HOMEWORK 4

- Show all the work/derivation with neat writing (this counts for the score). Engineering paper should be used. Each student should finish the homework independently.

1. **Deformation gradient**: Consider a unit cube of material (represented as a square since the depth dimension does not change) undergoing three different types of deformation as shown in Figs. 1 a, b and c. The coordinates of the corners of the undeformed and deformed square (in red) are indicated. The z coordinates remain the same in deformed and undeformed configurations, i.e. z = Z. Determine the deformation gradient **F** and write it as a 3x3 matrix for each case. Using your answer, compute the change in volume of the deformed cube using your answer. Based on your answer for change in volume for deformation (a), physically reason why the change in volume for (c) is different from (b).

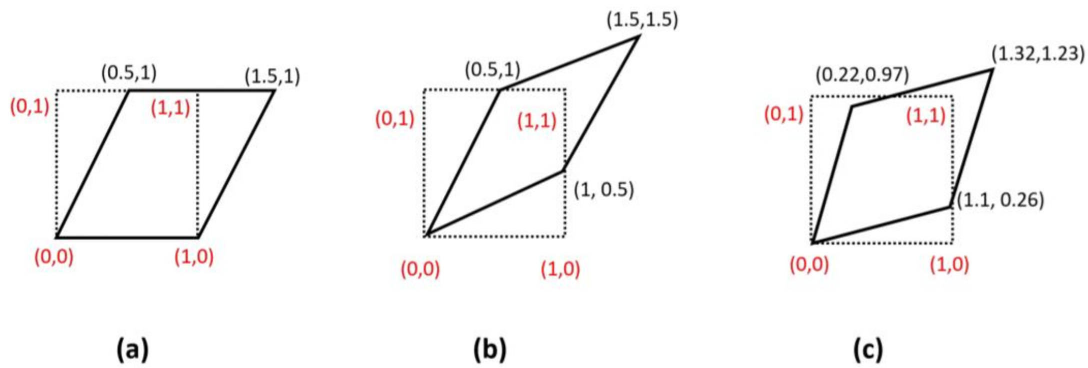

Figure 1: Deformation of a unit cube (represented by square with constant non-deforming depth dimension) for Problem 1

2. **Distribution tensor and eigen values**: Let us consider an isotropic distribution of chains such the probability density function of the vector in 2D $\boldsymbol{r} = (r_x, r_y)$ is given by

$$p_0(\boldsymbol{r}) = \frac{1}{2\pi\langle r_0^2\rangle}\exp\left(-\frac{r^2}{2\langle r_0^2\rangle}\right)$$

Let there be macroscopic deformation such that the probability density function of chains after deformation becomes a multi-variate Gaussian distribution of the form

$$p(\boldsymbol{r}) = \frac{1}{2\pi\sqrt{\det(\boldsymbol{\Sigma})}}\exp\left(-\frac{1}{2}\boldsymbol{r}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{r}\right)$$

where $\boldsymbol{\Sigma}$ denotes the covariance matrix given by

$$\boldsymbol{\Sigma} = \frac{1}{3}\begin{pmatrix}5 & 4\\ 4 & 5\end{pmatrix}$$

Given that $\langle r_0^2\rangle = 2$,

(i) Plot the probability density function p(r) as contours.

(ii) Find the eigenvalues of the tensor $\boldsymbol{\Sigma}$