Confronting the challenges of global EoR detection

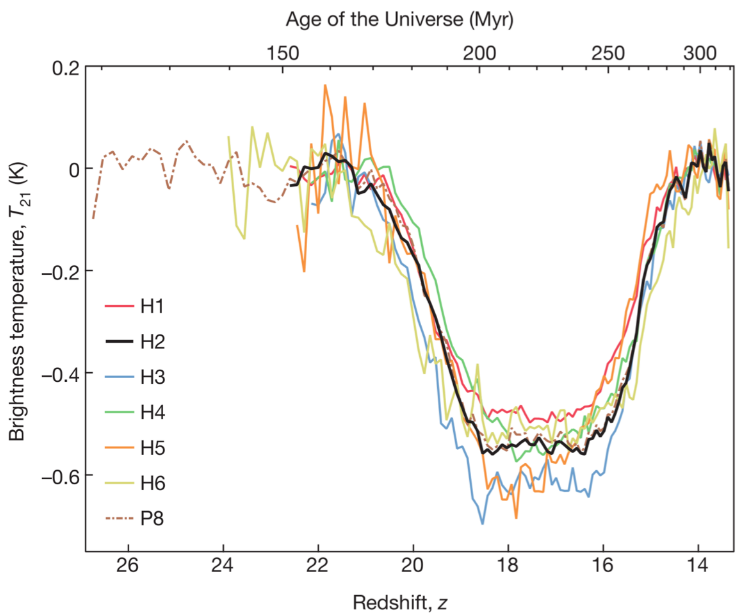
KEITH TAUSCHER

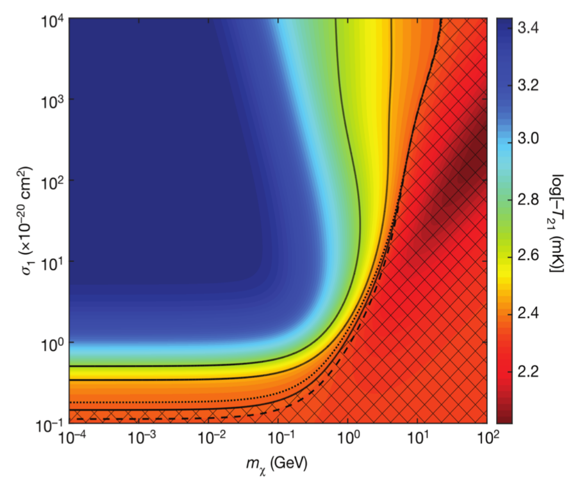
IN COLLABORATION WITH:
DAVID RAPETTI, JACK BURNS, RICHARD BRADLEY

Global 21-cm signal analysis: two distinct problems

1) Extracting shape of global signal

2) Model-dependent science



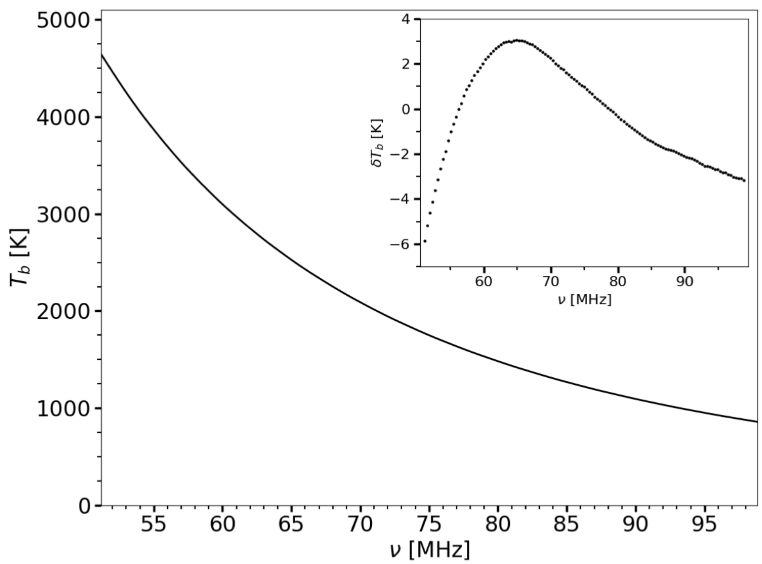


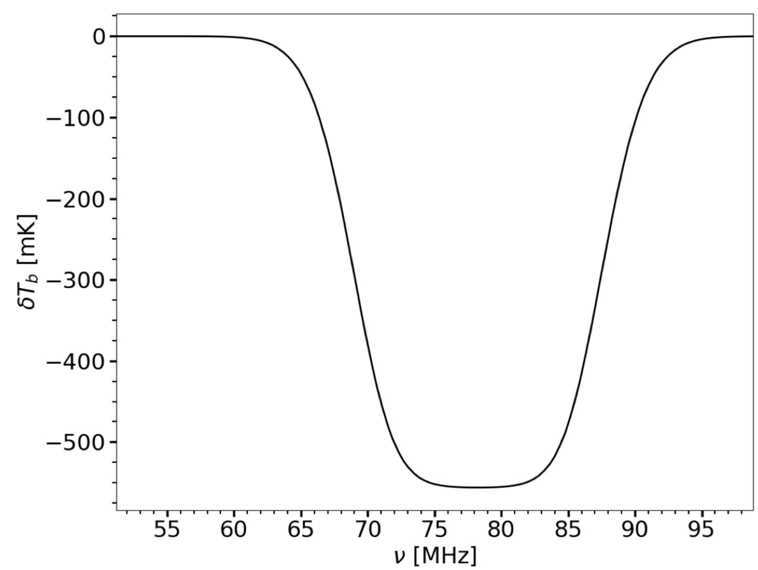
21-cm spectrum shape reported by Bowman et al. (2018)

Dark matter constraints by Barkana (2018)

Recent background: EDGES

EDGES released a single sky-averaged spectrum (50-100 MHz) in which they have fit a flattened absorption trough.





Data from Bowman et al. (2018). Fit and figures from Bradley et al. (2018).

Main problems of shape extraction

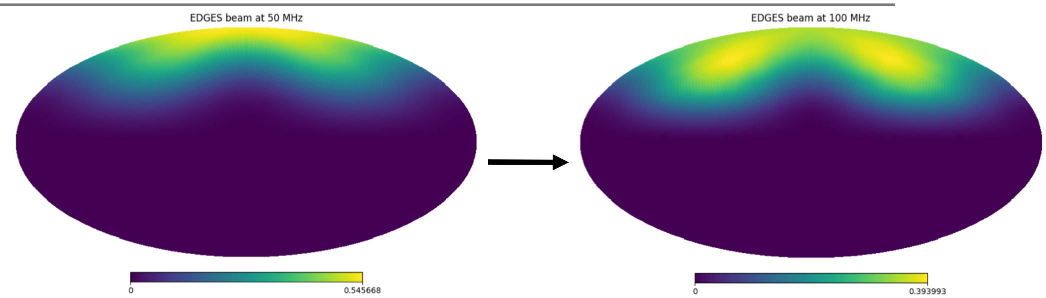
A. Beam effects: simple spectral shapes of individual foreground sources are distorted.
   a. For a deterministic (pre-modeling) correction of the data for beam-averaging to succeed, the beam and foreground must be known to the -50 dB level or better.

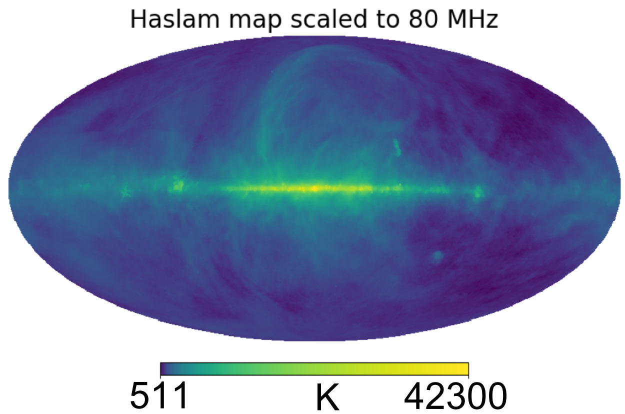
B. Observations must be statistically limited: high confidence extraction requires every $\gtrsim$1 mK effect be modeled.
   a. Models must be shown to accurately describe each effect individually.
   b. Specific model$\Leftrightarrow$effect relationships are preferred over generic models.

Beam effects: above the horizon

- Beam effects are traditionally thought of as affecting observations above the horizon through spatial averaging properties varying by frequency.
    - To correct for these effects with a priori knowledge, very precise beam and foreground models are required.

- Even if beam chromaticity is removed, spatial averaging still distorts the spectral shape of the foregrounds.

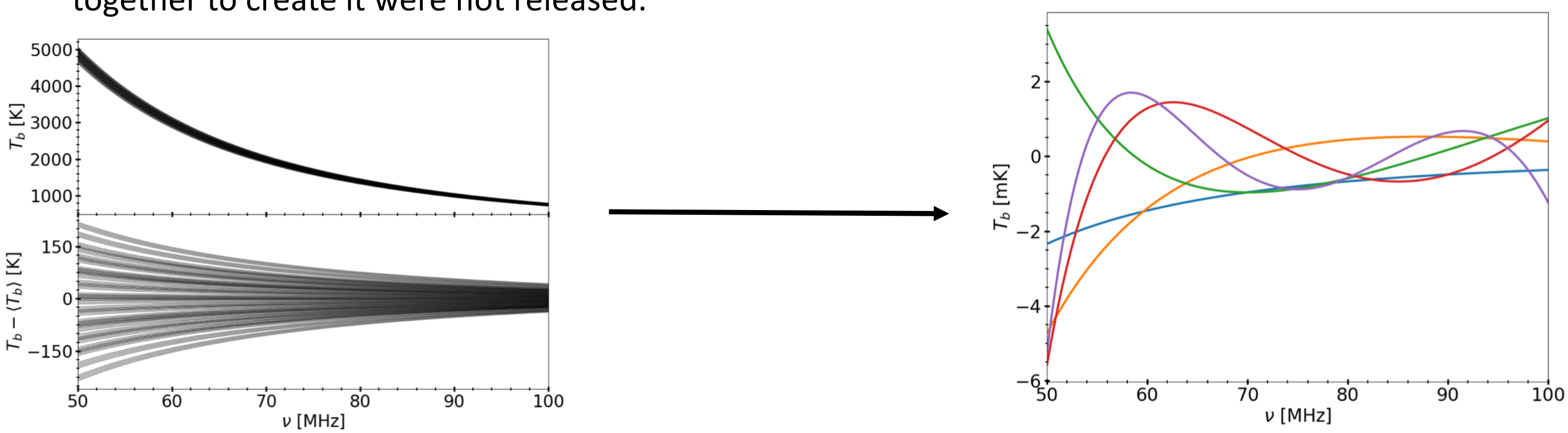




Beam patterns courtesy of EDGES team. Foreground map from Haslam et al. (1982)

Beam effects: above the horizon (continued)

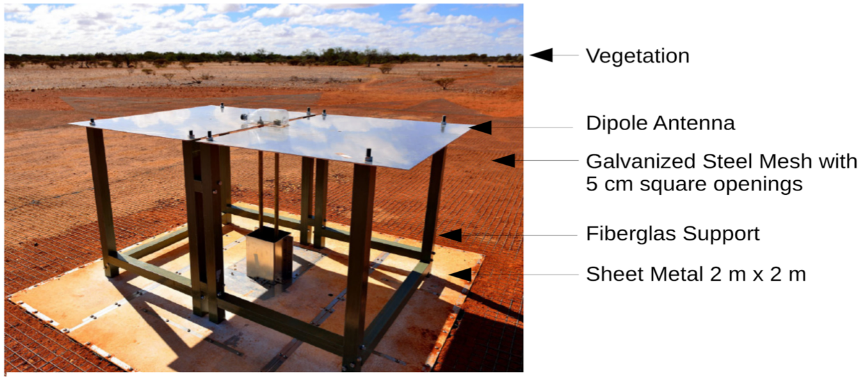
- Since a priori beam/foreground knowledge is insufficient, specific models must be formed with training sets and e.g. Singular Value Decomposition (SVD).
  - This essentially assumes that the beam weighted foreground spectrum is of a form which can be described by modes of variation in a training set instead of assuming a single spectral dependence.
  - No training set yet exists for the publicly released EDGES dataset since the times that were averaged together to create it were not released.



Beam effects: below the horizon

- Small or imperfect ground planes can act as patch antennas, absorbing sky radiation in a spatially and spectrally dependent manner.

- Time dependence is expected to differ from a multiplicative effect because of offset locations of primary antenna beam and resonance beams.



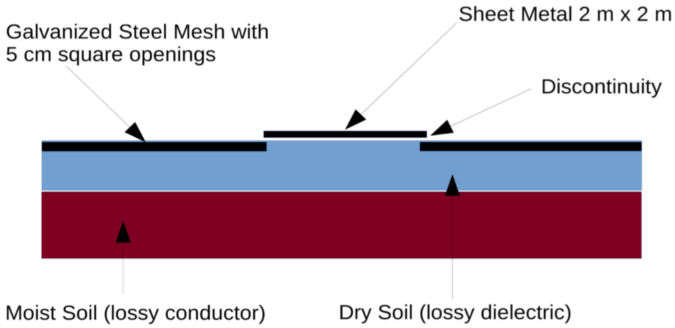
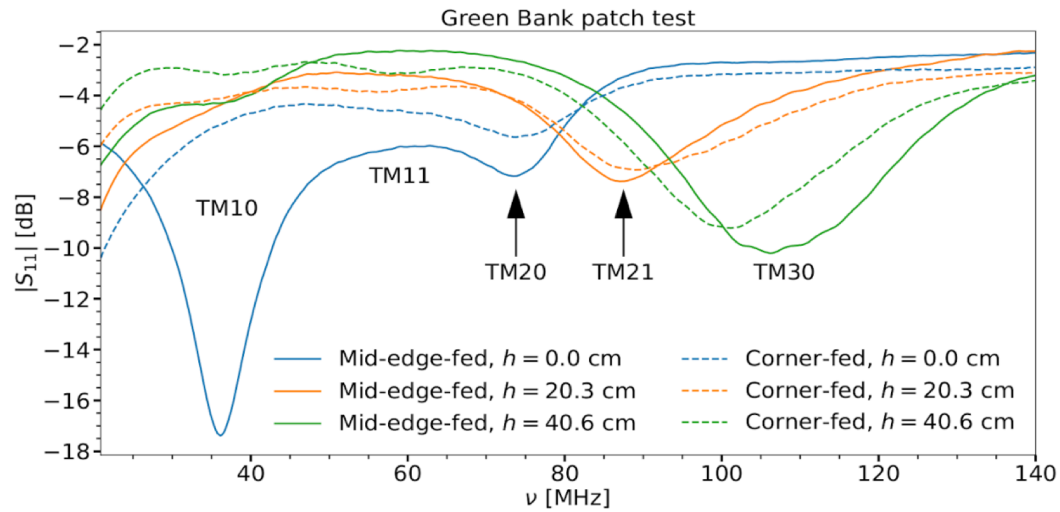


Photo courtesy of EDGES team. Cross-section sketch from Bradley et al. (2018).

Beam effects: below the horizon (continued)

- Physical patch antenna placed on wet soil at Green Bank Observatory
  - Variant fed from middle of edge of patch is illustrated in left panel
  - Difference between feed points is related to spatial dependence of resonant absorption



Described in Bradley et al. (2018).

EDGES fit: flattened Gaussian

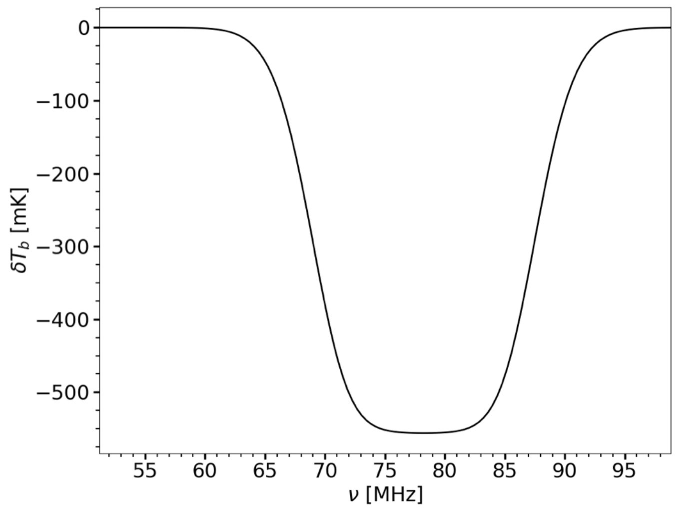
Parameter	Value
A	-556 mK
ν_0	78.2 MHz
w	18.8 MHz
τ	5.8

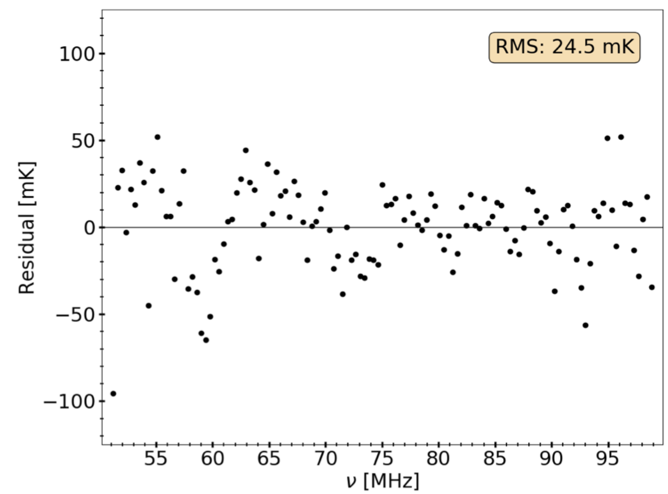
Signal:

$$T_{21} = A\left(\frac{1 - e^{-\tau e^B}}{1 - e^{-\tau}}\right) \quad \text{where} \quad B = \frac{4(\nu - \nu_0)^2}{w^2} \ln\left[-\frac{1}{\tau}\ln\left(\frac{1 + e^{-\tau}}{2}\right)\right]$$

Foreground:

$$T_{\rm fg} = a_0\left(\frac{\nu}{\nu_{\rm ref}}\right)^{-2.5} + a_1\left(\frac{\nu}{\nu_{\rm ref}}\right)^{-2.5}\ln\left(\frac{\nu}{\nu_{\rm ref}}\right) + a_2\left(\frac{\nu}{\nu_{\rm ref}}\right)^{-2.5}\left[\ln\left(\frac{\nu}{\nu_{\rm ref}}\right)\right]^2 + a_3\left(\frac{\nu}{\nu_{\rm ref}}\right)^{-4.5} + a_4\left(\frac{\nu}{\nu_{\rm ref}}\right)^{-2}$$





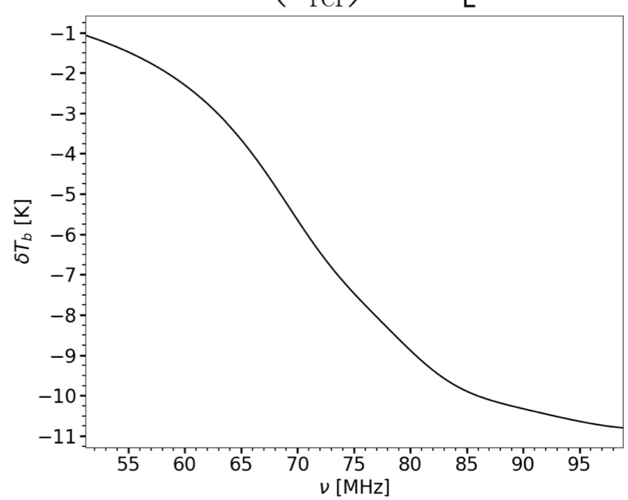
Data and models from Bowman et al. (2018). Fit and figures from Bradley et al. (2018).

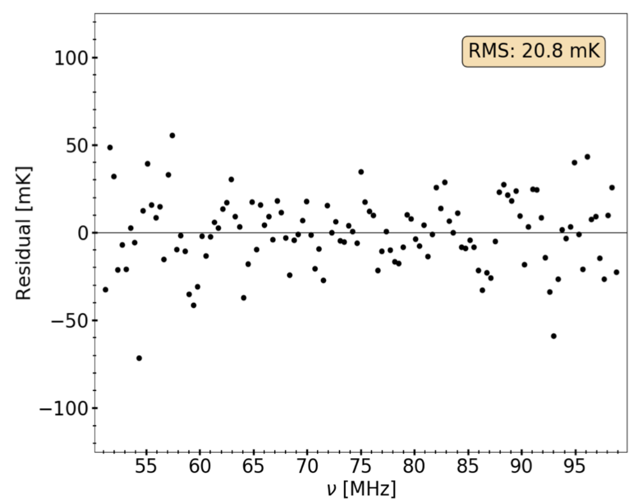
EDGES fit: resonances

Mode	ν_0 [MHz]	A [mK]	Q
TM20	73.8	-2235	3.9
TM21	84.2	-2469	3.8
TM30	111.8	-9403	1.3

3 Resonances: $$T_{\rm res} = \frac{A\nu^3\nu_0}{\nu^4 + Q^2(\nu^2 - {\nu_0}^2)^2}$$

Foreground: $$T_{\rm fg} = \left(\frac{\nu}{\nu_{\rm ref}}\right)^{-2.5}\left[a_0 + a_1 \ln\left(\frac{\nu}{\nu_{\rm ref}}\right)\right]$$

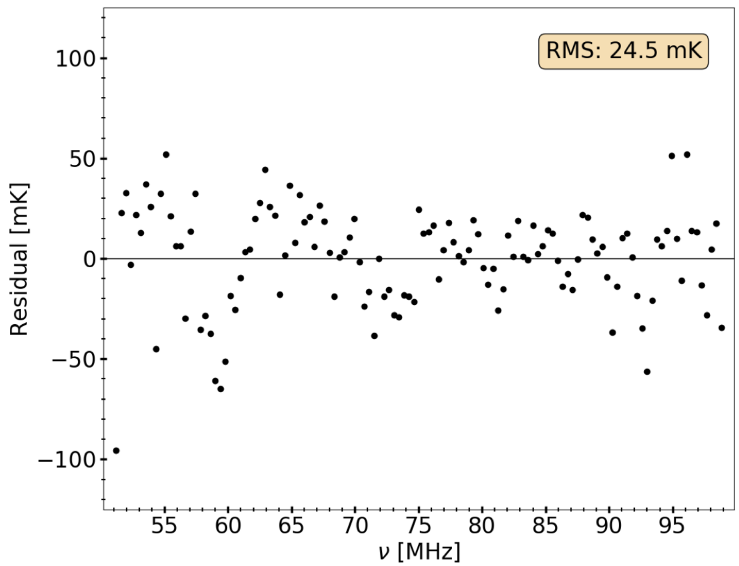




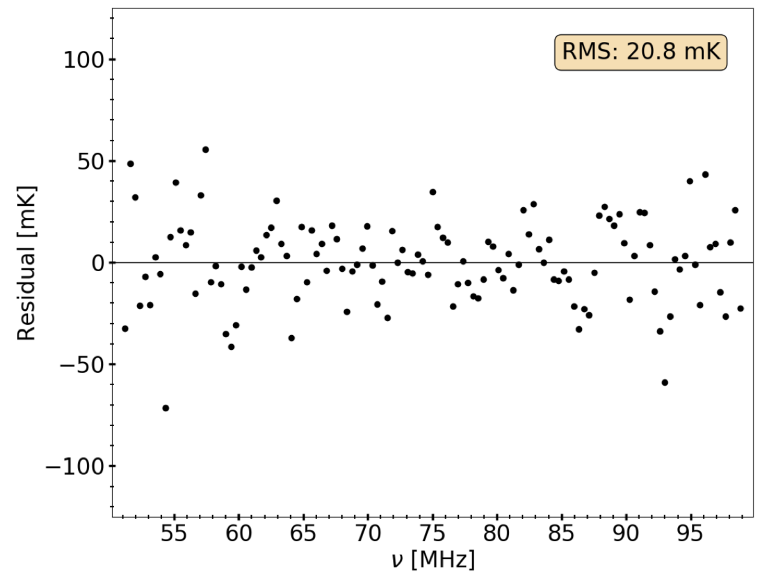
Data from Bowman et al. (2018). Fit and figures from Bradley et al. (2018).

EDGES fit: residual comparison

Flattened Gaussian: 9 parameters



Three resonances: 11 parameters



Statistically limited observations

- If observations are systematically limited, then:
  - Fit residuals are not noiselike.
  - Biases in modeling confuse the extraction of the signal, casting doubt on results.
  - The data provide no power to discern between different models.

- Goodness-of-fit statistics ascertain whether fit residuals are noiselike.
  - Significance thresholds on these statistics determine whether observations are statistically limited.
  - In order to measure the absolute goodness-of-fit, the noise covariance must be computed.
  - Goodness-of-fit statistics are not currently computable for the released EDGES data because no precise noise level is known.

Averaging pros and cons

More averaging

- Analysis computationally simpler
- Lower noise level ➞ easier visualization
- Noise distribution more Gaussian
- Averages out differences, losing constraining power

Less averaging

- Analysis more computationally intensive
- Higher noise level ➞ must rely more on statistics
- Noise distribution less Gaussian
- More constraining power due to greater ability to differentiate between components

- **Averaging should be performed only to the extent that:**
  - **Noise is sufficiently Gaussian for statistical purposes.**
  - **There is no significant difference between data being averaged.**
  - **Available computers can perform analysis.**

A new goodness-of-fit statistic for 21-cm cosmology

- Traditional chi-squared statistic is insensitive when S/N is too low, even with large amounts of data:

$$\chi^2 = \frac{1}{N}\sum_{k=1}^{N}\left(\frac{y_k - \mathcal{M}_k}{\sigma_k}\right)^2$$

Data Error Model

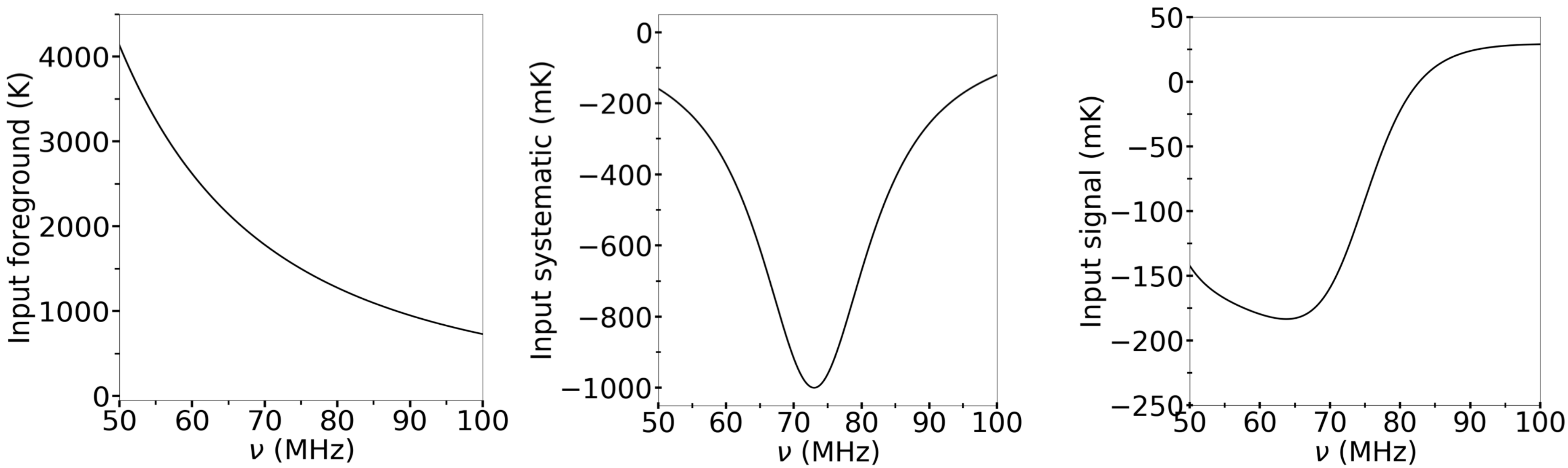
- New psi-squared statistic designed to look at the squared channel-to-channel correlations of residuals instead of their absolute value:

$$\psi^2 = \sum_{k=1}^{N-1}\left(\frac{N-k}{N-1}\right){\rho_k}^2 \quad \text{where} \quad \rho_k = \frac{1}{N-k}\sum_{q=1}^{N-k}\left(\frac{y_q - \mathcal{M}_q}{\sigma_q}\right)\left(\frac{y_{q+k} - \mathcal{M}_{q+k}}{\sigma_{q+k}}\right)$$

*See Tauscher et al. (2018) for more general equations which allow for noise covariances.

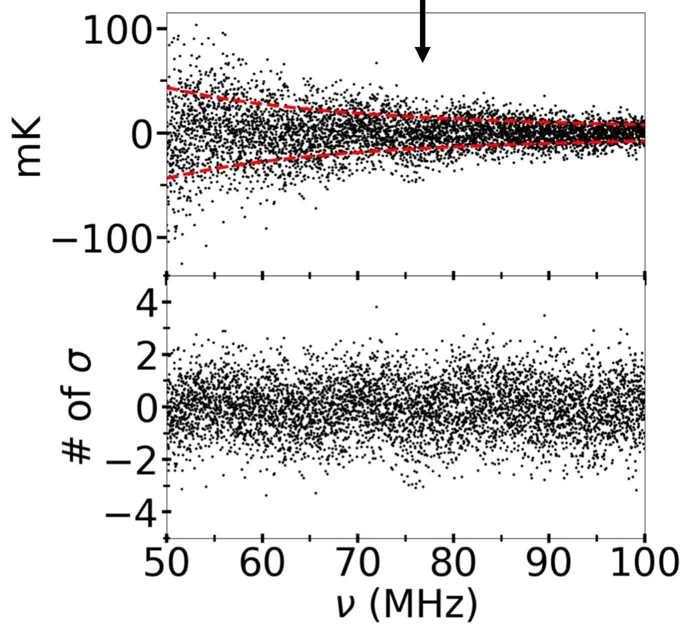
Utility of psi-squared statistic

We performed fits on simulated datasets containing 1) a foreground, 2) a ground plane resonance, and 3) a 21-cm signal, 4) Gaussian noise following the radiometer equation.



Tauscher et al. (2018).

Effect on statistics of not fitting all data components



F.G.+Sys. Fit

$$\chi^2 = 1.002 = \mu_{\chi^2} + 0.12\sigma_{\chi^2}$$

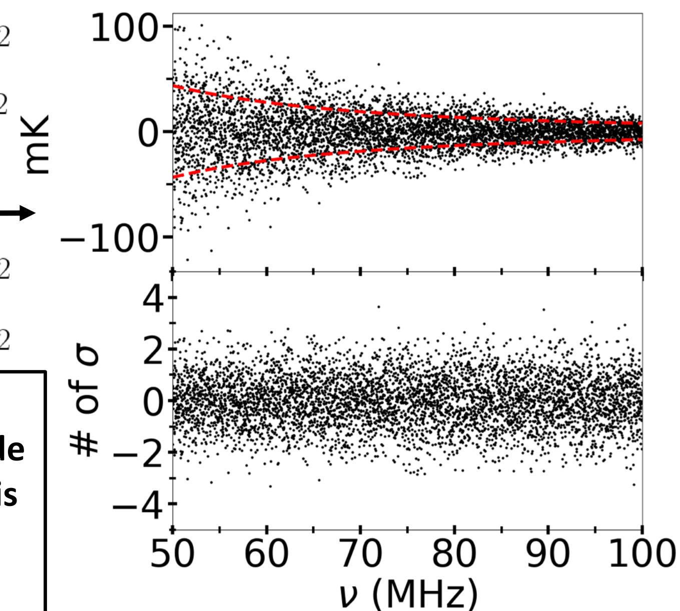
$$\psi^2 = 1.261 = \mu_{\psi^2} + 4.94\sigma_{\psi^2}$$

F.G.+Sys.+Signal Fit

$$\chi^2 = 0.982 = \mu_{\chi^2} - 0.88\sigma_{\chi^2}$$

$$\psi^2 = 0.961 = \mu_{\psi^2} - 0.73\sigma_{\psi^2}$$

psi-squared can better detect problems when a low magnitude part of multi-component data is not modeled than can chi-squared, while both retain ability to indicate good fits.



Figures from Tauscher et al. (2018).

Conclusions

- While sensitivity to the signal is an instrumental problem, extracting it is an analysis problem, which is complicated greatly by the vast difference in the magnitudes of the foreground (few thousand K) and the signal (few hundred mK).
  - Small biases in the modeling of components of the data (e.g. foreground, receiver biases) can, through covariances of different component models, lead to extraction errors of greater magnitude.
- The publicly released EDGES data is of high quality, but how it should be analyzed and the nature/presence of absorption trough(s) remain unclear.
  - Preferring either the flattened Gaussian or resonances fit is unfounded because the data's noise level is unknown.
- The new psi-squared statistic should allow for more unaveraged/unbinned data to be analyzed at once without losing goodness-of-fit discerning power.
  - It is designed to detect low-level, wide-band features that typify 21-cm signal experiments' residuals.
  - When used for this purpose, psi-squared is more sensitive than chi-squared.

References

Barkana, R., *Possible interaction between baryons and dark-matter particles revealed by the first stars*, Nature **555** pp. 71-74 (2018).

Bowman, J.D., Rogers, A.E.E., Monsalve, R.A., Mozdzen, T.J., Mahesh N., *An absorption profile centred at 78 megahertz in the sky-averaged spectrum*, Nature **555** pp. 67-70 (2018).

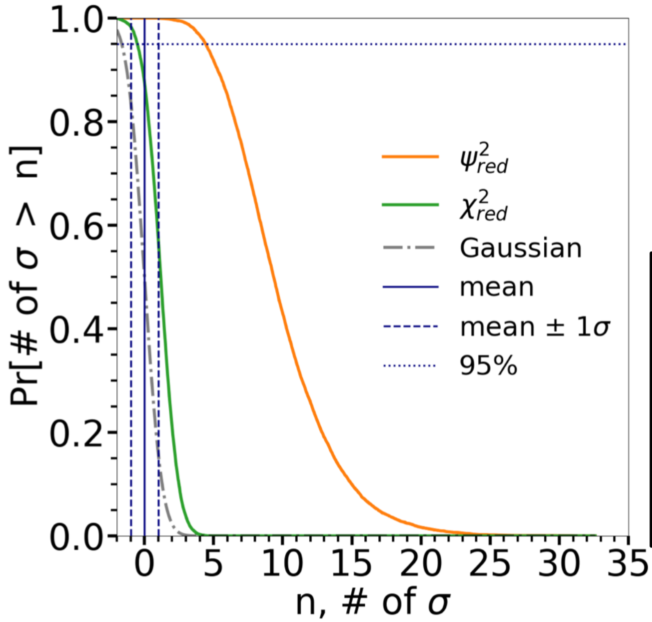
Bradley, R.F., Tauscher, K., Rapetti, D., Burns, J.O., *A Ground Plane Artifact that Induces an Absorption Profile in Averaged Spectra from Global 21-cm Measurements - with Possible Application to EDGES*, arXiv:1810.09015 (2018).

Haslam, C.G.T., Salter, C.J., Stoffel, H., Wilson, W.E., *A 408 MHz all-sky continuum survey. II - The atlas of contour maps*, A&AS **47** (1982).

Tauscher, K., Rapetti, D., Burns, J.O., *A new goodness-of-fit statistic and its application to 21-cm cosmology*, JCAP **2018** 12 015 (2018).

Extra slides

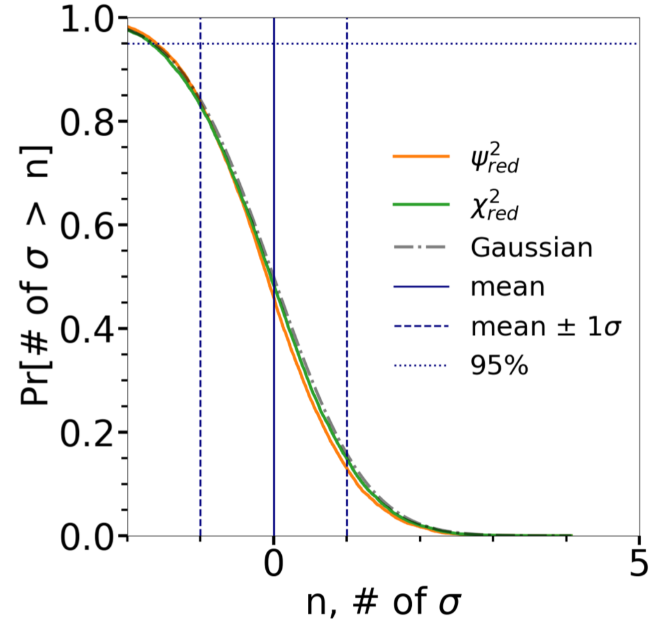
Rejection probability for chi-squared and psi-squared



← F.G.+Sys. Fit

F.G.+Sys.+Signal Fit →

psi-squared can better detect problems when a low magnitude part of multi-component data is not modeled than can chi-squared, while both retain ability to indicate good fits.

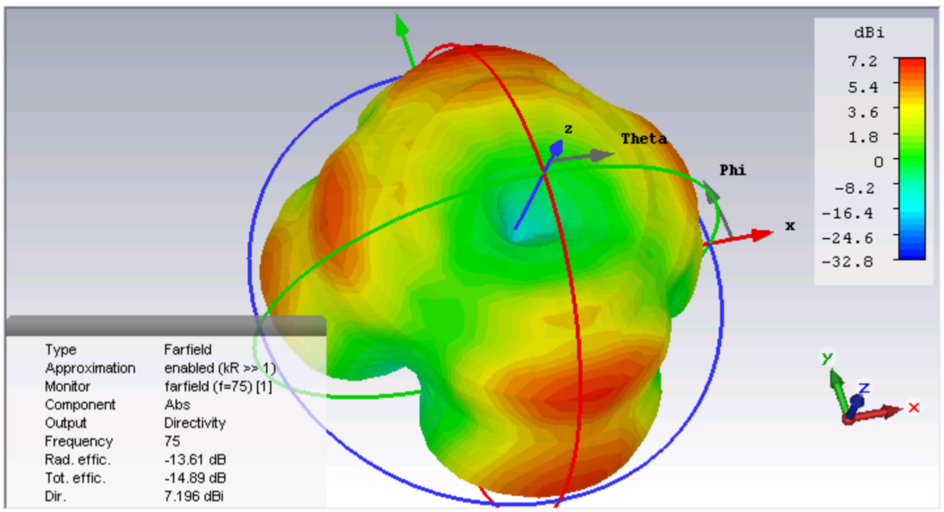


Figures from Tauscher et al. (2018).

Resonance beam patterns

Beams of ground plane resonances are located off-zenith and differ from mode to mode

75 MHz, TM20



85 MHz, TM21