at the time of learning but opted in later.[16]. In that case, the peer effect on the equilibrium of the SAR model will be more accurate.

That said, these results do not imply that these two approaches are exclusive. The SAR model has a limitation: it relies upon the strong assumption of the rational expectation and does not reflect the change

14. See Table A3.
15. See Table A3 for the two-step estimates with a richer set of covariates. The result is similar.
16. This is not verifiable with the published data because the panel data is only available as overall participation rates over time and the participants are not identifiable.

of the take-up rate over time. Instead, estimation is easier to perform and does not require panel data. Meanwhile, the diffusion model explains the diffusion pattern over time better but the probability of passing information must be estimated as well. The additional computational burden and theoretical assumptions will be the cost that must be incurred.

Table A3: The two-step and the SAR estimation with additional covariates

	Two-step estimation	SAR
	N=995	N=7919
Constant	-1.186***	-2.050***
	(.260)	(.116)
Roof material	.251	.143**
	(.166)	(.060)
Number of rooms	-.131**	-.063**
	(.067)	(.030)
Number of beds	-.063	-.032
	(.076)	(.030)
Islam	1.290***	1.046***
	(.404)	(.138)
No private latrine	.337*	.200**
	(.187)	(.080)
House not owned	-.159	.057
	(.308)	(.096)
House not owned	-.159	.057
	(.308)	(.096)
No electricity	.022	.214***
	(.190)	(.067)
General caste	-.153	-.409***
	(.221)	(.113)
Minority caste	.559	.153
	(.659)	(.231)
Scheduled caste	.385*	.396***
	(.198)	(.068)
Scheduled tribe	-.390	.223*
	(.448)	(.131)
Leader		.564***
		(.085)
	N=7919	N=7919
Peer effect	-.122***	.082***
	(.015)	(.012)