Analogical Scaffolding:

Making Meaning in Physics through Representation and Analogy

by

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Abstract

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Making Meaning in Physics through Representation and Analogy

Thesis directed by Assistant Professor Noah D. Finkelstein

This work reviews the literature on analogy, introduces a new model of analogy, and presents a series of experiments that test and confirm the utility of this model to describe and predict student learning in physics with analogy. Pilot studies demonstrate that representations (e.g., diagrams) can play a key role in students' use of analogy. A new model of analogy, Analogical Scaffolding, is developed to explain these initial empirical results. This model will be described in detail, and then applied to describe and predict the outcomes of further experiments. Two large-scale (N>100) studies will demonstrate that: (1) students taught with analogies, according to the Analogical Scaffolding model, outperform students taught without analogies on pre-post assessments focused on electromagnetic waves; (2) the representational forms used to teach with analogy can play a significant role in student learning, with students in one treatment group outperforming students in other treatment groups by factors of two or three. It will be demonstrated that Analogical Scaffolding can be used to predict these results, as well as finer-grained results such as the types of distracters students choose in different treatment groups, and to describe and analyze student reasoning in interviews. Abstraction in physics is reconsidered using

Analogical Scaffolding. An operational definition of abstraction is developed within the Analogical Scaffolding framework and employed to explain (a) why physicists consider some ideas more abstract than others in physics, and (b) how students conceptions of these ideas can be modeled. This new approach to abstraction suggests novel approaches to curriculum design in physics using Analogical Scaffolding.

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Preface

Before I joined the Physics Education Research group at the University of Colorado, I conducted research in geophysics. For the most part, I created models of processes occurring in the solid earth and turned these models into computational algorithms. I attended a number of conferences and published one paper during my two years in the geophysics group. The earth seems pretty solid and unmoving to humans, but on geological timescales of millions or billions of years, its motion and flow are a lot like a fluid. You could think of the earth as a wax sphere that is just hot enough to be molten on the inside, but cool enough on the edges to create a hard, cracked crust. But it is a lot more complicated than that. Everything that happens on the earth affects everything else. It is a highly complex and dynamic system. It is inherently non-linear, and in many respects, the way the earth behaves depends on what you are doing to it. It is a visco-elastic system. It is like a substance that Dr. Seuss named "oobleck".[1] Take a lot of corn starch and mix it into a little water and you get oobleck. Oobleck is weird. If you stir it very slowly with your finger, it acts like a viscous fluid – like honey. If you poke it hard and quickly, it bounces back like a piece of rubber. The earth, like oobleck, is not exclusively a solid or a fluid – it is whatever it needs to be at a particular place and time, depending on the circumstances. And most importantly, the earth's processes cannot be completely isolated from one another, or even from the effect of the moon or other bodies in the solar system.

However, in order to make progress in understanding the earth, one has to make certain assumptions. I wrote a paper that modeled a tectonic plate moving over the upper mantle as a rigid plate moving over a viscous fluid.[2] This is called *Couette* flow, and it is a well known system in fluid mechanics. This worked very nicely and earned me a publication. Then I tried to use a similar method to model the flow near a subduction zone. This is where a tectonic plate collides with another plate and slides underneath the second plate, eventually plummeting deep into the earth. All of a sudden, my computer codes would not work – the differential equations would not converge. The well known *Couette* flow model could tell me what oobleck would do when it was behaving nicely and going in one direction. But when I tried to figure out what oobleck would do when it turned a corner, the dual processors in my Linux workstation basically ran around like two chickens with their heads cut off for about a day and a half and then spat out garbage. When I switched from geophysics to education research, I realized early on that the problems I would face would be a lot like trying to figure out what oobleck would do when I tried to make it turn a corner.

The systems we work with in education are of the most complex, dynamic, non-linear kind. Further complicating matters, these systems are made up of individuals that, sometimes to the woes of researchers, are capable of thinking for themselves. Nonetheless, it is possible to carry out rigorous research in educational environments, and it is surprising how reproducible some research results can be. Like all scientists, education researchers rely on certain models of the way students learn, whether these are used implicitly or explicitly. Often, these models do a good job describing or explaining some empirical findings. And like nearly all scientific models, as science advances, these models come up short and new ways of explaining phenomena need to be sought. The work described in this volume is about analogy, representation, and learning physics. What I hope to convey here is that while existing models of analogy have been fairly productive within certain bounds, a call for new ideas about the way students think and learn may be approaching a tipping point. My contribution to this new thinking will be in the areas of representation and analogy.

In a recent talk, Neil deGrasse Tyson, the well known astrophysicist and host of the PBS educational TV show *Nova scienceNOW*, pointed out that throughout history, when scientists have come up against problems that they found intractable, these scientists often threw up their hands and claimed the problems were simply too complex for the human mind to comprehend. And almost every time, human minds have eventually figured these problems out. I feel the same way about education research. The problems we face, the questions we are trying to answer, and the changes we might be trying to affect are exceedingly difficult. Many of them may be intractable, for now. But science has always found ways of thinking to make sense of the world, whether that world is made up of atoms or people. The only way this has ever been done is by small steps, a willingness to believe that progress is possible, and a sort of faith that any step forward, no matter how large or small, is still a step forward. Central to this pursuit is the sharing of ideas. This is my motivation for the work that follows.

Chapter 1 – Introduction

> "Instead of using the analogy of heat, a fluid, the properties of which are entirely at our disposal, is assumed as the vehicle of mathematical reasoning...The mathematical ideas obtained from the fluid are then applied to various parts of electrical science."
>
> – James Clerk Maxwell[3]

> "It was as if you fired a 15-inch shell at a sheet of tissue paper and it came back to hit you."
>
> – Earnest Rutherford[4]

Some of the most celebrated advances in physics have been generated by analogies.[5] The first quote above exemplifies James Clerk Maxwell's mastery of analogical reasoning as a means of generating new scientific knowledge.[6] When Earnest Rutherford fired alpha particles at gold foil, he conveyed his utter surprise at what happened with the second quote above. To *explain* his findings, Rutherford used another analogy – the atom is like a tiny solar system – and, with this seemingly simple idea, ushered in a dramatic shift in the way physicists thought about the structure of atoms. Consider further the analogy relating Coulomb's law for electric force to Newton's law for gravitational force and it is hard to imagine atomic physics without analogies.

But physicists do not generally speak explicitly of analogies when they talk about atoms. When a physicists talks about electron "orbitals", the analogy is hidden.[7] It is tucked away in some mental closet and forgotten, leaving only the idea of electron orbitals available to the conscious mind. As David Brookes and Eugenia Etkina[8] point out, words like "orbitals" indicate the implicit use of an analogy.[*] In their research on students learning quantum mechanics, Brookes and Etkina analyzed student reasoning about a potential step function, and found that the word "step" prompted these students to think of a real step, that is, a concrete object like a stair step that a particle had to get up and over. Physicists are often surprised that students make such literal interpretations – after all, we know that *step function* is just what we call this thing (if it is a thing at all). To think of it as a stair step is so naïve as to be almost inconceivable. But when it comes to the meaning of symbols, for instance words or graphs, students are generally at a very different place than trained physicists. Because these expert interpretations are often so ingrained, it can be difficult for expert physicists to unpack how these particular ways of thinking were learned. The question for scientists is this: can we use a combination of theory and experiment to do some of this unpacking? Can we identify, at least in part, the structure and process of learning abstract ideas in physics? If so, we might make significant progress towards better understanding student learning, why students sometimes have difficulty with abstract physics concepts, and why students often hold such different (often surprisingly so) notions about physics than those of experts.

[*] Brookes' research is specifically on metaphor. Analogy and metaphor are very similar, and there may be a considerable degree of overlap between the cognitive mechanisms responsible for both. See, for example, ref. 14. For our purposes, the two terms can be used interchangeably.

It is indeed an understatement to say that the chain of experience and learning that brought most physicists from birth to Ph.D. is complex, and I won't be able to completely answer the questions posed above. However, in scientific tradition, I will try to take a small, but significant, step forward by focusing on analogical reasoning. While physicists often treat analogies implicitly in practice, physics instructors often make explicit use of analogies when teaching. For instance, Coulomb's law is often taught in introductory physics courses as analogous to Newton's law of gravitation. The Rutherford analogy (atoms are like tiny solar systems) is canonical, and will serve as a bedrock example in the chapters that follow. Electric current is often likened to water flowing through a pipe. Water waves, waves on a string, and sound waves are common lead-ins to teaching electromagnetic waves. Innumerable analogies are used in physics textbooks.[9,10] Table 1.1 lists some of the many analogies found in one popular physics text.[11] These analogies are commonly used in textbooks, but have not necessarily been rigorously tested with students. While this list may appeal to physics teachers, numerous studies have shown that analogies are not nearly as effective for teaching as we might hope, and students may not use analogies in the ways instructors intend.[*] Why is this? Are analogies simply not as useful as we think, or do we perhaps need to reexamine the ways analogies are used to teach physics?

Though understanding how analogies facilitate learning is a rich area of study, research on analogies *in physics* is relatively sparse. Significant efforts have contributed theoretical frameworks describing analogies, discussed in depth in the next chapter. Most of this work is not in physics, and only a handful of studies focus

[*] Consider the last analogy in the list. While the physicist may understand the particular mapping, a student may have a different interpretation – for instance, that waves travel faster than particles.

Table 1.1 Analogies used in a popular physics textbook.

Coulomb's law is like Newton's law of gravitation.
The electric field is like a temperature field.
Storing energy in a capacitor is like stretching a spring (or lifting a book).
The flow of electric current is like water in a garden hose.
An *emf* device is like a charge pump.
The magnetic field is like the electric field (they are both vector fields).
The earth is like a huge bar magnet.
An inductor, capacitor, resistor circuit is like a mass, spring, viscous system.
Particles are like sending a letter, while waves are like making a telephone call.

specifically on physics content. Nonetheless, experimentalists have asked specific research questions about the use of analogy in general. One very basic, but important, question is: do students use analogies when solving problems? If so, how? If not, why not? The short answer is, in general, no (even when students are given an analogy, a really good one that should make the problem a snap) and researchers are still trying to understand why. Slightly more advanced research questions are of the following type: which analogy leads to better student learning about electric circuits – water in a pipe, or a moving crowd? And, generally, the answer is it depends. I pose similar questions in my own research, but I also ask questions of the following type: what factors can we identify that promote (or hamper) student analogy use? How can we use these factors productively to teach physics with analogies? We will return to these questions shortly. Making progress towards answering these questions requires that researchers hypothesize and test possible mechanisms of student analogy use that are both descriptive and predictive. Ultimately, I will describe a model of analogy use which extends prior models and is supported by empirical evidence demonstrating that the underlying mechanisms of this model are both descriptive and predictive of student analogy use. But first, we need to understand some of the existing theoretical underpinnings of analogy, largely due to work in cognitive science.

* * *

In this work, I review the literature on analogy, introduce a new model of analogy, and describe a series of experiments that test and confirm the utility of this model to describe and predict student learning in physics with analogy. The next chapter reviews the literature on analogy, focusing particularly on analogy research related to physics. In this literature review, I describe several theoretical frameworks for analogy that have been developed by prior research and follow with a description of experimental efforts on analogy. Though much work has been done on analogy, many questions remain that call for further experimental and theoretical work. Chapter 3 describes a pilot study conducted to explore the role of representations (e.g., diagrams) in the use of analogy. The findings in this study call for a new explanatory model, which I have developed and is described in Chapters 4-5. I describe the essence of this model somewhat informally in Chapter 4, then provide a detailed and formal description of the model in Chapter 5. I call this model Analogical Scaffolding.

Chapter 6 introduces a transition from theory to experiment. Chapter 6, describes an experiment meant to test Analogical Scaffolding in a large-scale (N>200) physics course. This experiment, the first of its kind, will demonstrate that students taught with analogies, according to the Analogical Scaffolding model, outperform students taught without analogies on pre-post assessments focused on electromagnetic waves. Following this result, Chapter 7 describes further large-scale experiments designed to test applications of Analogical Scaffolding to predict student learning under different conditions. I show that the representational forms used to teach with analogy can play a significant role in student learning, with students in one treatment group outperforming students in other treatment groups by factors of two or three. I demonstrate that Analogical Scaffolding can be used to predict this result, as well as finer-grained results such as the types of distracters students choose in different treatment groups. Chapter 8 will return to theory and introduce new mechansims to augment Analogical Scaffolding to explain the notions of abstraction and salience, and distinguish between students' individual understanding and the community consensus in physics. The utility of this augmented model of Analogical Scaffolding is demonstrated in an analysis of student reasoning during an interview situation.

Chapter 2 – Literature Review

> "The similarity is a similarity between relations, not a similarity between the things related."
>
> – James Clerk Maxwell[12]

The word *analogy* gets thrown around a lot. Analogies are used in advertisements, political speeches, everyday conversations, and, of course, physics. What, exactly, is an analogy? Loosely stated, an analogy is a comparison between two similar things, or the use of something familiar to convey or understand something unfamiliar. Straightforward as they seem, these definitions do not accomplish very much. We could describe many flavors of constructivist learning this way – the central tenet of modern learning theories is that people use what they know to learn those things that they do not know. In order to have a working theory of analogy that is more specified than, say, a constructivist perspective or a *resources* framing,[13] we will need to formalize and operationalize analogy.* In this chapter, we will seek and explore models of analogical reasoning that are well beyond a one or two sentence definition. By analogy, consider the following definition of an atom.

> Atom [at-*uh* m] - The smallest component of an element having the chemical properties of that element, consisting of a nucleus and one or more electrons.

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* On the other hand, one could argue that all human cognition is inherently analogical. In this case, we might say that the most appropriate definition of analogy is "the way humans think." This is rather tautological, but alas these are the sorts of paradoxes we run into when we humans try to think about our own thinking. For now, I will be less bold and avoid paradoxes by considering analogy a subset of human cognitive activity. This is a safer position, though not as much fun.

Not entirely useless, but not very useful either for understanding how an atomic system works. Cognitive systems like analogical reasoning similarly reach beyond the scope of definitional classification.

This chapter will be divided into two main sections, theory and experiment. There exists a great body of literature on analogy, both theoretical and experimental. Much of this work has been done by cognitive scientists, psychologists, and linguists, with a few notable contributions from physics education researchers. Here, I will provide a review of the most often cited and influential work on analogy from cognitive science and physics education research. In particular, I will focus on work in analogy that can be related directly to physics teaching and learning at the high-school and college levels.

Part I: Theory

Physicists have theories to explain observations and phenomena that we experience every day. The Greeks looked at the material world around them and hypothesized that it was made of atoms. As modern physicists, it is clearly not enough to name something an atom – we need to understand its structure, how it works, and what it might do under different circumstances. Once an alpha particle recoils and hits you like a 15-inch shell, as a scientist you want to explain what just happened. The same is true of analogy. Human beings use analogies on a daily basis, and these cognitive tools have the power to change the way we think (or so it is assumed). How do analogies work? What is the structure? How can scientists dig below the surface to understand what is going on? In this chapter, I will describe the key theoretical efforts related to modeling the cognitive process of analogy.

The Contemporary Theory of Metaphor

A useful starting point for a discussion of analogy is with metaphor, a closely related cognitive process.[14] George Lakoff, a scholar in linguistics and cognitive science, has developed an elaborate theory of metaphor.[15] Lakoff's theory is quite rich, and I will limit the discussion here to some key features of his framework. (For more see ref. 15; see also ref. 16.) In Lakoff's view, the word metaphor means a *cross-domain mapping in the conceptual system*.* Objects in a *base* domain are mapped to objects in a *target* domain.† Domains, in this sense, are loosely equivalent to mathematical sets containing descriptive features of an object or idea, for instance, the hydrogen atom. One of Lakoff's key ideas is that with a metaphor, a familiar situation is used to ground understanding of an unfamiliar situation. In Lakoff's terminology, Rutherford's planetary model of the atom might be called the *Atoms are Tiny Planetary Systems* metaphor. In Table 2.1, I have adapted Lakoff's model to represent the Rutherford model. The base domain is the solar system and the target domain is the atom. This metaphor suggests a particular, albeit rudimentary, model of the atom. The mapping is shown in Table 2.1 below. Elements are mapped from the

* This definition of metaphor can also serve as a basic definition of analogy. Both processes can be modeled as mappings with subtle distinctions. One view posits that metaphors indicate analogies which have become hidden to the conscious mind.[7] The *Atoms are Tiny Planetary Systems* might be one such example. Another example is the *Electrons are Waves* metaphor in quantum mechanics. From a theoretical standpoint, analogy generally receives a more prescriptive and formally structured treatment, as discussed later on.

† The word domain here denotes a mental space, conceptual packet, or possibly a content area (e.g., atoms). Incidentally, other names for base and target are, respectively, *source* and *target*, or *vehicle* and *tenor*.

solar system domain on the left to the atom domain on the right. Note that additional mappings may be present in this metaphor. Electrons are conceptualized as tiny hard spheres orbiting a larger, and more massive, spherical object. The reader familiar with set theory might recognize this mapping as an isomorphism.[*] Each element on the left in Table 2.1 is mapped to exactly one element on the right. This framework presents one way of formalizing metaphor and teasing out the structure of this cognitive process that we use every day, often with little effort or awareness.

Table 2.1 The Atoms are Tiny Planetary Systems Metaphor. Arrows represent mappings.

Solar System (Base Domain)		Atom (Target Domain)
Sun	→	Nucleus
Planets	→	Electrons
Sun attracts planets	→	Nucleus attracts electrons
Sun is more massive than planets	→	Nucleus is more massive than electrons

Lakoff and Nunez[17] have applied this cognitive framework to mathematics, contributing the idea of *layering* metaphors. Briefly, metaphors can build upon other metaphors, explaining how people come to understand very abstract mathematical ideas. Target domains become base domains for new metaphors, and sets of metaphors layer upon one another to create richer, more complex, and more abstract ideas. Here is an example of the layering process presented by Lakoff and Nunez. Infants are able to engage in a cognitive process known as subatizing. They can recognize the difference between one object and two or three (up to about 4, more than which usually requires counting, say on the fingers). Psychologists have determined this by, for example, placing a doll in front of an infant. The

[*] An isomorphism is a mathematical mapping from a set A to a set B which is both one-to-one and onto. That is, each element in A is mapped to exactly one element in B, and every element in B is mapped from an element in A.

psychologists cover up the doll and then remove the cover to expose the same doll, or sometimes two dolls. By carefully observing the infant's expressions (it is easy to tell if an infant is surprised or bored with what they see) the psychologists can tell if the infant recognizes the difference between one doll and two. When one doll is revealed, infants stop paying attention. When two dolls are revealed, the infants stare in surprise. The activity of subatizing soon becomes counting, which, according to Lakoff and Nunez, depends on the metaphor *Numbers are Groups of Objects.*

Continuing from the early childhood activity of counting objects and building up to more complex and abstract mathematics, Lakoff and Nunez plot an extensive course of metaphors layered over years of learning and experience to explain how people can come to make sense of Euler's equation,

$$e^{i\pi} = -1$$

where $i = \sqrt{-1}$. This equation, at first glance, appears absurd. The symbolic interpretations required for this equation to make sense are substantially different from the ordinary ones. At a most basic level, symbols are simply symbols – shapes on a page. The notion that these symbols stand for numbers is fairly advanced. Even with algebra, the usual behavior of an exponential function is to increase without bound, or decrease asymptotically to zero, and there is no obvious indication that the expression on the left in Euler's equation should equate to the *negative* value on the right. It only makes sense after the symbols and their relations have been endowed with certain meanings via a long and complex learning process.

Following Lakoff and Nunez, we could imagine plotting a course over numerous layered metaphors to explain how people learn increasingly complex, and abstract, physics ideas like models of the atom. One stage of many along this course might be the conceptual leap from the hard sphere conception of the atom to the planetary model. Between these two models lies an intermediate layer, the plum pudding model for instance, and possibly many others. In layering these metaphors, some structural features of the plum pudding model (constituting one layer) are retained, e.g. the existence of electrons and positive charge, but the arrangements of these objects are modified in the planetary model (a subsequent layer).

Remaining on the topic of mathematical symbols for just a moment, consider the standard base-10, or *decimal*, number system with which we are all familiar. This is not the only possible system – numbers can be base-2, or base-5, etc. Lakoff's notion of grounding in experience suggests that the overwhelming use of base-10, compared to other bases, is likely due to the fact that humans have 10 fingers on which to count. So the mathematical abstraction is grounded in our human physiology. Interestingly, this abstraction allows us to carry out certain calculations very rapidly, but not others. For example, try to divide the number 7960.0 by the number 10. How long did it take? Probably less than a second to get 796 – you simply move the decimal point. Now divide the number 7960.0 by the number 16. (No calculator allowed.) This probably took a lot longer. The process is different. The base-10 representation of 7960.0 does not allow for rapid division by numbers other than 10. However, if we were working in a base-16 representation, so called *hexadecimal*, we could carry out the division by 16 very rapidly by just moving the decimal point.* Likewise, if we wanted to divide by 2, we could do this very quickly

* In hexadecimal, 7960 is rewritten 1F18, and 16 is rewritten 10. 1F18 / 10 = 1F1.8 (= 497.5 in decimal).

in a base-2 or *binary* system. Note that, in the hexadecimal and binary systems, dividing by 10 becomes much more difficult. The point here is that the way we represent abstract ideas does work for us, and some representations are better for accomplishing certain tasks than others. However, in order to take advantage of these representations, we have to somehow give them meaning. As Lakoff suggests, meaning can be built up over a series of layered metaphors. We will return to this idea of representation and layering in later chapters.[*]

I have suggested that Lakoff's model can be applied to metaphors in physics. However, Lakoff is mostly interested in everyday metaphors that have become so ingrained in our culture, language, and thought as to be almost invisible. Lakoff uses his conceptual framework to expose these hidden metaphors. For instance, if you were to say "my relationship has hit a road block", you would be using what Lakoff calls the *Love is a Journey Metaphor*. The abstract idea of love is conceptualized in terms of the concrete idea of a journey, in which you can have smooth sailing, road blocks, wrong turns, etc. I suggest, however, that the notion of metaphor and layering is essential to understanding how complex and abstract ideas are constructed and learned in physics, especially for physics ideas that are "invisible", such as atoms or electromagnetic waves. I will argue for this idea in the pages that follow. However, due to the particular nature of physics, its complexity and formalism, we will need a more sophisticated model to describe the nature of domains and cross-domain mappings.

[*] The meaning of symbols may depend on particular cultural influences as well. Consider the following equation: $d(x,y) = x^2 + y^2$. What is $d(r,\theta)$? If you said $d(r, \theta) = r^2$, chances are you are a physicist. If you said $d(r, \theta) = r^2 + \theta^2$, you are probably a mathematician.

Structure Mapping – A Formal Theory of Analogical Mapping

Deidre Gentner's work[14,18,19,21,37] is often cited as foundational in the literature on analogy. Gentner's work was part of a paradigm shift in analogy theory that occurred in the early 1980's. This new line of analogical modeling is related to Lakoff's view of metaphor, but introduces a formal approach with complex structures more familiar to mathematicians and computer scientists.[*] The analogical models developed in this era followed on the heels of ideas from cognitive science such as neural networks and connectionist models of human cognition. (Ref 20 is the seminal capstone of preceding work in connectionism.) Not surprisingly, these formal models of analogy also coincided with the advent of personal computers which became widely available and were relatively easy to program.[†]

According to Gentner's *Structure Mapping Theory*,[21] "The analogy 'a *T* is like a *B*' defines a mapping from *B* to *T*." *B* is called the *base* domain and serves as a knowledge source. *T* is called the *target* domain, and is the subject to be learned. Symbolically, the analogy is the mapping *M*,

$$M{:}b_i \rightarrow t_i$$

where the subscript *i* denotes objects (*b* and *t*) in the base and target (*B* and *T*). Gentner illustrates this idea with a canonical example, the planetary model of the atom. The base domain is the solar system, while the target domain is the atom.

[*] I do not mean to imply that Lakoff's theory is simpler or more rudimentary than Gentner's, it is simply a different approach in terms of the formalism. The two approaches were actually developed simultaneously with somewhat different purposes.

[†] Again, this follows the theme of tools allowing for new kinds of thought. I grew up in the era of personal computers. My father likes to tell me about how he completed his mathematics degree programming computers with paper punch cards. You would bring one of these cards in to a person at a desk and leave it for them to run your code. If it worked, you were happy. If the code did not work, you were left with an error code and a paper punch card from which to debug your program. The advent of widely available and (relatively) easy to program computers allowed for new types of cognitive models to be pursued and for new ideas about cognition to flourish.

FIGURE 2.1 shows a schematic representation of the mapping from the solar system to the atom, with structures for each analogical domain.

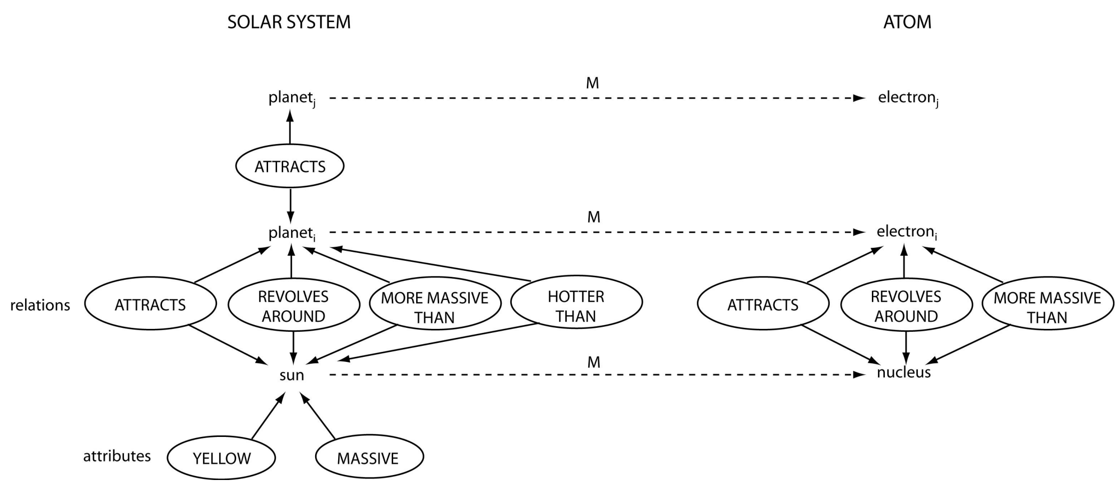

FIGURE 2.1 Structure-map for the Rutherford analogy: "The atom is like the solar system."

Gentner's theory allows for objects, e.g. *electron*, and two types of predicates, or descriptors, that can map across domains. *Attributes* are predicates that take one input, e.g. MASSIVE(sun), while *relations* take two inputs, e.g. ATTRACTS(sun , planet). Certain attributes and relations can be mapped (e.g. ATTRACTS) while others might not map (e.g. HOTTER THAN or YELLOW).[*] Gentner's argument is that in an analogical mapping, a large number of relations are mapped, while few attributes are mapped. To differentiate analogies from other mappings, Gentner breaks out domain comparisons into three categories:

[*] Note how similar Gentner's syntax is to a computer language. Not surprisingly, work of this type is the foundation of many artificial intelligence (AI) efforts in cognitive science. Numerous analogy processing algorithms have been developed, but none of these can currently reproduce the fluidity with which humans use analogies in any substantial way.

1. Literal similarity – a large number of both attributes and relations are mapped (e.g. the X12 star system is like our solar system).

2. Analogy – a large number of relations, but few attributes, are mapped (e.g. the hydrogen atom is like our solar system).

3. Abstraction – the base domain is an abstract relational structure (e.g. the hydrogen atom is a *central force system*).

Since the majority of mappings in FIGURE 2.1 are relations, rather than attributes, the planetary model of the atom is considered an analogy, according to Gentner. The correct way to complete the mappings in the Rutherford analogy should be obvious to any physicist. However, it may not be obvious to students. An important, and still debated, issue in analogy research is how people select which elements to map, and which elements not to map. Gentner posits the following mechanism of selection for mappings. She suggests that elements that are part of *higher-order relations*, i.e. relations between relations, are more likely to be mapped in an analogy. For instance, MASSIVE should map since it is part of the higher-order relationship CAUSE[MORE MASSIVE THAN(Sun , Planet) , REVOLVES AROUND(Planet , Sun)]. Since YELLOW is not part of this higher-order relation (or any other), it should not be selected in the mapping. Abstractions, as defined above, consist of a collection of higher-order relationships.

This model is indeed compelling and consistent with the way physicists often use analogies, both in teaching and generating new knowledge. However, as we will see, certain paradoxes arise when we attempt to unify structure mapping with other contemporary models of student knowledge and learning. Furthermore, experimental evidence demonstrates that applying structure mapping to inform teaching practice is insufficient for generating any significant student learning.[37] Possibly, structure mapping is necessary but not sufficient for promoting student learning by analogy. It is also plausible that, while productive for describing *expert* use of analogy, structure mapping may not be the most productive model of *student* learning by analogy.

Multi-constraint Theory

Holyoak and Thagard present an approach to analogy, *multiconstraint theory,*[22,23] which closely parallels Gentner's structure mapping theory. Their theory posits three constraints on analogy use: similarity, structure, and purpose. Holyoak and Thagard define an analogy as a mapping between two domains, again an isomorphism, based on these three constraints. First, some similarity between the domains guides the use of the analogy. Similarities in the structures of two domains place constraints on the analogical mappings that are possible. Second, the analogy user is constrained to maximize the structural parallels between the base and target domains. For instance, in the Rutherford analogy, planets map to electrons, rather than the nucleus, since these two are the smaller objects in each domain. Finally, the analogy user is constrained by some purpose, or goal, for the analogy at hand. The third constraint of purpose is important in understanding the selection of mappings –

the selection of a base domain and the particular mappings that are selected can depend on the intended goals of using an analogy. Holyoak and Thagard demonstrate the utility of their model using analogies for the 1991 Iraq war as follows. Subjects in a study were asked to relate the 1991 war between the U.S. and Iraq with World War II. Here is an example of the reasoning observed:

> "Similarity at the object level favored mapping the United States of 1991 to the United States of World War II, simply because it was the same country, which would in turn support mapping [George H.W.] Bush to Roosevelt. On the other hand, the United States did not go to war until it was bombed by Japan, well after Hitler had marched through much of Europe. One might therefore argue that the United States of 1991 mapped to Great Britain of World War II, and that Bush mapped to Winston Churchill (because Bush, like Churchill, led his nation and Western allies in early opposition to aggression). However, other relational similarities supported mappings to the United States and Roosevelt; for example, the United States was the major supplier of arms and equipment for the Allies, a role parallel to that played by the United States in the Persian Gulf situation. These conflicting pressures made the mappings ambiguous."

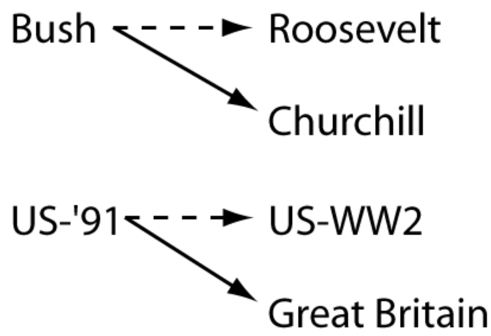

FIGURE 2.2 Bi-stable mapping for the WW2 to 1991 Iraq war analogy. A subset of possible mappings is shown here for illustrative purposes. Adapted from ref 22.

In this “bi-stable mapping”, the purpose of the analogy can promote one or the other mapping, shown schematically in FIGURE 2.2, depending on which role the reasoner wishes to emphasize for the United States. One might use the analogy from Bush to Roosevelt in order to emphasize the heroic stature of Roosevelt and the role of the U.S. in defeating Hitler. On the other hand, one might want to analogize Bush to Churchill in order to convince someone that early opposition to aggression is necessary in the present case of Iraq. Interestingly, Spellman and Holyoak[24] found that if reasoners have limited knowledge of the domains (i.e., reasoners are not very familiar with World War II or the 1991 Iraq war), the reasoners can be influenced to select one or the other mapping depending on the information they are provided. Two sets of reasoners, both given historically accurate yet different sets of information about World War II, were found to preferentially select different mappings in the analogy quoted above, depending on the information they received. Thus, subjects’ prior knowledge not only plays a role in the structure of the base domain, but can influence the mapping process as well.

Applying multiconstraint theory to the Rutherford analogy, we might hypothesize that a particular model (or analogy) of the atom will be guided, or constrained, by a certain purpose. For many cases in physics, the purpose might be explaining an experimental result. Rutherford chose the planetary model of the atom to explain the results of firing alpha particles at gold foil, while de Broglie contributed the wave analogy for electrons based on the purpose of explaining quantization. Consider teaching about electromagnetic (EM) waves. A physics instructor might choose a wave on a string analogy with the goal of teaching the transverse nature of EM waves. On the other hand, an instructor might choose a sound wave analogy with the goal of teaching the three dimensional (3D) nature of EM waves (EM waves spread throughout space as they propagate from a source, similar to sound waves propagating from a loudspeaker).

These prior frameworks for analogy seem promising, but certain concerns and limitations come to light in the context of student learning. Both structure mapping and multiconstraint theory belong to a class of theories that might be categorized as *abstract transfer*.[25] Applying an abstract transfer theory to instruction rests on three assumptions: First, analogies are assumed to be inherently linear and unidirectional – mappings are made directly from one structure to another. A second assumption is that students possess a mostly complete understanding of the base domain and little or no understanding of the target domain. (Even in a bi-stable mapping, as described above, knowledge of the base domain is assumed, either already held by the reasoner or provided to the reasoner.) The third assumption is that students will accept that the analogy relation is valid and be able to complete the mapping correctly. Rather than focus on students' ability to use an analogy, abstract transfer theories tend to rate an analogy's effectiveness based on the robustness of the analogy itself, the structure of which has been defined *a priori* by researchers.[26] In other words, analogies are framed from the expert physicist's point of view, not the student's. While these models are useful for framing our understanding of analogies, they fall short of explaining how analogies are used by students, or how to use analogies productively for teaching.

Bridging Analogies

One response to the abstract transfer approach to analogy is that of *bridging*.[25, 27,28,52] Even when an analogy seems obvious to an instructor, students may not be able to make sense of that analogy when it is used in instruction. Bridging analogies provide intermediate steps to help students make sense of an instructor generated analogy. Rather than assume students have little or no understanding of the target domain, bridging strategies assume students have some conceptions about the target, albeit students' ideas may be incomplete or in error. The bridging analogy is intended to promote conceptual change through the use of the following procedure:

1. A misconception is made explicit by means of a target question.
2. The instructor suggests an analogous case which will appeal to the student's intuitions.
3. If the student is not convinced of a valid analogy, the instructor attempts to establish the analogy relation. The student is asked to make an explicit comparison between the base and target.

If the student still does not accept the analogy, the instructor attempts to find a "bridging analogy" (or series of analogies) conceptually intermediate between the base and target.

One bridging strategy is shown schematically in FIGURE 2.3. This bridging strategy involves teaching Newton's third law. A teacher puts a book on a table and asks a student if the table exerts a force on the book (FIGURE 2.3, bottom right). A common student response is that the table does not exert a force – it simply "gets in the way" to prevent the book from falling. As a bridging analogy, the teacher proposes that the book is resting on a spring instead (FIGURE 2.3, bottom left). The student accepts that the spring exerts a force on the book, but does not yet accept that the table exerts a force (maintaining that the table only gets in the way). The "bridge" that the teacher suggests is that the table itself bends under the weight of the book – the table is "springy" (FIGURE 2.3, top). The bridge provides a way for the student to

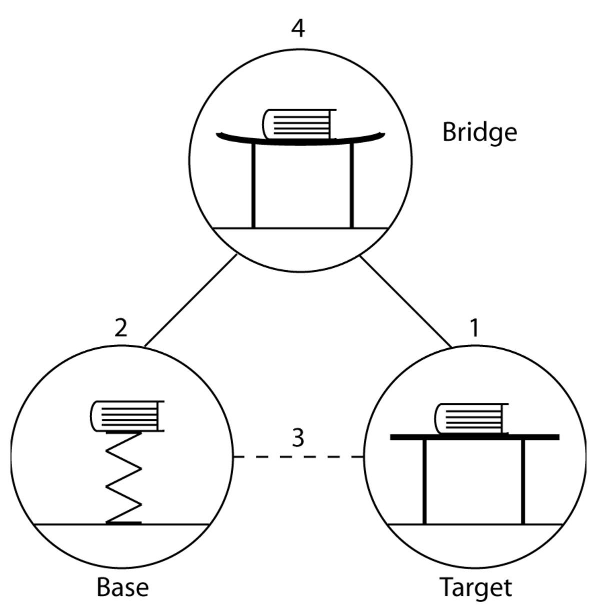

FIGURE 2.3 A bridging strategy for teaching Newton's third law for a book resting on a table. Adapted from ref. 25.

grasp the idea that the table does exert a force back on the book. This approach to analogy use appears to differ from Gentner's in a subtle way. Rather than framing the analogy in terms of a direct mapping between two domains, a number of intermediate mappings are employed which build on students' prior conceptions. The sequence of mappings is not necessarily linear, but often forms a richly interconnected network with many bases and targets connected in multiple ways to others. The end result of such a sequence is that the teacher attempts to combine the relevant features of different domains – in this case, tables and springs. Brown and Clement refer to this result as *enrichment of the target domain*. A complementary perspective, one that I will describe in detail later on, is that domains mix into a *conceptual blend* as described by Fauconnier and Turner.[29,82] (See ref. 17 as well.) The *springy table* blend can be considered an abstract mental construct created by combining the objects *table* and *spring*. The springy table construct is a useful instructional tool because students may readily accept the idea that the springy table exerts a force on the book. This construct gives students a *mechanism* for understanding (and accepting) Newton's third law for the situation of a book on a table.

Conceptual blending will be a central theme in my work, to be described in detail in a later chapter. For now, I claim that structure mapping, bridging analogies and blending are mutually supportive theoretical frames. Structure mapping is a formal cognitive model which provides a useful way of describing analogies in terms of mapping, and allows instructors to identify mappings in the analogy to be taught. Bridging analogies describe a particular way teachers can lead students to construct and then use multiple analogies, and blending provides one underlying mechanism of this multi-domain comparison process.

* * *

The main theme of this section has been that currently accepted models define an analogy as a mapping, like an isomorphism, of objects, attributes, and relations from a base domain to a target domain. It is worthwhile to contrast the contemporary views of metaphor and analogy, in terms of cognitive function, with historical beliefs. Prior to the 20th century, the prevailing view was that analogy provided a convenient language, but that an exact, scientific language should exist free from metaphor and analogy.[30,31] What this view missed was that metaphor and analogy, and language itself, are not merely "surface terminology" or flowery rhetoric – they are a part of the human conceptual system. In contrast to the historical view, Lakoff and others strongly advocate for the contemporary notion that analogies are an inseparable part of human cognition – they are part of the way we make sense of the world around us. In the next section, I will describe a number of experiments that confirm this contemporary point of view. Researchers today consider analogy a mechanism of thought, emphasizing the notion of grounding in experience, both in the day-to-day use of analogy and in more formal learning, for instance in physics courses (e.g., refs. 7, 32). Lakoff defines a metaphor (or analogy) as a *cross-domain mapping in the conceptual system*. The consequence of this framing is that, when using an analogy, students' conceptualization of the target domain will depend critically on the structure and content of the base domain, as well as the mapping selected.* Gentner takes a formal approach to analogy, describing a framework that moves towards computational modeling, accompanied by experimental verification. She and other researchers formulate testable (albeit abstract) models of analogy which can be productive for understanding analogies in highly formal and abstract domains such as physics (e.g., refs. 22, 33, 34). The implication of these analogy models for learning physics is that students' *prior knowledge* will play a key role in the *correct* (or incorrect) understanding of new concepts.

What is the structure of prior knowledge, how and when is it used, and what do we mean by *correct* understanding? Keep these questions in mind in the next section as we survey some of the major experimental findings on student analogy use in physics.

* Note that here, the base domain is viewed as relatively static, complete structure, the mapping is viewed as a process, and the target domain is viewed as a product of the mapping. It may well be that the structure and content of both base and target, as well as the mapping used, change more fluidly and dynamically than suggested by most current models.

Part II: Experiments

> SOCRATES: Do you realize that what you are bringing up is the trick argument that a man cannot try to discover either what he knows or what he does not know? He would not seek what he knows, for since he knows it there is no need of the inquiry, nor what he does not know, for in that case he does not even know what he is to look for.
>
> MENO: Well, do you think it a good argument?
>
> SOCRATES: No.
>
> - Plato, *Meno*

With a rigorous theoretical framework for analogy in hand, we can ask how the theory is borne out by experiment. Additionally, we will explore how this way of thinking about analogy is productive for learning and instruction. The following questions are important to keep in mind. Do these theoretical frameworks accurately describe how people use analogies? Further, do these frameworks inform how analogies can be used to teach?

Substance Based Conceptions – Analogies are Grounded in Experience

Reiner et al.[35] conducted experiments which probed students' thinking about physics concepts such as electricity, light, and heat. Drawing on a range of experimental results, they determined that students often used *substance based* conceptions. That is, students assigned material properties to non-material physical concepts. For example, heat flow was often conceptualized as fluid flow – heat took on the material properties of water.* Based on experimental results, Reiner et al. defined a generalized knowledge of substances which includes properties such as: substances are pushable, frictional, containable, etc. These findings seem to support Lakoff's notion of grounding, and promote Gentner's mapping theory as a tool for analyzing analogical learning. Abstract ideas in physics are often thought of in terms of experiences in the real world, i.e. with material substances. Analogy provides the bridge from the material world to the abstract physics domain. In fact, expert physicists also tend to use substance based conceptions, but somehow these experts know when these incomplete models are useful, and when they are not. For example, Maxwell conceptualized electricity as a fluid, and, not surprisingly, diagrams showing electric field lines near positive and negative charges *look like* sources and sinks in fluid flow. The mathematical formalism of electricity and magnetism parallels fluid dynamic models surprisingly well. In the planetary model of the atom, Rutherford found it quite productive to think of the electrons and nucleus as tiny hard spheres, a substance-based view. However, other mappings are not productive in these substance-based analogies. For instance, planets attract each other – experts know to ignore this relational mapping to electrons, which repel each other. Some may be worried by these complications and perhaps avoid teaching with analogies. (See for example ref. 36.) Our hope is that analogies may still be productively used to teach. The drive to teach with analogies rests on the assumption that analogies will be productive for student learning, and subjecting this assumption to experimental

* While this may seem to be a naïve conceptualization of heat, note the parallels with Maxwell's use of analogy as quoted earlier.

verification will reveal some surprising results. Notably, the way some students *know* a thing (concept or idea) may not necessarily correspond directly to the optimal way to *teach* that thing.

The Cognitive Nature of Analogy – Surface Terminology or Generative?

One of the first questions asked by modern analogy researchers examined the cognitive nature of analogy. According to Gentner and Gentner,[37] two hypotheses can be stated about the role of analogy in understanding physics concepts.

- Surface Terminology Hypothesis – analogies are merely convenient terminology. (Historical view.)

- Generative Analogy Hypothesis – analogies are used in generating inferences. (Contemporary view.)

In order to show that analogies are generative, experimentalists were faced with the task of demonstrating that the inferences people make on a certain topic vary according to the analogies they use. Gentner and Gentner approached this problem using analogies for electricity.

Structure-mapping schematics of water and moving-object analogies for electricity are shown in FIGURE 2.4. Relations (indicated in ellipses, connected by arrows) map directly from one domain to the other (top to bottom). These analogies should be familiar to most physicists – wires correspond to pipes or tracks, current

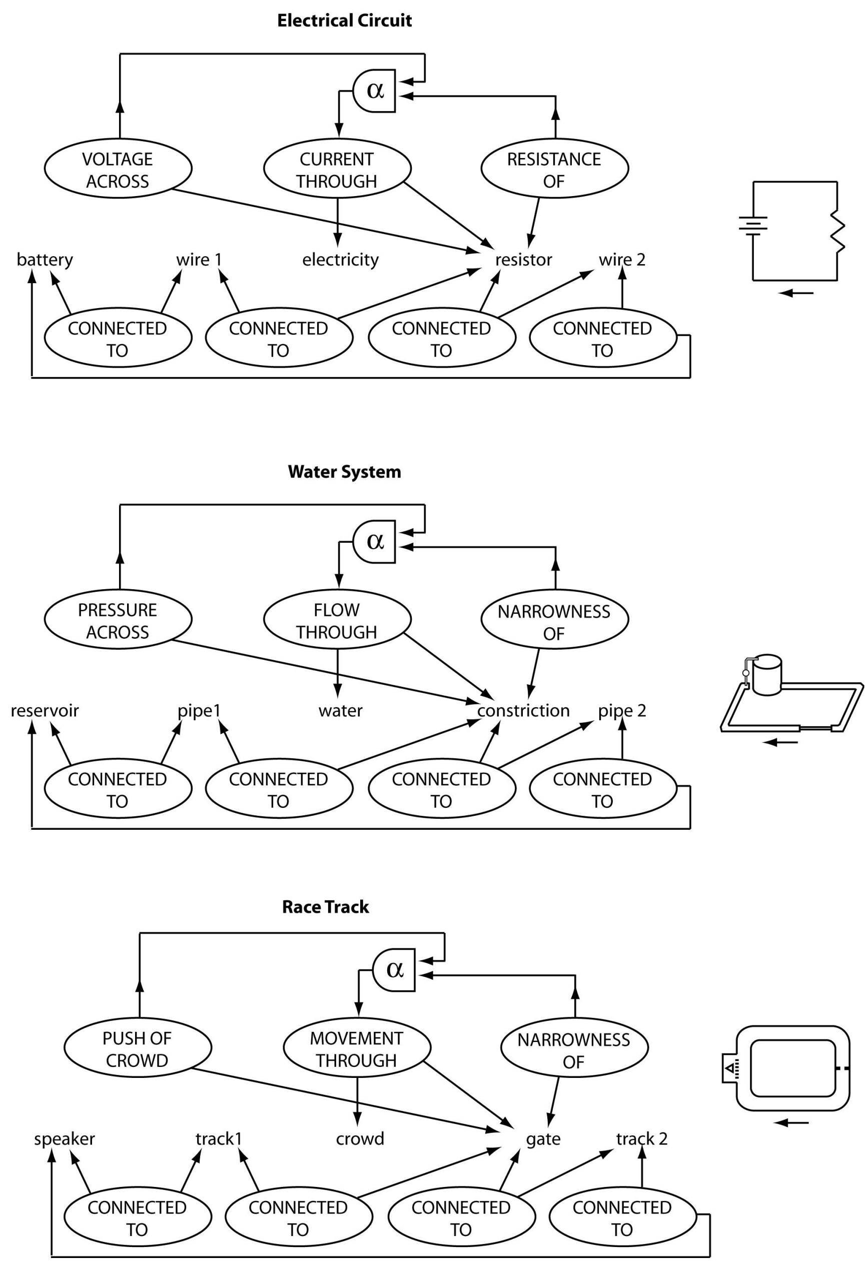

FIGURE 2.4 Structure mapping schematics for electric circuit and water system, adapted from ref. 37.

corresponds to flow of water or movement of the crowd, etc. The relation labeled α symbolizes a higher-order qualitative relation that transcends the individual domains. Gentner and Gentner point out that this relation constitutes Ohm's law, but that "naïve users of the analogy may derive only simpler proportional relations such as 'More force, more flow' and 'More drag, less flow'".[*] To the right in FIGURE 2.4 are pictorial representations of the intended models for an electric circuit, water system, and race track (i.e. moving object). Note that no single analogy has all of the correct properties of electric circuits, nor do these two exhaust the possible analogies for electric circuits (e.g., consider a moving bicycle chain mapping to electric current).

In their first experiment, Gentner and Gentner gave a multiple choice exam on series and parallel electric circuits to 46 high school and college students. They then asked the students to elaborate on how they thought about electricity, and from this determined whether individual students used a water analogy or moving object analogy[†]. Gentner and Gentner made the following predictions:

1. Students who use a water analogy should demonstrate a better understanding of batteries compared to students using a moving-crowd analogy. The reasoning is that water reservoirs are a more robust mapping to batteries than the analog in the moving-crowd model (i.e., loudspeakers urging on the crowd).

[*] These relations apply to what diSessa[71] terms *phenomenological primitives*, or p-prims. For instance, "More force, more flow" may be a particular instantiation of the p-prim *more is more*.

[†] Only subjects that consistently used a water or moving-crowd analogy were included in the original results, leaving N=7 subjects using the water model and N=8 using the moving-objects model.

2. Students who use a moving-object analogy should demonstrate a better understanding of resistors compared to students using a water analogy. The reasoning is that gates provide more robust mapping to resistors than constrictions.

FIGURE 2.5 shows student performance on an assessment targeting electric circuit concepts in Gentner and Gentner's study. Students who used a water analogy performed significantly better on questions about batteries than resistors. The opposite was true for the moving-objects group. This result supported the Generative Analogy Hypothesis.

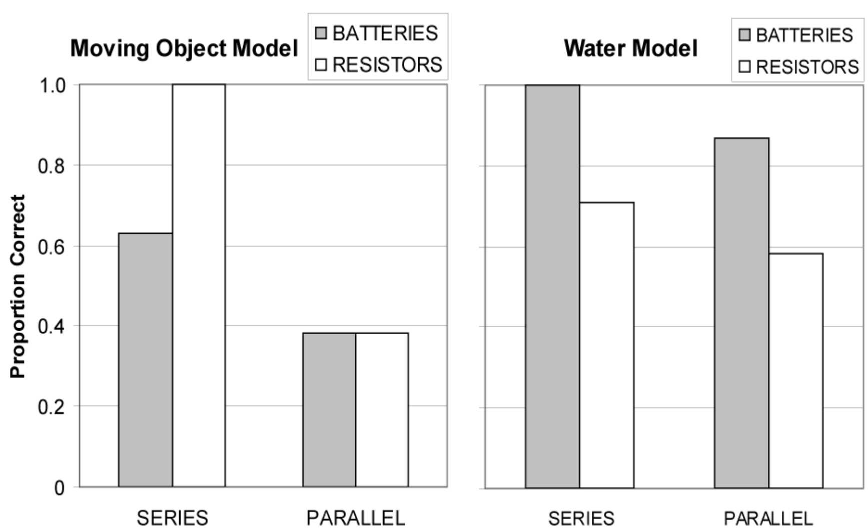

FIGURE 2.5 Data from questions about circuits for two analogies for electricity. Moving object (left) and water (right).

In Gentner and Gentner's first experiment, students generated their own analogies. In a second experiment, students were *taught* either a water analogy or moving crowd analogy. The predictions were similar to those in the first experiment. Students taught the moving crowd analogy did demonstrate a slightly better understanding of resistors, supporting the first prediction. However, the second prediction for understanding batteries was not supported. Gentner and Gentner suggested two explanations for this discrepancy:

1. Students may not have sufficient understanding of the base domain. For example, students may not understand how water systems work.

2. Students may not accept the model they are taught. For example, if a student is predisposed to use a moving crowd analogy, a single teaching session using a water analogy may not convince the student to use the new analogy.

Note that while students may sometimes use analogies spontaneously, often they do not. In their first experiment, Gentner and Gentner began with 46 subjects, but only 15 generated an analogy on their own. The data in FIGURE 2.5 are drawn from these 15 students. This result suggests that student generated analogies may be rare. Experts, however, generate analogies fairly often and with relative ease. Possibly, if we better understand expert use of analogy, we might make headway understanding the conditions under which students do, or do not, use analogies.

Spontaneous Analogies

In their first experiment, Gentner and Gentner assumed students would generate analogies on their own. To get a more detailed view of this process, Clement[38] explored the generation of "spontaneous" analogies by expert problem solvers (advanced graduate students and professors in technical fields). A *spontaneous* analogy is one generated without provocation or prompting.[38,39] 10 experts were shown the two springs shown in **FIGURE 2.6** (with equal masses attached) and asked which spring would have a longer stretch. In explaining their thinking, 7 of the 10 experts generated significant analogies (analogies that were useful to the solution process). 31 significant analogies were generated in total. For instance, thinking of a bending diving board helped some of the problem solvers toward a solution.

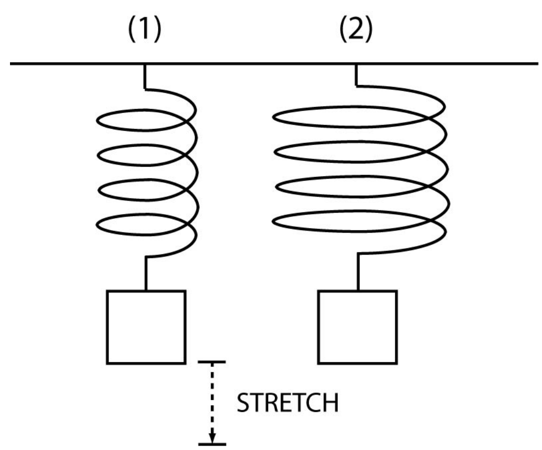

FIGURE 2.6 Springs with equal masses attached.

From observations of these experts, Clement suggested the following four processes in making productive use of a spontaneous analogy:

1. Generating the analogy.
2. Establishing confidence in the analogy relation.
3. Understanding the analogous case.
4. Applying findings.

Similar step-wise processes have been suggested elsewhere as methods of teaching with analogy.[40] Alternative methods exist, avoiding step-wise procedures in favor of more non-linear approaches to teaching with analogy, such as simultaneous comparisons of two domains.[33,41,42] However, these alternatives represent the cutting edge of experimental work on teaching with analogy. Clement defined three methods for generating spontaneous analogies, listed below. The number of times each method was observed, out of the 31 significant analogies, is indicated*. The three methods above are shown schematically in FIGURE 2.7.

1. Generation from a Formal Principle – a single equation or formal abstract principle (e.g. conservation of energy) applies in two or more different contexts. **1 Observed.**

2. Generation via a Transformation – an analogous situation *B* is created by modifying the original situation *A*. **18 Observed.**

* The total adds to 27 observed significant analogies. The remaining 4 were classified *Method Unclear*.

3. Generation via an Association – the subject is “reminded” of an analogous case *B* in memory, rather than transforming *A* into *B*. **8 Observed.**

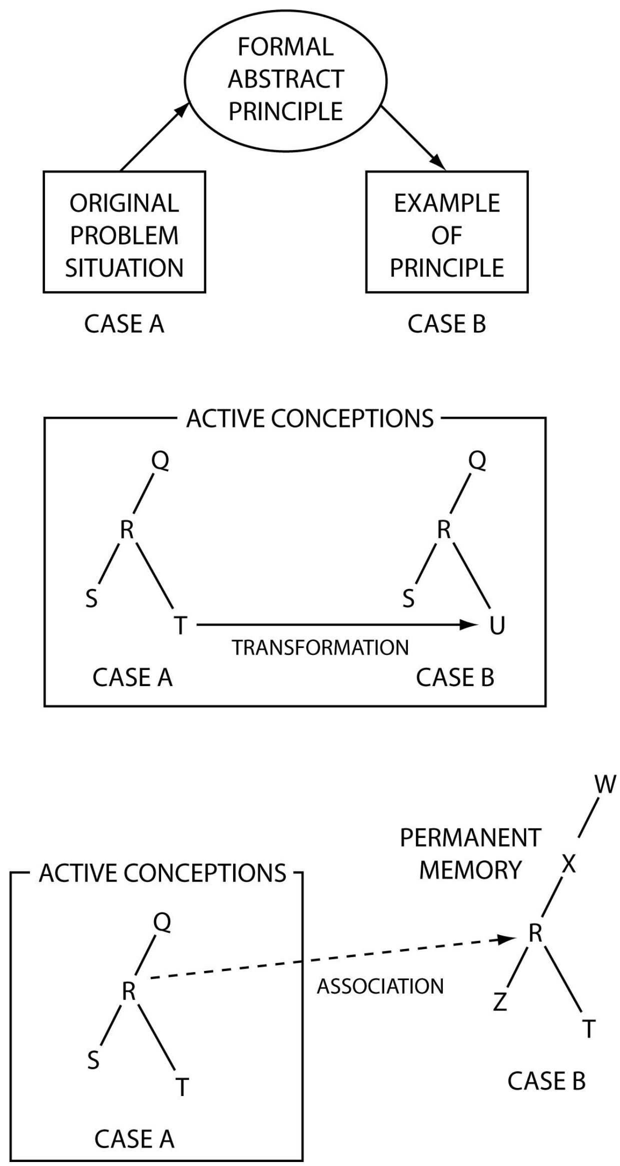

FIGURE 2.7 From top to bottom: Generation via Formal Principle, via Transformation, via Association. Adapted from ref. 38.

This categorization of analogy represents a complimentary view to Gentner's categorization (abstraction, analogy, literal similarity). However, Clement's categories differ in terms of how the mapping connects the two domains. According to Clement, anywhere from one to many features of the target may promote a particular mapping from a selected base domain. Mapping YELLOW from the sun to the nucleus is not necessarily disallowed. This suggests that there are many ways of recognizing and using analogies. Clement showed that spontaneous analogy use could be observed quite readily, and in fact, his subjects were quite adept at generating creative and productive analogies and adapting these analogies to the problems at hand. But these subjects were experts in their domain. What might we expect from non-experts?

Dunker's Radiation Problem

Dunker's radiation problem[43] is a central theme in a series of important analogies studies.[44, 45,46,47] The problem is the following.

> It has been discovered that x-rays of high enough intensity can destroy a cancerous tumor. The complicating issue is that if a patient has a tumor in a particularly sensitive area inside their body, the tumor must be somehow destroyed without harming the surrounding tissue. How can the tumor be subjected to a beam of high intensity x-rays while leaving the surrounding tissue unharmed?

Gick and Holyoak[44,45] conducted experiments to test whether students could solve Dunker's radiation problem using analogies.

In an early experiment, students were given a story about a general invading a fortress. The general's army could attack the fortress from a single direction, but this would leave the army too susceptible to counter attack from the defending forces. The general's solution was to send his troops to battle along multiple lines of attack, thus dividing the opposition's defenses. The attacking soldiers would then *converge* inside the fortress with sufficient force to take the fortress handily. Gick and Holyoak wanted to see if students could spontaneously apply the fortress convergence solution to Dunker's radiation problem.

What do prior models predict about students' performance in this analogy task? A structure mapping analysis of this analogy reveals strong structural consistencies in the mapping. Soldiers map to x-rays, the fortress maps to the tumor, and the dividing and convergence of the attacking army maps to the dividing and convergence of x-rays. Further, students should be guided by the constraint of solving the radiation problem – the use of the fortress analogy has a clear purpose. Finally, prior knowledge should not be a stumbling point since students were given the fortress problem. Students were not initially told to use this analogy. Gick and Holyoak wanted to test whether students would spontaneously use the analogy given all of the favorable conditions listed above.

First, Gick and Holyoak had students solve the radiation problem without giving these students any analogy to use. Only about 10% of these students were able to come up with the convergence solution. This number is somewhat surprisingly low, but these students had very limited resources for solving the problem (i.e., no analogy provided).[*] Next, Gick and Holyoak gave a different set of students the fortress analogy and had them solve the radiation problem. What Gick and Holyoak found was surprising – only 30% of these students were able to come up with the convergence solution. With all of the favorable conditions given, we would have expected a significant fraction of students to use the analogy provided, yet the vast majority did not use any analogies. Interestingly, when students were given a hint to use the story they had been told earlier, the fraction of these students who came up with the convergence solution then shot up to 75%. Indeed, students could use the analogy productively, but they could not make this connection on their own.

Gick and Holyoak followed this study with a second in which students were given two analogs instead of one. Both analogs used a convergence solution, but in different domains. With two analogs to compare, students were much more likely to produce the convergence solution spontaneously (but still a majority of students did not). Gick and Holyoak attribute this to the formation of an abstract *problem schema*, i.e., the general idea of convergence, which was formed more readily when comparing two analogs than when only a single analog was given. In a third experiment, Gick and Holyoak gave students the fortress problem supplemented by a diagram indicating convergence. They found that with the diagram, spontaneous use of the analogy was also increased, and that analogy use after a hint to use the analogy was increased as well. These results were further investigated by other researchers. Pedone et al[46] found that diagrammatic representations of convergence could promote

[*] Note that these students were asked to spontaneously *generate* and use the analogy. The other groups were asked to spontaneously *use* an analogy that had been generated for them.

spontaneous analogy use by students, both when these representations were static pictures and even more so when the representations were animated. Keane[47] found that enhanced similarity of the base domain to the target also promoted significant spontaneous use of analogy – when the base story was about a surgeon using the convergence solution to solve the tumor problem, subjects were able to recall this story a week later to solve the radiation problem again. Keane was also able to replicate the lack of spontaneous analogy use found by Gick and Holyoak when the base domain used the fortress story instead. According to Gentner's definitions, subjects in Keane's study were able to solve the radiation problem when the domain comparison required was a literal similarity (surgeon A is like surgeon B) but not when the comparison required was an analogy. However, Keane also found that after a hint was given to use the previous story, most subjects were able to productively use the fortress analogy to solve the radiation problem.

Thus, we draw the following conclusions from this series of studies. First, the explicit spontaneous use of analogy may be relatively rare for students. However, when students know to use an analogy, these students are often capable of using the analogy productively. Comparing multiple base domains, rather than using a single base, can produce a more generalized schema, here the convergence idea, that students are more likely to use spontaneously in problem solving.[*] Notably, diagrams can play a significant role in student use of analogy.

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[*] This notion of a generalized schema is posited by researchers, but there may be other points of view. For instance, students may become better at recognizing certain patterns of relations or developing particular *habits of mind*, and it is an interesting and open question whether these other views can be distinguished empirically from the generalized schema hypothesis. As an aside, it may be worthwhile to consider the very notion of abstraction, what an abstraction *is*, and whether abstractions can be considered separately from their phenomenological grounding. As I said, this is

What is interesting about these studies is that, apparently, while students sometimes know how to use analogies, but they may not know when to use analogies. These studies show quite clearly that giving students a hint to use an analogy can be a very productive method of inducing student analogy use. However, many analogies in physics are more subtle than those described above. We would like to better understand how students might use analogies without such explicit prompting. Further, students may often use analogies implicitly (possibly without being aware they are doing so), and we would like to understand the underlying mechanisms of this process. After all, if Lakoff's view is correct, then analogical reasoning pervades thought at many levels and across circumstances, whether we are aware of it or not. Consider, among many possible contexts, students answering a physics exam question. These students might use analogical reasoning, implicitly or explicitly, to answer this question. What features of the problem might lead students to use an analogy (or not), or to choose a particular analogy among several? When we teach physics, what are the teaching strategies or particular material features that might lead students to use particular analogies?

Strategies for Teaching – Bridging Analogies and Conceptual Change

Thus far, we have examined mostly clinical studies designed to test certain hypotheses about analogy. Gentner and Gentner attempted to teach electricity concepts using analogy, but found their teaching method largely ineffective. Clement's process for using analogy, while drawn from experts, could be applied to

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an interesting question, but I am not interested in arguing for, nor against, either position at this point.

teaching students. However, studies have shown that experts and novices do not always categorize problems in the same way,[48] e.g., the inclined-plane problem for the novice may be a conservation-of-energy problem for the expert. If different caliber students draw on different base domains, a specific strategy may be necessary to enable students to use a particular analogy successfully. Brown and Clement[25] explored the use of analogies in overcoming student misconceptions[*,49] in the target domain. As suggested by Gentner and Gentner, the success of an analogy-based teaching method depends on student knowledge of the base domain (i.e. prior knowledge) and student acceptance of the analogy. Brown and Clement, claiming that students have both useful and detrimental preconceptions, explored the use of a bridging strategy to build on students' useful conceptions in order to bring about conceptual change.

In one small-scale experiment, teaching interviews were conducted with four students.[25] The four interviews involved conceptual questions relating mostly to Newton's laws and forces exerted between objects, listed in Table 2.2 below. The physical scenarios are listed on the left and the *anchoring* analogy for each scenario is listed on the right. The anchoring analogy is the first analogy suggested, from which a series of bridging analogies follow in order to help a student connect the physical scenario (i.e., target) and the anchor (i.e., base).

[*] According to Redish,[49] a misconception is a particular mental model or line of reasoning that is robust, but is in contradiction to scientific theory, and found in a significant fraction (20% or more) of students.

Table 2.2 Four cases of analogy used in Brown and Clement's interviews.

Physical Scenario (Target)	Anchoring Analogy (Base)
1. A book resting on a table.	Book resting on a spring.
2. A shuffleboard puck sliding on the floor.	Two brushes rubbing the bristles together.
3. One roller skater pushing off of another roller skater.	Two train cars pushing off of each other via springs between the cars.
4. A moving billiard ball colliding with a stationary billiard ball.	A man standing on the end of a train car that collides with another car carrying a long log.

Using the bridging strategy, the interviews achieved noticeable conceptual change in cases 1 and 2, but failed in cases 3 and 4. Brown and Clement suggest that the differences were due to the type of analogy used. In the failed analogies, the base shared only abstract form with the target. For instance, in case 3, skaters push off in the target while train cars push off in the base. There are few surface similarities between the base and target domains in this case. In the successful analogies, the base and target shared some material features, instead of only abstract form. For instance, case 1 involves a table and a book in both the base and target. This may mean that what Gentner refers to as an *analogy* is more productive for conceptual change than an *abstraction.* Further, Brown and Clement's method revisits Lakoff's idea of layering – for a sufficiently abstract concept, a series of intermediate steps may be necessary. More formally, consider an abstract mapping A→B. For the analogy to be successful, it may be necessary to provide an intermediate domain C, resulting in the series of mappings A→C→B. Notably, both surface and deep structural features may play important roles in the use of analogy.[50][51]

One limitation of the study by Brown and Clement is the small number of students involved (N=4). Following this small-scale study, Clement[52] carried out a large scale study to test two of the successful bridging cases discussed above, a book resting on a table and an object sliding on a surface with friction, and an additional third case, that of two moving automobiles that collide. The physics content areas corresponding to these three cases were, according to Clement, *static normal forces*, *friction forces*, and *dynamic third law* (i.e., Newton's third law). The anchoring analogies were the same as discussed above for the first two cases. The anchor for the colliding cars was a person compressing a spring between both hands.

The subjects in this study were 205 high school students taking a first-year physics course. 150 of these students, the experimental group, received lessons using bridging analogies. The remaining 55, the control group, received standard instruction without analogies. Students in both groups were given pre- and post-tests covering the three content areas. Positive gains were found for both groups, but the experimental group achieved pre-post gains two to three times those of the control group across all three content areas. This study demonstrated that bridging analogies could be used to teach in a large-scale physics course.

Limitations of Prior work

The most glaring way in which prior work appears limited is in the experimental results. While researchers have documented many cases where students largely fail to use analogies (productively or at all), very few studies have described cases where student do use analogies productively. Moreover, few studies have identified reliable factors that can promote student use of analogy. The most reliable method found to promote student analogy use is to tell students to use an analogy.

Framed another way, however, we might view analogy as a resource that students regularly use, but that usually flies under the radar. Prior analogy research has focused on specific analogies and how to get students to use them, but very little on how analogies are used by students when instructors are not actively trying to promote the use of a particular analogy. So called spontaneous analogy use can have two forms – spontaneous use of an instructor generated analogy and spontaneous generation of analogy by the student. Both of these processes have been observed in use by experts in physics, but both are rarely found in use by physics students. Notably, this lack of findings for students may be partially due to the sorts of analogy use that researchers have been looking for.

What I will attempt is to seek factors that may promote analogy use by students (including cases where students do not explicitly recognize their own analogical reasoning). I will ask, what factors can we identify that lead to students using resources that we, as researchers, can identify as analogical in nature? In other words, I will take a step back. It may be that researchers' focus on getting students to use particular analogies is actually exploring a higher cognitive function than these researchers seemed to be assuming. It may be fruitful to tease out students' use of analogy from students' meta-skills with analogy, i.e., students' ability to consciously and actively observe and critique their own use of analogies. One framing on prior analogy work is that these studies were seeking observations of student meta-skills with analogy, rather than the more basic function of analogical thinking that does not include the more conscious awareness of how or why one is using an analogy. In my work, I am interested in how, when, and why students use analogical reasoning even when these students are not explicitly cued to use analogies and are possibly unaware of their own thinking processes on a meta-level.

To be sure, I do not mean to imply that there have not been successes. Notably, the success of Clement's large-scale study in promoting conceptual change using bridging analogies may be attributed largely to substantial empirical efforts, researchers' intuitions about student learning, and perhaps a bit of luck in finding an analogy that was productive for students. While the bridging strategy can be a useful heuristic for informing teaching practice, it may not be a sufficient model for predicting specific experimental outcomes. That is to say, bridging provides an excellent avenue towards developing curricular materials, but does so at the cost of a greater specificity that is provided by other models of analogy. Abstract transfer models provide one possible alternative, describing analogical reasoning as a specific mapping from one structure to another. These structures are generally more descriptive than bridging, at least at a finer conceptual grain size. Structure mapping, for instance, specifies objects, attributes, and relations. However, models that only employ a unidirectional mapping between two relatively static, abstract, expert-like structures may not accurately (or sufficiently) model student reasoning.

Structure mapping might have utility to inform teaching in a way, say, if an instructor were to employ the model to create curricular materials based on specific mappings to be completed by students. Bridging, on the other hand, draws on a more student-centered approach which recognizes the complex and often non-linear ways students actually learn. My contention is that neither of these approaches alone is sufficient to substantially explain students' use of analogy, nor to productively inform teaching practice. The model I will present, Analogical Scaffolding, is an attempt to unify existing theories of analogy as well as incorporate other models of cognition in order to generate a model of analogy with greater specificity, utility, and, importantly, predictive power for cases of student learning with analogy.

Studies on bridging have demonstrated that student learning can be highly complex, involving a set of more than two analogical domains which are not necessarily accessed in a linear fashion. Cognitive processes in addition to mapping may be involved in learning complex and abstract ideas. For instance, two domains may combine (or *blend*) into a new domain which is not a perfect isomorphism from either of the original two domains.[17,29] Lakoff emphasizes that abstract ideas are rooted in metaphors, which in turn are ultimately rooted in concrete human experience in the material world. However, Lakoff's focus is on a relatively long time-scale and may not be sufficient for describing the dynamics of student reasoning on the scale of a semester, a week, or a few minutes (let alone the second to millisecond scales on which many cognitive processes may function).

Importantly, existing models tend to focus on cognitive structures while largely ignoring salient factors of the environment, for instance diagrams that are often used to teach physics. Physics education researchers have established that student reasoning can be highly dependent on the representational forms used to teach physics and assess student learning of physics.[8,53,54,57,75,85,86] In coming chapters, I will demonstrate that representations, such as the sine wave representation, can play a significant role in students' use of analogy. Students' use of representations can be analyzed at two levels (possibly more). One is student *representational* *competence*,[54,55] or simply how students use and interpret representations. A higher level is *meta-representational competence*, [*,56] the ability to invent, critique, and learn new representations.[57,58] Ochs et al.[59] found that practicing scientists were adept at this skill, while diSessa et al.[60] also found indications of strong meta-representational competence in sixth-graders (see also ref. 61). Nonetheless, students do not always demonstrate this skill, and researchers continue to examine its development, as well as how/if it can be taught. While this meta-level skill is certainly important, student representational competence (or simply representation use) in the context of analogy has been largely unexplored, and current models of analogy do not explicitly include the entailments of representation. In the following work, I will focus mostly on the role of representations in student learning by analogy. Investigating or teaching meta-skills explicitly may lead to potentially fruitful research lines in the future.

* * *

This chapter has surveyed the literature on analogy in physics learning and instruction. Other researchers have covered a good deal of ground on the use of analogy, but there remains significant motivation for further studies that will add to our understanding of analogy use by students. Early studies in this area showed that analogies were more than convenient terminology, but actually generated inferences between base and target domains. When applied to physics, this means that the analogies students generate will affect their understanding of physics concepts.

[*] If we agree that representations can be considered analogies of a sort, then we could also consider *meta-analogical competence* as a valuable scientific skill.

Brown and Clement explored the spontaneous generation of analogies by experts, while Gick and Holyoak examined spontaneous *retrieval* by students, and a few studies have examined spontaneous generation of analogy by students. However, prior studies have found that explicitly teaching a certain analogy may not directly affect student learning – students' prior knowledge can interfere (constructively or destructively) with these teaching strategies. Effective teaching strategies seem to involve analogies that are at a level that students can understand – that is, the analogy cannot be too abstract. While analogy is a rich area of research for cognitive scientists, there remain many open questions and unexplored avenues in physics education. These efforts all contribute to the broad effort by the physics education research community to enhance instruction through a better understanding of student learning.

All of these prior findings and models may be pieces of a larger puzzle, or some may simply be inappropriate for the purpose of describing student reasoning and learning.[*] There remain open questions about the dynamic nature of analogical thinking. Meta-skills, such as the ability to choose a productive analogy, and experts' knowledge of which relations to map, are still poorly understood. Most studies assume a one-way mapping from the base to target. So called *simultaneous* analogies may also be considered, where two analogous situations work in unison to construct knowledge of both domains.[33] Another exciting area of research asks how students learn abstraction. Simultaneous domain comparisons may lead students to extract abstract structure and develop conceptual knowledge. One of the principal skills of

[*] A broad range of literature describing pieces of this puzzle is provided in the Annotated Bibliography, Appendix B.

expert physicists is to think in terms of abstractions. To understand abstraction, Lakoff's ideas will prove useful – students may develop the skill of abstraction by building upon lower level analogical thinking skills.

Chapter 3 – Identifying Critical Questions on Analogy and Representation*

"I see nobody down the road," said Alice.

"I only wish I had such eyes," the King remarked, in a fretful tone.

"To be able to see Nobody! And at that distance too!"

– Lewis Carroll, *Through the Looking Glass*

Previous researchers have made claims that analogies are generative, and that teaching with analogy can sometimes be productive. Questions remain as to how analogies can be used in a teaching mode, and whether previous results can be replicated with larger numbers of students (N>200) in an authentic learning environment (i.e. an introductory physics course). We now wish to test the hypothesis that analogies generate inferences with large N in a college setting, to study how they affect student reasoning, and to seek mechanisms leading to a model of how analogies are used. In this chapter, I will describe a series of preliminary experiments which address these issues by focusing on analogies for teaching and learning electromagnetic (EM) waves.[62] These pilot studies will provide a foundation for theoretical work and further experiments, discussed in the coming chapters.

Of the concepts taught in introductory physics, the EM wave is one of the most abstract. We never really experience electromagnetic waves *as waves*. Humans

* Chapters 3 and 4 are largely drawn from published work, Podolefsky, N.S. & Finkelstein, N.D. (2006) The Use of Analogy in Learning Physics: The Role of Representations. *Phys. Rev. ST - Phys. Educ. Res.* 2, 020101.

see in the visible spectrum, but we see shapes and colors, not waves fluctuating at several hundred terahertz. Contrast the experience of seeing light with seeing a wave on a string. The wave on the string is directly observable by the human sense of sight. Under the right conditions, a wave on a string can look like a perfect sinusoid. Consider sound waves, which fall somewhere in between strings and EM waves in terms of direct experience as waves. Humans experience sound as vibrations, sometimes quite apparently for low frequencies at high decibel levels. However, humans do not, in general, conceptualize sound waves as sinusoidal waves. Though sound waves vibrate our bodies, the sinusoidal interpretation requires some scientific resources – some additional layers (in Lakoff's terminology) for this particular formal interpretation of the sound wave. EM waves, I hypothesize, require even more layers for the scientific interpretation due to the large gap between the human experience of EM waves and the physical model that describes the waves.

It should not be surprising, therefore, that students have difficulty interpreting graphical representations[*] of EM waves and relating them to the physical phenomena.[63] Unlike more concrete wave phenomena (e.g., oscillations of a string), EM waves are variations in fields – the fields represent potential forces on charges. Furthermore, students are taught that, for a plane wave, the fields exist everywhere in space and the wave propagates even in the absence of a medium.[†] For these reasons, EM waves can provide a potentially fruitful content area for studying analogy. In this

[*] Throughout this work, we will use the word *representation* to refer exclusively to external representations (e.g. graphs), rather than internal representation (e.g. mental models).

[†] Students are also taught that the fields exist in space even if the charges (and, hence, forces) are not present.

study, we ground the abstract concept of EM waves with more concrete phenomena: sound waves or waves on a string.

Drawing on the dominant prior theoretical frames of students' use of analogy, we designed a study to examine whether, how, and when students use analogies. Recall that structure mapping defines an analogy as a mapping $M{:}B \rightarrow T$, where B is the base domain and T is the target domain. To use the analogy is to complete a mapping from one structure to another. For instance, in applying this theory to Rutherford's analogy, the sun maps to the nucleus, the planets to the electrons, the gravitational force to the Coulomb force, etc. In addition, relations between these objects, such as "revolves around", map across domains. This analogy is commonly employed to teach a particular, though rudimentary, model of the atom to students. This model has certain features that are useful for understanding how atoms work: the nucleus is at the center with electrons orbiting, held in by some central force; electrons are tiny compared to the nucleus, and most of the atom is empty space. This model lays in contrast to other analogies, such as the "plum pudding" or "electron cloud" models, which ascribed very different characteristics to atoms.

In the last chapter, I explained that structure mapping belongs to a class of theories that might be categorized as *abstract transfer* with certain inherent assumptions about students learning and analogy use.[25] The important point was that rather than focus on students' ability to use an analogy, abstract transfer theories tend to rate an analogy's effectiveness from the expert physicist's point of view, not the student's. While these models are useful for framing our understanding of analogies, they may fall short of explaining how analogies are used by students, or how to use analogies productively for teaching.

Expert physicists commonly use analogies productively, even when solving basic physics problems.[38] In fact, physicists seem to be experts at using analogies: they know when an analogy generates correct inferences, and recognize when it fails. Students, unfamiliar with the content to be learned, are not necessarily able to make such productive use of analogies, especially when using analogies to learn about concepts that are very abstract or unfamiliar.[25] Structure mapping delineates what is means for an analogy to "work", and what it means for an analogy to "break down" in terms of mapping. For example, in the planetary model of the atom, the attribute MASSIVE should map from the sun to the nucleus, but not the attribute YELLOW. The relation ATTRACTS(sun , planet) maps to ATTRACTS(nucleus , electron), but ATTRACTS(planet , planet) does not map to electrons (which use the relation REPELS(electron , electron)). Structure mapping predicts that attributes and relations are more likely to map if they are tightly integrated into a hierarchy of connected ideas. For instance, structure mapping predicts that MASSIVE will map because it is a key component of a higher order structure, i.e., a central force system. YELLOW is not as likely to map because it is not a necessary component of a central force system. In this study, we use structure mapping to identify analogical mappings, but not as a mechanism by which students use analogies in their learning. Brown and Clement distinguish between expert and novice usage of analogies, and address the challenge of novice usage specifically with a bridging strategy that builds on students' prior conceptions. We build on the ideas of mapping between domains, layering of more complex or abstract ideas, and blending domains to examine how and when students successfully use analogies.

In the large-scale (N>200) studies described below, we demonstrate that different analogies can lead to varied student reasoning. When different analogies were used to teach EM waves, we found that students explicitly mapped characteristics either of waves on strings or sound waves to EM waves, depending upon which of these analogies were taught to students. We extend these results by investigating how students use analogies. Our findings suggest that representational form plays a key role in the use of analogy.

In this chapter, we address the following questions. Does the use of different analogies lead to different student reasoning in a large physics class at the college level, and, further, do analogies support the generation of inferences when taught in this environment?* These questions lead us to examine some of the key mechanisms by which students productively use analogies. Our findings in these pilot studies are summarized as follows: Analogies can lead students to generate different ideas depending on the analogies used (i.e. analogies are generative in the sense used by Gentner and Gentner[37]). Further, these analogies can be generative when taught. We demonstrate these results in a large-scale study focusing on undergraduate physics. In addition, we begin to explore key mechanisms by which students use analogies, and find that representations are crucial to student reasoning and the promotion of certain analogical mappings.

* In particular, we follow the work of Gentner and Gentner who provided evidence supporting the generative analogy hypothesis in a small scale experiment.[37]

Overview of Experiments

Our pilot studies were carried out in two introductory physics courses at a large university. Both courses were calculus-based introductory physics, one taught in spring 2005 (N=249) and one in fall 2005 (N=353). Both courses were the second semester of a two semester sequence, primarily covering electricity and magnetism. Each course consisted of three one-hour lectures per week in a conventional lecture hall and one hour per week in a small recitation setting (N~25) led by two teaching assistants. The lectures made extensive use of *Peer-Instruction*[64] and personal electronic response systems (PERS, also know as "clickers").[65] Otherwise, lectures were the traditional style, with the instructor lecturing from the front of the room with chalk, overheads, and an occasional demonstration. During the recitations, students worked in small groups using the *Tutorials in Introductory Physics*.[66] Students generally completed a Tutorial pre-test online, submitted before the start of recitation.

The initial run of our experiment (spring) was designed as a preliminary study. We modified the follow-up study (fall) based on results of the first study in three key ways. (1) The sole focus of the recitation was restricted to paper and pencil based tutorials. The first run included other activities. (2) Based on results of the initial study, we refined the tutorial used in the follow-up study. (3) We more tightly coupled the post-test to the treatment by administering the post-test sooner after the treatment, and by placing the experiment so no other relevant instruction other than the tutorial occurred before the post-test. The follow-up study was meant to demonstrate that our initial findings were repeatable, and that our approach to teaching with analogies could be refined. Our studies were conducted in two parts.

Part I examined the generative use of analogies. Part II examined mechanisms behind the use of analogy, focusing on representations.

Part I: Teaching with Analogies

Methods

In the first portion of our study, students learned about EM waves from a tutorial which borrowed heavily from the *Tutorials in Introductory Physics*.[66] Students completed the tutorial in recitation sections. We modified the original tutorial to include a front section that focused on analogies. In each course, students were randomly assigned by recitation to one of three groups, denoted as the *string analogy*, *sound analogy*, and *no-analogy* groups. All the students in a given recitation section were placed into the same group, and the differential treatments in recitations were evenly distributed among different teaching assistants and times of day. Table 3.1 lists the numbers of students in each treatment group for each course. Each group completed the modified EM waves tutorial, which consisted of three parts. For the analogy groups, part 1 covered basic wave concepts, such as amplitude, wavelength, and frequency, in the context of either sound waves or waves on a string. Part 1 for the no-analogy group was isomorphic to the analogy groups, but used EM waves instead of one of the analogies. Part 2 was substantially identical for all three groups and covered basic wave concepts for EM waves. Part 2 also used more sophisticated representations than part 1, described in more detail below. Part 3 was unmodified from the original version from the *Tutorials*, covering concepts related to forces on charges from electric and magnetic fields. The tutorials for the three groups differed only in the use of analogy, and were made as isomorphic as possible.* In both courses, this tutorial provided students' first formal instruction on the content of the analogies used (string and sound waves).

Table 3.1 Two treatment groups (string and sound) and control group (no-analogy) for the initial (spring) and follow-up (fall) studies.

Course	Group	N
Spring 2005	String Analogy	72
	No-Analogy	90
	Sound Analogy	87
Fall 2005	String Analogy	91
	No-Analogy	112
	Sound Analogy	95

In addition to covering the basic wave concepts listed above, the tutorials in the follow-up study (fall) were tailored to address specific concepts about the propagation of waves through space. In part 1, students were presented with one of the pictures shown in FIGURE 3.1. The string and sound analogy groups were presented with the string and sound pictures, respectively, and the no-analogy group with the EM wave picture. In the string group, three beads are labeled 1-3. Students were asked to describe the motion of each bead as the wave propagates to the right. The intention of this exercise was to cue on two features of traveling waves on strings: (1) since these are traveling waves, bead 2 moves; (2) since these waves are two dimensional (2D oscillations confined to a vertical plane), bead 3 does not move. In the sound group, students were asked to describe the pressure at points 1-3. Here, the intention was to cue on the three dimensional (3D) nature of sound waves (sound propagates as spherical wave fronts, and, hence, the pressure is nearly equal at all

* The teaching materials used are available in Appendix A.

three points, very nearly like a plane wave). In the no-analogy group, students were asked to describe the magnitude of the electric field at points 1-3. Here, the intention was to cue on the 3D nature of EM waves (since this EM wave is a plane wave, the field is equal at all three points). Note that both the traveling and 3D wave characteristics are critical for understanding EM waves.

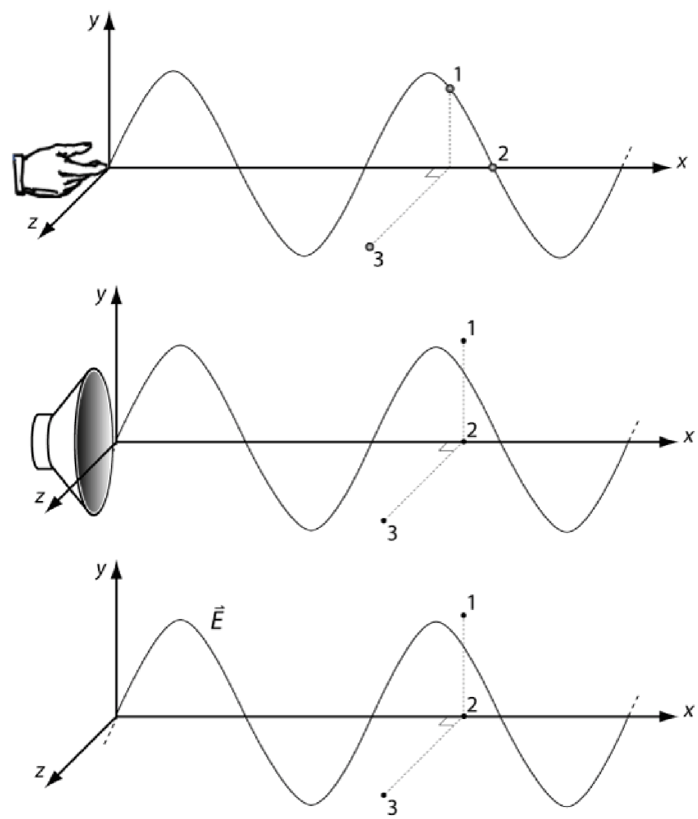

FIGURE 3.1 String (top), sound (middle), and EM (bottom) wave pictures from part 1 of the tutorial. These were added to the tutorial in the follow-up (fall) study.

Although each picture in FIGURE 3.1 uses a sinusoid to represent the wave, the sinusoid carries different meaning for each. Specifically, the sinusoid for the string represents a string oscillating up and down – a transverse wave constrained to the x-y plane. For the sound wave, however, the sinusoid represents the sound pressure (or motion of air particles) – a longitudinal wave propagating throughout space in three dimensions. Finally, for the EM wave, the sinusoid represents the magnitude of the electric (and/or magnetic) field – a transverse wave propagating throughout space in three dimensions. Referring to FIGURE 3.1, students were explicitly taught about traveling waves in the context of strings, and about 3D waves in the context of sound.

In part II, for both the initial and follow-up studies, students in all three groups were presented with the picture shown in FIGURE 3.2, drawn from the *Tutorials*, and told that it represented an EM wave at one instant in time. Students were asked to rank the magnitude of the electric and magnetic fields at each point (1-4) in the image. Note that this exercise addresses the behavior of the electric and magnetic fields at four points in the x-y plane, but not at other z-positions. Students in the two analogy groups were given a hint to use an analogy to sound or wave on a string in answering this question. However, the tutorials did not instruct students about which mappings to make (e.g. that an EM wave is like a sound in that it is 3D).

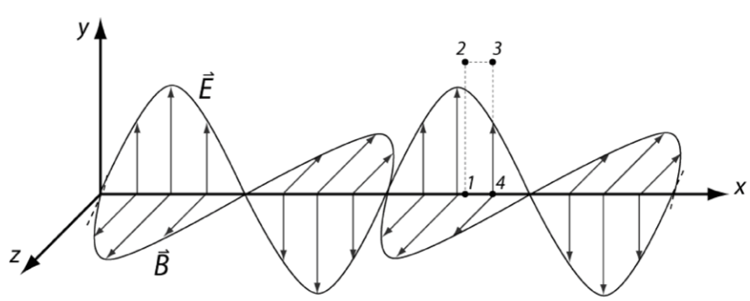

FIGURE 3.2 EM wave picture from part 2 of the tutorial. This picture was used in both the initial and follow-up studies.

In each study, we assessed the effects of teaching with different analogies by giving a post-test which included a question that drew directly on the concepts covered in the tutorials. In the initial study, we gave an EM wave question on the final exam, given five weeks after the EM wave tutorial. The exam question presented students with a representation of an EM wave (FIGURE 3.3) and explained that it showed a plane wave propagating to the right. The multiple choice question asked students to rank the *time-averaged* signals received by each antenna (labeled 1-4). In the follow-up study, we posed the same EM wave question as a concept test in lecture on the day following the recitation in which students completed the EM wave tutorial. The concept test was given at the beginning of lecture, and students were instructed not to discuss the question before answering. Thus, for students in the follow-up study, the tutorial provided the only formal instruction prior to the post-test.

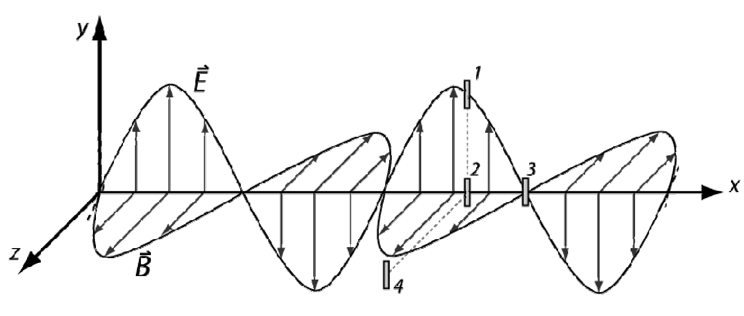

FIGURE 3.3 EM wave as presented on the post-test in both the initial and follow-up studies. Vertical antennas are labeled 1-4.

We anticipated particular outcomes from student responses to the post-test questions. While many studies examine only whether students answer questions correctly or incorrectly, here we attend to the information contained in which wrong answers (distracters) students choose.[67,68] In the analysis of post-test data, we look for effects of different analogies by examining student responses across different treatment groups (sound, string, or no analogy). We do not focus specifically on whether students chose the correct answer, but rather whether we can observe differential response patterns on the post-test for the three treatment groups (string, sound, and no-analogy). We hypothesized that students in the string group would be more likely to choose distracters associated with "traveling" and "2D" characteristics, since these characteristics are also associated with a wave on a string (a wave on a string is, generally, confined to a single 2D plane, e.g. the *x*-*y* plane). We similarly hypothesized that students in the sound group would be more likely to choose distracters associated with "3D" characteristics, since this characteristic is associated with sound waves (sound waves spread from a source, extending outside of the *x*-*y* plane in the *z*-direction).[*] Note that an EM wave, as presented in FIGURE 3.3, includes both "traveling" and "3D" characteristics. To be clear, this does not mean that we anticipated more students in the string group, for example, to choose the "traveling,2D" distracter over other distracters. There may be many reasons students answer in a particular way, and one distracter may prove stronger overall because of these reasons. Nonetheless, finding differential response patterns between treatment groups on the post-test would support the hypothesis that analogies generate inferences when taught.

[*] Sound waves are traveling waves, but this was not explicitly taught in the tutorial.

Results

Of the five possible student responses, three dominated in both studies (>82% in spring, >90% in fall), and no other distracter received a substantial fraction of responses (<11% in spring, <5% in fall).* The top three responses are shown below the horizontal axis in FIGURE 3.4 (from left to right: 1=2=3>4, 1=2=4>3, and 1=2=3=4). We consider the first distracter (1=2=3>4) to be associated with "traveling" and "2D" properties, and the second distracter (1=2=4>3) to be associated with "3D" properties (as defined above). The third answer (1=2=3=4) is correct and includes both "traveling" and "3D" properties.

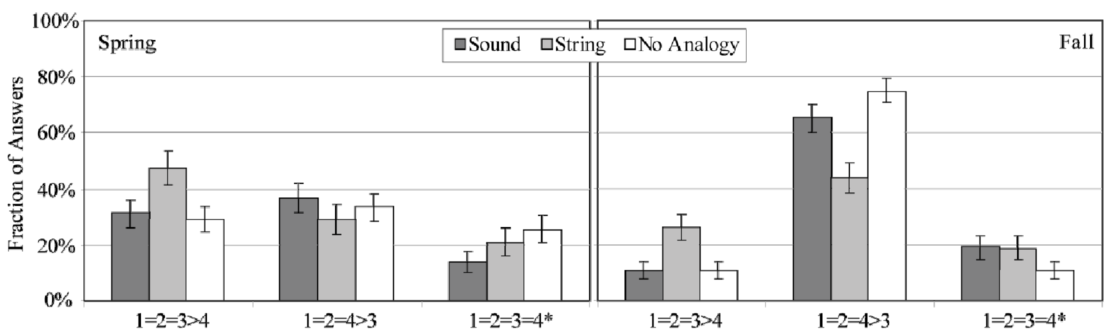

FIGURE 3.4 Post-test results for the initial study (spring, left) and follow-up study (fall, right). The top three answers (1=2=3>4, 1=2=4>3, 1=2=3=4) are shown below the horizontal axis. The analogy groups (sound, string, and no-analogy) are indicated in the legend. Error bars are the standard error ($\sigma / \sqrt{n}$). The "*" indicates the correct answer.

The results of the post-test in the initial study (spring) are shown at the left of FIGURE 3.4. Overall, only 20% of students chose the correct answer (1=2=3=4).

* The two other answers were revised in the follow-up study to be more attractive, but neither proved to be strong distracters.

Students from the string group were most likely to choose 1=2=3>4 ($p<0.05$).[*] There was no significant difference ($p>0.1$) between the sound and no-analogy groups on the distracter 1=2=3>4, nor were there significant differences ($p>0.1$) between any of the groups on the distracter 1=2=4>3. Students in the no-analogy group were slightly more likely to answer correctly compared to the sound group ($p=0.05$). Notably, the post-test in the initial study assessed the effectiveness of a single recitation conducted five weeks prior.

We found clear evidence that the different analogies affected student responses in the follow-up study (fall). The results are shown at the right in FIGURE 3.4. Students from the string group were more likely than sound to choose 1=2=3>4 ($p<0.01$), while students from the sound group were more likely than string to choose 1=2=4>3 ($p<0.01$). There were no significant differences ($p>0.1$) between the sound and no-analogy groups on either distracter, nor were there significant differences between any of the groups on the correct answer (1=2=3=4).

In summary, in both courses, students who chose the ”traveling,2D” distracter were most likely from the string group, while students who chose the ”3D” distracter were more likely from the sound and no-analogy groups. The ”3D” distracter describes a wave extending throughout space in three dimensions, which is characteristic of both sound and EM waves.[†] Notably, students in the no-analogy group were explicitly taught the 3D characteristic of EM waves. However, students in the sound group were only taught this characteristic of sound waves; these students were never explicitly taught this characteristic of EM waves. Thus, we believe

[*] Post-test results were analyzed using a two-tailed z-test of pair-wise comparisons.

[†] Since the 3D characteristic was taught explicitly for EM waves in the no-analogy tutorial, students may well have chosen the distracter 1=2=4>3 more often based on memorization or recall.

students in the sound group were mapping this characteristic from sound to EM waves.

Part II: Representation and Analogy

In early student interviews, we found that representations could cue students to focus on different characteristics of EM waves. In order to study one possible mechanism for using analogies, we developed an assessment meant to probe student understanding of wave representations and associated phenomena. The assessment, described below, was given online, consisting of multiple choice and long answer questions. Students completed the assessment prior to recitation.

Methods

In both studies, fall and spring, all students were divided evenly into two groups, denoted by string representation and sound representation groups. Table 3.2 shows the numbers of students in each group for each course. The representation assessment presented students with a pictorial representation of either a hand moving a string or a speaker and dust particle, shown on the left of FIGURE 3.5. Students were asked to choose the representation of the sound or string wave that made the most sense to them and then asked to explain their choice as a long answer. The choices students selected from are the iconographic representations shown on the right in FIGURE 3.5. Students were told that there was no correct answer to the choice of representation. Students were then asked a multiple-choice follow-up question about the motion of either a dot on the string or the dust particle. The top three responses[*] to the motion question were "up and down", "to the right", and "side-to-side".

Table 3.2 Two groups for the wave representation assessment (string and sound) for the initial (spring) and follow-up (fall) studies**.**

Course	Group	N
Spring 2005	String Representation	122
	Sound Representation	122
Fall 2005	String Representation	170
	Sound Representation	168

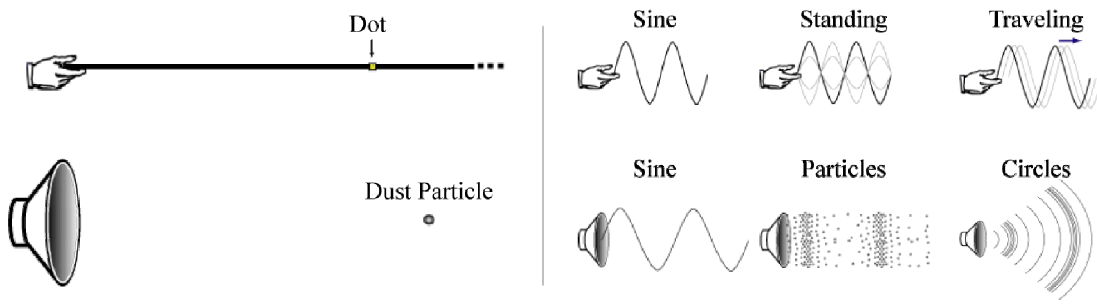

FIGURE 3.5 Left: Pictorial representations from the string (top) and sound (bottom) wave representation assessments. Right: Iconographic representations from the wave representation assessments for strings (top) and sound (bottom). The representations are labeled only for reference in this chapter – the labels did not appear in the representation assessment.

Results

To analyze the results of the representation assessment, we first examine the reasons students provided for choosing a particular iconographic representation. The reasons given were binned into categories based on an emergent coding scheme, described in

[*] Out of 6 in the spring, 8 in the fall.

more detail below. We next examined the relationship between which iconographic representations students choose and their answer on the motion question. Again, we look for information in the distracters as well as the correct answers. While students who choose different representations do not actually constitute different treatment groups, we look for associations between choice of representation and answer selected on the motion question.

Three dominant categories emerged from coding the reasons students gave for choosing a particular representation of sound. These categories are shown in Table 3.3 along with sample statements from students. "Formalism" is characterized by reference to mathematical objects associated with wave physics (e.g. usage of the word "sinusoidal"). "Medium" is characterized by reference to the medium through which the wave propagates (e.g. reference to "air particles"). "Spreads" is characterized by reference to sound as spreading, or traveling in multiple directions, as it propagates (e.g. in circles away from the speaker). Table 3.4 presents the number of statements in each reasoning category sorted by selected iconographic representation. Approximately 15% of statements contained elements from two categories, and these statements were counted in each of these two categories. To test for the reliability of the coding scheme, two individuals (the lead author and a researcher unrelated to the study) coded a subset of the students' statements. Coding agreed to better than 87%, and the patterns shown in Table 3.4 were extremely consistent for the lead author's and other researcher's coding. There is a statistically significant relationship between students choice of representation and reasoning given (χ^2; $p<0.001$). The bulk of responses fall along the diagonal, suggesting association between representation and reason stated. Nearly all students who chose the sine or particle representation fell into the formalism or medium category, respectively. Students who chose the circles representation fell dominantly into the spreads category, with additional statements falling into the medium category. This result is similar for both semesters.

Table 3.3 Long answer coding for sound waves.

Category	Sample Statement
Formalism	"This makes sense to me because when I think of waves I think of sinusoidal waves."
Medium	"It's a compression wave that is moving air particles."
Spreads	"I think that sound waves spread out from a source, such that you can hear them in any position in front of the speaker."

Table 3.4 Data for students' long answer reasoning for sound waves.

	Spring			Fall		
	Formalism	Medium	Spreads	Formalism	Medium	Spreads
Sine	14	1	0	23	0	0
Particle	1	15	0	1	10	1
Circles	6	18	39	7	12	71

Four dominant categories emerged from coding the reasons students gave for selecting representations of the wave on a string. These categories are shown in Table 3.5 along with sample statements from students. “Formalism” is defined in the same way as the category for sound. “Transverse” is characterized by reference specifically to up/down motion of the string. “Traveling” is characterized by reference specifically to propagation along the length of the string. “Generic motion” is characterized by reference to motion of the string with no specific direction. Table 3.6 presents the number of statements in each reasoning category sorted by selected iconographic representation. Again, approximately 15% of statements were counted in two different categories. Agreement between two separate coders was better than 82% in this instance. Examining the rows of Table 3.6, we find each iconographic representation is associated with a different pattern of reasoning categories. This relationship is statistically significant (χ^2; $p<0.01$). However, the data in Table 3.6, for the string group, are more distributed among reasoning categories than the data for sound (Table 3.4).

We make the following claims based on the results above. We found that students focused on different characteristics of sound, and that these associations were strongly coupled to their choice of representation. On the other hand, while students focused on different characteristics of oscillating strings, a single representation was associated with multiple characteristics, unlike the case for sound.[*] These results were similar for both semesters.

[*] At this point, we do not know whether representations influence student conceptions of sound, conceptions influence the choice of representation, or if these two influence each other.

Table 3.5 Long answer coding for waves on a string.

Category	Sample Statement
Formalism	"It is like a sine function which I understand."
Transverse	"String moves in wave motion, each point goes up and down."
Traveling	"The wave will travel down the rope."
Generic motion	"You can see the whole motion of the string and how it changes."

Table 3.6 Data for students' long answer reasons for waves on a string.

	Spring				**Fall**			
	Formalism	Transverse	Traveling	Generic motion	Formalism	Transverse	Traveling	Generic motion
Sine	8	3	3	8	10	3	2	2
Standing	7	13	5	9	16	11	2	13
Traveling	4	4	15	13	16	13	21	9

The results on the motion question were similar in both initial and follow-up studies. FIGURE 3.6 shows the fractions of students choosing a particular representation that selected a particular answer to the motion question. For example, in the spring semester, of the students who chose the "sine wave" representation of sound, 43% answered up/down. The "*" indicates the correct answer for each group. In the string group, the majority of students (>83%) chose the correct answer (up/down). This choice was independent of their choice of representation (χ^2; p>0.3). In contrast, the responses in the sound group were varied. We found a relationship between students' choice of representation and response to the follow-up question (χ^2; p<0.01). Students who chose the "sine wave" representation were mostly likely to choose vertical motion (up/down) compared to students choosing other representations, and were least likely to choose the correct answer (side-to-side). Students who chose the "particles" representation (middle) were most likely to choose the correct answer, and students who chose the "circles" representation were most likely to choose "to the right". Thus, we find that for sound, students' choice of representation is associated with a particular answer, while for string there is no such association between representation and answer.

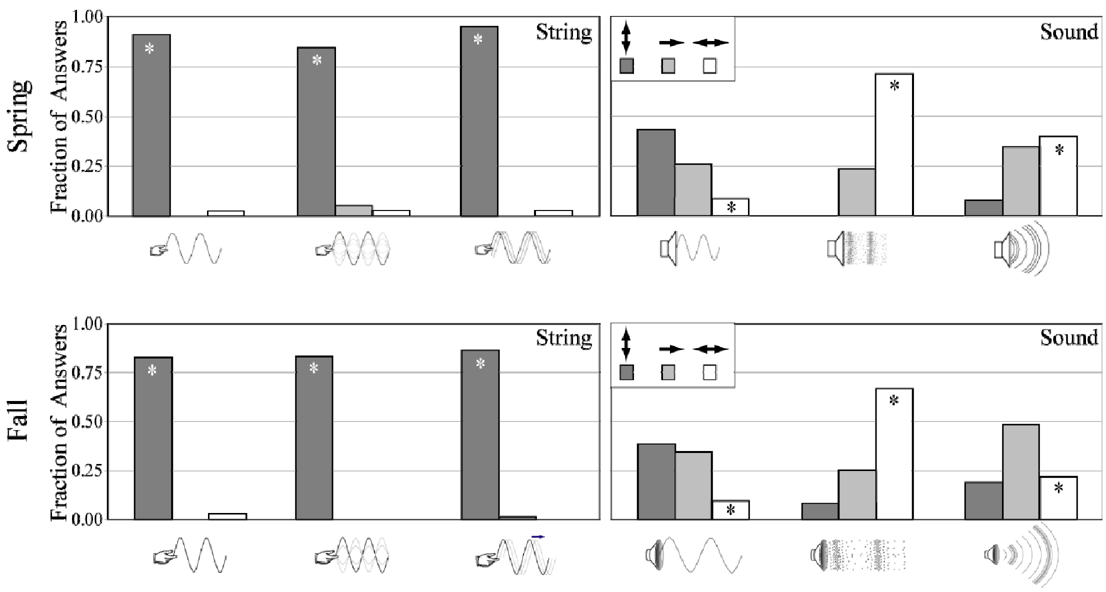

FIGURE 3.6 Student responses to the motion question on the representation assessment. String group (left) and sound group (right). Initial study results (spring) are shown in the top two graphs; follow-up study results (fall) are shown in the bottom graphs. The choice of iconographic representation is shown below the horizontal axis. The three top answers (up/down, to the right, side-to-side) are represented by the directional arrows in the legend. We look for patterns of association between representation and answer. There was no association for string ($\chi 2$: $p>0.3$), but significant association for sound ($\chi 2$: $p<0.01$). The "*" indicates the correct answer.

* * *

This chapter has described a series of studies examining students' use of analogy and representations in physics. While previous researchers have demonstrated that analogies generate inferences, they have involved only small numbers of students and demonstrated that teaching with analogies is only sometimes productive. In this study, we found evidence that analogies generate inferences when taught in a large scale introductory physics course. These findings lead to the following implications for instruction: (1) Analogies can generate inferences for students when these analogies are taught in a large enrollment physics course; (2) when teaching physics with analogies, instructors should attend to the myriad ways representations can be interpreted by students.

These implications lead us to develop a theoretical perspective that will guide design of materials and will help to explain experimental findings. What are the possible underlying mechanisms of analogy use at play here? How might we capitalize on these to productively teach with analogy? I will draw on several lines of research that have investigated student use of representations and analogy, relating this prior work to my own findings on students' use of analogy.

Chapter 4 – Towards a Model of Analogy: The need to include representations

"…some breaking news. A series of concentric circles have begun emanating from this glowing red dot in the big blue area over my left shoulder. The circles are colliding with the multiple green misshapen objects approximately two inches away from the pulsating dot."

- video news correspondent from *The Onion*[69]

What happens when someone sees a diagram or other representation? What does that person perceive and how is meaning made? As we saw in the last chapter, when students see a sine wave that represents a sound wave, many of these students may interpret this representation quite literally as air moving up and down (as opposed to the correct scientific model of air moving left and right, a longitudinal wave). On the other hand, students who selected a picture of air particles appeared to hold ideas closer the correct scientific model. Thus, student reasoning was heavily representation-dependent for sound waves. This was not the case for students' interpreting representations of a wave on a string. How can we account for these particular ways of student reasoning? In this brief chapter, I will describe the flavor of model we would like to use in order to explain the prior experimental findings. This chapter will serve as a bridge to a more formally structured model to be described in Chapter 5.

The studies described in the last chapter demonstrated that analogies are generative for a large college physics class, and further demonstrated that analogies can be generative when taught. Here, we will use our findings on students' use of representations to begin to understand how and why analogies may be taught. Results of the representation assessment suggest that, for these students, different string and sound wave representations may be associated with particular characteristics of these wave phenomena, but only in certain cases were particular representations associated with students' reasoning about the phenomenon. For instance, while different representations of a wave on a string seem to associate with particular characteristics, the vast majority of students answered the motion question correctly, and there was no correlation between choice of representation and answer to the follow-up question on strings. Thus, students demonstrated a correct understanding of the phenomenon regardless of representation choice. Although the representations of strings associate with different characteristics, they are not vastly different – all three are variations on the sinusoid. This result points to an important characteristic of a wave on a string, namely that it is a *concrete* phenomenon. By concrete, we mean that students have direct access to waves on a string via visual input channels – students have seen a string moving up and down. Thus, a model of a wave on a string is based on direct experience, and this model constrains the forms of representation that are appropriate.* Further, we might hypothesize that, since a model of a wave on a string is concrete, students may already possess certain phenomenologically grounded knowledge of strings, and are able to project this knowledge over any of the three representations.

Unlike the case for strings, we found that students' reasons for choosing a representation of sound waves, as well as their answer to the motion question, were associated with the choice of representation. Students do not always demonstrate a correct understanding of this phenomenon, and their conceptions of sound appear to be coupled to particular representations. For instance, answering "up and down" on the motion question is associated with the sine wave representation, and answering "to the right" is associated with the circles representation. This result suggests that, compared to a wave on a string, sound is a more *abstract* phenomenon. By abstract, we mean that students' experience with sound does not necessarily lead to conceptions that easily map to a scientific model. We use abstract in contrast to concrete, or phenomenologically grounded, experience as described above. Students can hear sound, but they do not see the air moving, and they do not directly experience sound as pressure varying in a sinusoidal fashion. The scientific model of sound is air particles moving as a longitudinal wave, spreading out from a source. However, students may base their reasoning about sound on several models other than the scientific model.[70] Furthermore, it is well known that students' knowledge can be fractured, consisting of unstable bits and pieces rather than stable, robust mental models.[13,71] Without a firm understanding of the scientific model, students

* We could represent the string with some other representations, such as vectors, but this would not provide any additional important information about the string, nor would it be a typical representation used by an expert.

may turn to the resources at hand, such as representations, to make sense of the phenomenon. This finding suggests caution in the best use of analogy and representation, as it may often be difficult to predict the myriad ways students interpret representations. Recognizing this difficulty, we would like to better understand this finding and begin to explore mechanisms that explain our observations.

Expert physicists use multiple representations (including verbal, graphical, and gestural) and shift easily between representations.[59,72] To the expert, all three representations of sound are equivalent in that they all stand as proxy for the correct model. The ability to apply such *meta-representational* skills[*,58] is a defining characteristic of scientists, but the particular interpretations that scientists apply must be learned.[73] Students, new to the ways physicists think and communicate, appear to draw meanings that vary depending on the representations used.[54] Research has shown that students can rely on iconic interpretations of graphical representations.[74] Elby describes this as *What-You-See-Is-What-You-Get* (WYSIWYG).[75] WYSIWYG is one particular form of "read-out" strategy[76,77] and describes reasoning along the lines of *x* means *x* (e.g. *sinusoid goes up* means *object goes up*). WYSIWYG makes specific predictions for sound. Applied to various iconographic representations of sound, WYSIWYG may lead to different models of sound, some correct and some incorrect. Specifically, if students use representations to frame how they think about phenomena, WYSIWYG predicts that the "particles" and "circles" representations will be most closely aligned with the scientific model (e.g. *circles spread* means

[*] Meta-representational competence describes the "full range of capabilities that students (and others) have concerning the construction and use of external representations."[58]

sound spreads). Conversely, WYSIWYG predicts that the "sinusoid" will be aligned with a transverse wave model of sound (e.g. *sine wave goes up* means *particles go up*), with the wave propagating in a straight line from the source (does not spread). In fact, this result is just what we observed on the representational assessment for sound as described in the previous chapter. Applied to various iconographic representations of strings, WYSIWYG predicts varying models for strings. However, since students' choice of representation did not correlate with their answer to the motion question, this suggests that all three string representations map to a transverse wave model of oscillating strings. Based on these observations, we posit that, to students, sound waves are more conceptually abstract than waves on a string. Thus, we might conclude that for abstract concepts, the WYSIWYG effect is more pronounced. How can we include interpretive mechanisms for representations, such as WYSIWYG, in an account of student learning of abstract ideas?

One potential mechanism by which people can learn abstract ideas is conceptual blending.[17] Recall the *springy table* blend, wherein characteristics of a spring were combined with a table in order to lead students to a more robust conception of Newton's third law. We extend this idea and suggest blending as a mechanism by which representations come to stand for scientific models. Suppose we blend the sinusoid with an iconographic representation of an oscillating string. Then the sinusoid can stand for the scientific model of the oscillating string (e.g. 2D, transverse wave). Suppose, instead, that the sinusoid is blended with an iconographic representation showing circular wave fronts. Then the sinusoid can then stand for a model of sound that carries with it several characteristics of the scientific model (e.g.

3D, longitudinal wave in air). Notably, the same representation may stand for two or more different models.

Returning to the data on teaching with analogy from the previous chapter, we may hypothesize that cueing[*] and blending can be used to teach about EM waves. Our prior results, which showed that the method used to teach with analogies was effective at generating different inferences about EM waves for students, are consistent with this hypothesis. For example, students taught a sound analogy to learn EM predominantly connected the three dimensional characteristic of sound to EM waves (See FIGURE 3.4). Conversely, students taught a string analogy were more likely than others to connect the two dimensional, traveling wave characteristics of waves on a string to EM waves. The different analogies were effective at promoting these connections even though we did not explicitly teach students which mappings to make. We therefore hypothesize that the analogies taught different ways of assigning meaning to the same sinusoidal representations. For a wave on a string, the sine wave stands for a wave confined to the x-y plane, while, for sound, the sine wave stands for a wave spread throughout space. When this sine wave is used to represent an EM wave, these different characteristics are cued. This cuing would explain why choosing the ”traveling,2D” distracter was associated with the string group, while choosing the ”3D” distracter was associated with the sound group.

Noting that both distracters contain elements of the correct answer to the EM wave propagation question, we might seek to combine the elements present in the ”traveling,2D” distracter (i.e. 1=2=3) with the elements in the ”3D” distracter (i.e. 1=2=4), in the hopes of achieving the correct answer (1=2=3=4). Thus, we suggest

[*] Cueing, as used here, is akin to priming, prompting, activating, or eliciting of particular student ideas.

that while either analogy alone is productive for teaching some useful ideas, neither alone is optimized for teaching all of the ideas necessary for a complete understanding of EM waves. It may be that a blend of wave on a string and sound waves will result in a more robust base domain for EM waves. We therefore hypothesize that teaching about EM waves using both analogies, layered and blended, may better prepare students to answer the post-test question correctly. At the same time, understanding why this might be the case suggests the need for an extended framework for analogy. Developing this framework is a subject of the next chapter.

* * *

In this chapter, we have started to delineate a mechanism by which analogies may be taught. We find that representations play a key role as a mechanism of analogy use. Representations are associated with particular characteristics of physical phenomena, and we therefore hypothesize that different representations may cue students to focus on associated characteristics. Because productive analogy use requires knowledge of which attributes and relations to map between domains, representations and cueing may be used to promote the appropriate use of analogies by students. While literal (or WYSIWYG) interpretations may seem naïve to experts, students tend to use such interpretations, and their productive (or unproductive) use depends upon a variety of factors, including instructional environment, framing of analogies,[78] and student resources. In the next chapter, I will extend these ideas, drawing on a range of existing frameworks to formulate a more formally structured model of analogy use. Our results suggest that this theoretical framework should include elements of mapping, blending, and read-out strategies such as WYSIWYG.

Chapter 5 – The Analogical Scaffolding Model[*]

"Our imagination is stretched to the utmost, not, as in fiction, to imagine things which are not really there, but just to comprehend those things which are there."

- Richard P. Feynman[79]

The last chapter revealed implications for the productive use of analogy and called for a new explanatory model.[80] Building on prior theoretical models of analogy, in this chapter we extend other researchers' efforts to study mechanisms of analogy and general features of teaching with analogy that lead to their productive use for learning physics. Spiro et al[81] suggest teaching with multiple analogies in order to circumvent the drawbacks of single analogies, (e.g., single analogies may be misleading or incomplete) especially when teaching complex and difficult topics. Broadly, our efforts build on prior work in order to better understand how, when, and why analogies can be used productively to teach physics, particularly in the context of using multiple analogies. This chapter presents a model of analogy, Analogical Scaffolding, which extends these prior efforts and examines sample data supporting this model.

Our research efforts address the need for further study of teaching with analogy in three ways. (1) The existing literature reports mixed success of teaching with analogy. Existing models of analogy suggest general approaches to, but do not

[*]Chapters 5 and 6 are largely drawn from published work, Podolefsky, N.S. & Finkelstein, N.D. (2007) "Analogical Scaffolding and the learning of abstract ideas in physics: An example from electromagnetic waves." *Phys. Rev. ST - Phys. Educ. Res.* 3, 010109

specifically inform, curriculum design. We make progress towards understanding successful cases, describe specific mechanisms by which analogies can be used productively to teach, and propose a model with direct implications for the design of learning materials using analogies. (2) Often, no single analogy is sufficient for teaching a new idea in physics, especially if that new idea is highly abstract. We propose a mechanism of student use of analogies to learn abstract ideas in physics, specifically by using multiple, layered analogies. (3) We believe models of student learning in physics should begin to explain the complex dynamics of student thinking, allowing for on-the-fly variations in students' thinking that depend crucially on salient factors in the environment. Our model proposes one such mechanism of student reasoning, focusing on how multiple analogies and representations used to teach those analogies interact to influence student thinking.

In this chapter, we broaden the scope from a strict definition of analogy to consider the idea of *domain comparisons*, of which analogy,[21] metaphor,[15] and conceptual blends[29] are particular cases. In all of these cases, mappings of object attributes and relations, as well as other types of connections, are made between two (or more) domains. In the literature dealing with domain comparisons, a domain is often represented by a mathematical set, and a comparison between domains is represented by a mapping, isomorphism, union or another related operation on the two sets. We draw on a range of prior work, focusing on Fauconnier and Turner's theory of conceptual blending,[29] to develop the Analogical Scaffolding model. As we will demonstrate, blending includes several features that extend traditional models of analogy. For instance, contrasted with traditional "two-domain" models of analogy, blending is a "multi-domain" model, making it a promising model for dealing with multiple analogies. According to Turner and Fauconnier, allowing for multiple domains "introduces a higher degree of variability and a loss of parsimony, but with a corresponding increase in sensitivity and generality. The two-domain model is in fact a special case of the many-space model."[82,83] At the same time we build on work in semiotics[73] to frame meaning making with representations, and on the work of layering meaning[17] to see how analogies can scaffold one another. More details on these prior theoretical frameworks will be provided in the next section. Following this section, we briefly describe a selection of the empirical work on which we base our model of Analogical Scaffolding. Next, we describe this model, which builds on prior models of analogy and addresses some outstanding questions on teaching with analogies.

Motivation

Before proceeding, we should take a moment to revisit some of the limitations of previous analogy theories. Existing models of analogy have difficulty explaining why analogies are only sometimes successful for affecting student reasoning. In particular, it is not well understood how one knows to make some mappings in an analogy and not others. For instance, in the Rutherford analogy, the attribute YELLOW does not map to the nucleus, whereas MASSIVE does. Structure mapping[21,37] proposes the *systematicity principle*, according to which the mappings selected tend to be those which fit into abstract higher-order relations, or relations between relations. For instance, as discussed in Chapter 2, the statement "sun is more massive than planet *causes* planet revolves around sun" uses higher-order relation CAUSE. It follows that, since YELLOW is not a part of this higher-order relation describing a *central force system*, it is less likely to map. However, this mechanism only makes sense if we assume the person using the analogy holds this higher-order relation in mind. It is well known that student knowledge can be fractured, consisting of unstable bits and pieces rather than stable and coherent conceptions or structures.[48,71] Thus, it is likely that while in the process of using an analogy to learn something new, students may not be able to apply or even to possess the higher-order relations necessary to use an analogy productively. Indeed, abstractions, such as *central force system*, may not exist for students prior to using an analogy. It follows that if students do not possess these abstract structures, it is unlikely that these students are aware of which relations to map from one structure to another. On the other hand, students may map surface features, such as shape or color, if these features are salient in the presentation of an analogy. One possible explanation for how people overcome this difficulty in using relations productively, also known as the bootstrapping problem,[41] is that abstractions emerge during the use of an analogy, being produced by the comparison of two domains. Our model builds on this mechanism of comparison in order to make productive use of salient information, leading students to apply increasingly abstract frameworks to learning new ideas. Our claim is that surface features may be productively used in an analogy, especially surface features of the representations presented when teaching with analogy. Therefore, the issue at hand is how students know how and when to use surface features (as well as more abstract relationships) productively and the role of representations in the process of using an analogy.

Certainly, the comparison process depends on a range of mechanisms. Focusing on possible mechanism, we examine representations. Several lines of research have recently found that student reasoning, whether comparing everyday objects,[84] learning complex systems by analogy,[85] learning abstract mathematical principles,[86] or solving physics problems,[53,54] can depend strongly on the representational forms presented to students in these activities. These findings were, in some sense, foreshadowed by Chi, Feltovich, and Glaser,[48] who demonstrated that representations can play a significant role in the ways physics experts and novices differently categorize physics problems. In their studies, novices tended to focus on surface features (e.g., problems with inclined planes), while experts focused on physics principles (e.g., conservation of energy) in their grouping of physics problems. Along these lines, other studies have found students may interpret some representations as *objects* even when the representations stand for something abstract (e.g., interpreting the arrows on electric field lines as signifying paths of motion).[*,35,87,88] Elby[75] terms these sorts of object-like interpretations *What-You-See-Is-What-You-Get* (WYSIWYG). Notably, students commonly invoke such object-like interpretations of electric field[89] and electromagnetic (EM) wave diagrams.[63]

While such findings may be troubling, these interpretations may also be resources[13] Our model builds on this idea, scaffolding learning of abstract ideas by focusing students' attention on surface level thinking that is productive, and using

[*] We note that this object-like reasoning may also apply when the sign is in the form of words, e.g., interpreting the phrase "step-function" as describing a stair-step-like object. See for example ref. 87.

multiple analogies as stepping stones toward more deeply structured abstract reasoning. To this end, the consequences of using representations of varying abstractness are pivotal to our model.

Building on Prior Results

In Chapter 3, we demonstrated that analogies can generate inferences when taught to students in a large-scale, calculus-based introductory physics course. In particular, we found that different analogies (e.g., waves-on-a-string or sound waves) were productive for teaching different associated features of EM waves (e.g., traveling waves or plane waves, respectively). The results of this study demonstrated two key findings. (1) String and sound analogies generate inferences about EM waves when taught in a large-scale introductory physics course, and the particular inferences generated depend on which analogy students are taught. (2) Representations couple to associated student reasoning, but this coupling is more pronounced for abstract ideas (i.e., sound waves) than for concrete ideas (i.e., waves-on-a-string).

Note that the traveling and plane wave features, both characteristics of EM waves, are individually key elements of string and sound waves, respectively. The study described above revealed that students applied one or the other of these features to EM waves depending on which analogy was taught. We hypothesized, therefore, that the optimal way to teach EM waves would be via both string and sound analogies. This hypothesis is compelling, but not self evident, hence motivating the Analogical Scaffolding model described in the next section. *

Analogical Scaffolding

Sign, referent, schema

Our previous large-scale findings suggested that for these students, iconographic representations coupled to associated ideas about physical phenomena (i.e., sound waves). In addition to the studies described previously, the development of our model draws on a range of data collected in ongoing large-scale studies as well as numerous interviews conducted with individual students. We limit the scope of this chapter to a detailed description of the model. More detailed empirical findings which support the model will be discussed in the following two chapters.

We begin a description of our model with a theoretical account of prior findings on the role of representation in an analogy.[80] We draw on the work of Roth and Bowen[73] to describe the relationship between a signifier, *sign*, the thing the sign refers to, *referent*, and a knowledge structure mediating the sign-referent relationship, *schema*. The sign-referent-schema relationship is represented by the diagram in FIGURE 5.1. We use the word *sign* to refer to external representations, such as text, graphs, equations, pictures, gestures, or utterances.† According to Rumelhart, [Ref. 90, page 37] "the central function of schemata is in the construction of an

* Alternatively, one might hypothesize that this approach would lead to more confusion for students. The string and sound analogies may interfere in problematic ways, or there may be simply too much information for students to learn at once.

† In common physics parlance, sign and representation share the same meaning. In other sciences, representation often refers to internal or mental representations, along the lines of mental models or schemata.

interpretation of an event, object, or situation”. Thus, *schemata* (plural of schema) can be considered knowledge structures or resources employed, among other things, to interpret sign-referent relationships. The sign, referent, and schema are represented by the vertices of the triangle in FIGURE 5.1 and the sides of the triangle represent connections between these three elements. We demonstrate the utility of the representation in FIGURE 5.1 by applying it to the canonical Rutherford analogy.

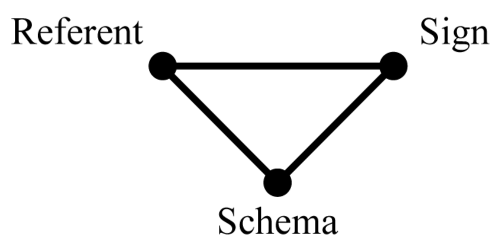

FIGURE 5.1 Relationship between sign, referent, and schema.

In FIGURE 5.2a, the referent is the solar system and the sign is the iconic representation at the upper-right vertex. This sign is associated with a particular schema for the solar system – spherical planets orbiting a yellow sun. A subset of the associated schema elements is shown at the lower vertex. Note that a different sign could be associated with a different schema for the same referent. For instance, depicting one of the orbiting objects in FIGURE 5.2a as yellow, and the central object black, would be associated with the sun orbiting a planet. FIGURE 5.2b shows a similar diagram for the atom. The sign used in FIGURE 5.2b cues certain analogical mappings from the solar system which are inherited by the atom schema. This cuing is similar to signs cuing reasoning strategies in the study described above[80] where different iconographic representations of sound were associated with different reasoning about the motion of a dust particle. The form of the sign can imply which schema elements to map, i.e. "revolves around", and which elements not to map, i.e. "yellow". Thus far, our model is consistent with the idea that an analogy is a mapping from base (solar system) to target (atom), but we go further and hypothesize that particular signs promote the selection of particular mappings. We also note that signs are not the only factors that can promote mappings.

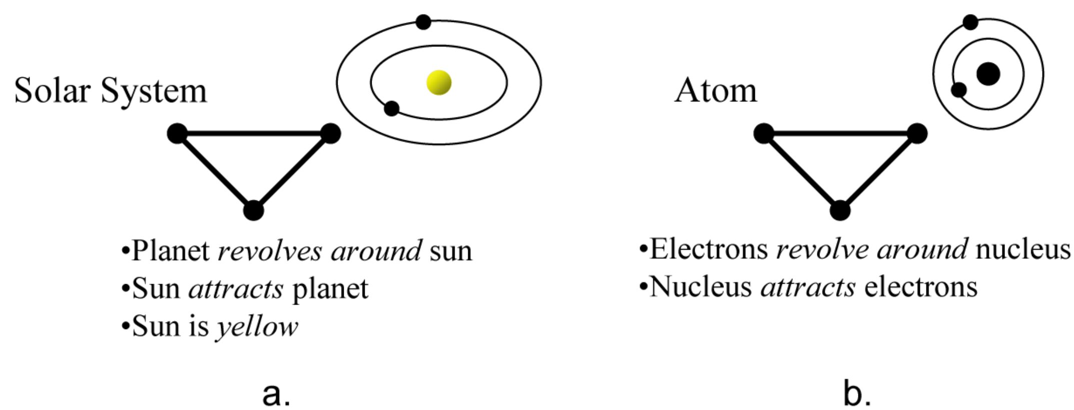

FIGURE 5.2 (color) Sign-referent-schema for (a) the solar system and (b) the atom.

A consequence of this hypothesis is that subtle changes to the sign can produce significant changes in the use of an analogy. For instance, if the large central sphere in FIGURE 5.2b were yellow, this would support, rather than inhibit, the inference that the nucleus is, in fact, yellow. On the other hand, if the sign in FIGURE 5.2b were a picture of an electron cloud, the analogy to the solar system might not be cued at all. We suggest that signs are key mechanisms by which analogies can be used productively, and are therefore a key part of teaching with analogy.

Conceptual blending

The structure mapping approach to analogy relies on students' prior knowledge of the base domain, but skirts the issue of students' prior knowledge of the target.[25] Turning again to the Rutherford analogy, it is likely that students learning about the atom will already have some preconceived ideas about atoms. Further, students may not possess a fully fledged model of the solar system to use as a base. Consider the hypothetical situation in which students' prior knowledge of atoms includes the idea that atoms are made of electrons and a nucleus, but with a number of possible arrangements besides the one given by the Rutherford model.[*] In this case, students using the solar system analogy are asked to compare the solar system and the atom, in so far as they understand each, and formulate a new conception of the atom based on this comparison.

Conceptual blending[29] provides a theoretical framework for describing such a process. In a conceptual blend, two *input spaces* are combined to produce a *blend space*. This process is represented schematically in FIGURE 5.3. Input and blend spaces are instances of *mental spaces*, defined by Fauconnier and Turner as "a small conceptual packet constructed as we think and talk, for purposes of local understanding and action." [Ref. 29, page 102] Elements in one input space (top left of FIGURE 5.3) have counterparts in the other input space (top right of FIGURE 5.3) and elements from each input space can be projected to the blend space (bottom of FIGURE 5.3). Projection is selective, and not all elements are necessarily projected from the inputs to the blend. Connections between input space elements are called

[*] This was the state of scientific knowledge prior to Rutherford.

vital relations. There are many vital relations – two examples are *space* and *representation*. For instance, the very different scales of the solar system and atom are connected by the vital relation *space.* Electron paths and concentric circles are connected by the vital relation *representation*.[91] Blending includes some mechanisms from traditional theories of analogy, such as connections between spaces.[82] However, an important distinction is that rather than enriching a target domain by mapping elements from a base, in a blend, elements from two input spaces project to a *new* mental space.[82] Importantly, input space elements combine in a blend such that, for instance, electron path and concentric circles are no longer separate, but become orbitals in the blend space. Finally, blends can have emergent structure not contained in the inputs, represented by the empty circles which appear below in FIGURE 5.3. For instance, electrons orbit the nucleus at fixed radii, and these radii can be associated with *energy levels*.

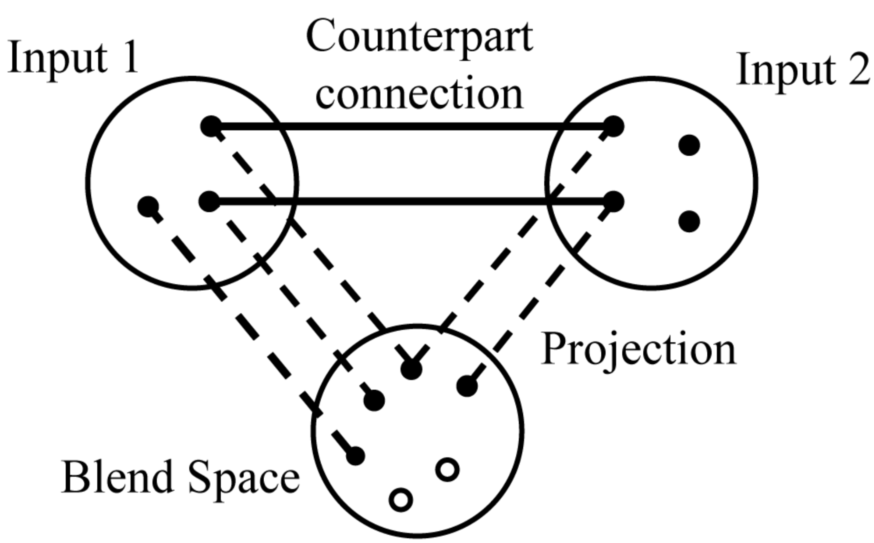

FIGURE 5.3 Conceptual blending diagram. Adapted from ref. 29.

Input spaces to a blend can provide structures, or *organizing frames*, which dictate the way elements in a mental space relate to one another. Different types of blends can be categorized according to whether the organizing frame for the blend comes from one input or from both inputs. When the organizing frame comes from one input, say input 1, the content in input 2 is organized in the blend by the frame from input 1. For instance, in the Rutherford analogy, the solar system space provides a frame that is applied to content from the atom space. In the case where both inputs contribute frames, the frames are often clashing and only selected elements of each frame are projected to the blend. For instance, a wave on a string evokes a transverse wave frame, while a sound wave evokes a longitudinal wave frame. Blending these clashing frames can produce a blend wherein a generic wave frame contains common elements such as amplitude, frequency, and wavelength. Alternatively, these frames may clash and not be resolved. Organizing frames are somewhat akin to relational structures in structure mapping, but, in some cases, students may not bear these frames in mind. Selecting and layering a series of blends establishes the scaffolding around which students may build particular (and generally effective) frames. In the next section, we discuss mechanisms of selection and layering.

Application to the atom

Conceptual blending can be applied to view the Rutherford analogy in a new light. In our model, we represent each mental space with a sign-schema-referent diagram. FIGURE 5.4 shows one input space containing the solar system and associated sign and schema, another input space containing a not fully developed

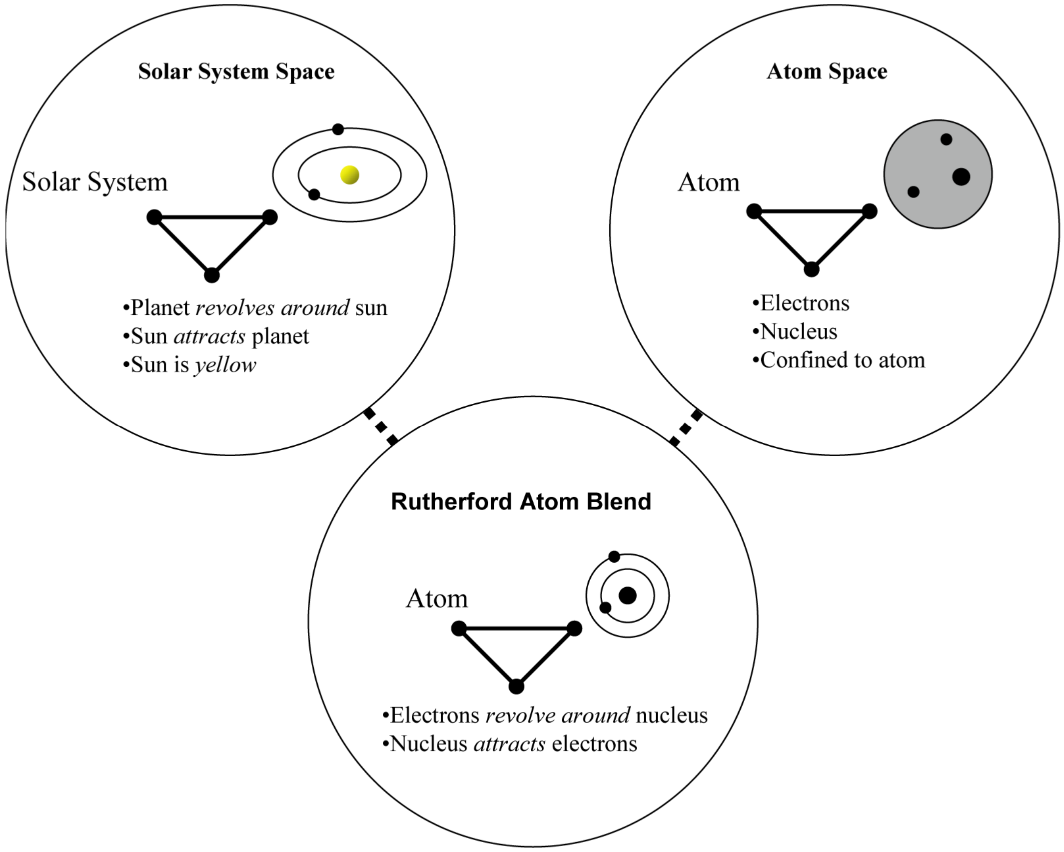

FIGURE 5.4 (color) Blending the solar system and a rudimentary model of the atom, resulting in the Rutherford model.

conception of the atom (e.g., a small material object that has protons and electrons) with associated sign and schema, and the blend space containing the Rutherford model of the atom.* At the level of input spaces, the atom schema has not yet inherited the frame from the solar system and could consist of any number of variants. In the blend, selected elements from the atom input space (i.e., electrons and nucleus) and from the solar system input space (i.e., *x* revolves around *y*) are

* One might ask why an external representation is contained within a mental space. We side with the view that artifacts of the environment, such as pictures on paper, are key components of cognition, and hence mental spaces. See M. Wilson, Six views of embodied cognition. Psychon. Bull. Rev. 9(4) pp. 625-636 (2002)

projected to the blend. Put another way, elements from the atom space are organized in the blend space by inheriting the solar system frame.*

There may be several mechanisms responsible for the selection of elements from each input space. One mechanism may be competition between schemata where the schema that is more tightly coupled to sign and referent (discussed below) is projected with higher probability. Here, the schema associated with the solar system is strongly tied to both the referent and the sign, whereas the input level schema for the atom is not strongly tied to either referent or sign. This would be the case if a student were learning about the atom for the first time. In the blend, the input level schema for the atom is easily discarded in favor of the more tightly coupled solar system schema. This mechanism may apply to individual schema elements rather than entire schemata. Note that unlike the high-order relation hypothesis of structure mapping, this mechanism does not require stable, large-scale structures, but instead relies on the strength of coupling between smaller-scale schema elements and signs. For example, our model predicts that since "revolves around" is tightly coupled to the concentric circles sign, this schema element likely projects to the blend. The nature of this coupling is discussed in more detail in the next section.

Another mechanism of selection may be the sign used to cue the blend. In FIGURE 5.4, the sign in the blend space is itself a blend of the signs from the input spaces. Its form is such that it can cue the projection of nucleus and electrons from the atom, but also includes the idea of planetary orbits from the solar system. According to Fauconnier and Turner's model, this cueing is a result of the vital

* In the language of blending theory, this is referred to as a "single scope" blend, where the organizing frame of one mental space (solar system) is applied to the elements of another (atom). For more see reference 12.

relation *representation*. The first mechanism described above explains how students might create blends based on their prior knowledge. The second mechanism explains how blends can be cued via the use of carefully chosen signs. This second mechanism is particularly relevant to creating learning materials that rely on promoting certain blends.

Blending theory provides an alternative and productive way of describing the Rutherford analogy. However, blending also describes cognitive processes that structure mapping does not. First, rather than employ a unidirectional mapping from one structure to another, a blend combines inputs from two input spaces to produce a new blend space. Second, blend spaces can become input spaces to new blends. Thus, a blend can be fractal in nature, itself the product of many input spaces, which are themselves blends of several input spaces, and so on.[*] Lakoff and Nunez[17] refer to this process of creating more and more complex blends as layering. For instance, one possible input space to the Rutherford blend is the plum pudding model due to J.J. Thompson. Far prior to Thompson, the prevailing model of the atom was an indivisible hard sphere. The progression of atomic models from hard sphere to plum pudding to solar system to electron cloud (quantum) could be viewed as a progression of layered blends.[†]

[*] Each of these sign-schema-referent triangles may be considered a blend, whereby the sign, referent, and schema are connected by vital relations. In the case of the solar system, a picture of the solar system (sign) may be connected to the real solar system (referent) via the vital relation *representation*. In the Rutherford atom blend, the size of both the solar system and atom referents are scaled to a human scale (the actual size of the sign) via the vital relation *space*. *Role* and *value* vital relations may be involved, whereby the role *central object* takes the values *sun* or *nucleus*, and the role *outer object* takes the values *planet* or *electron*.

[†] In this case, the historical progression happens to match a pedagogy that may be productive. We do not mean to suggest, however, that pedagogy should, in general, follow historical accounts of discovery.

In the prior example, blends were layered by changing simultaneously the sign and the schema for a given referent (i.e., the atom), wherein a new sign cued a new schema. We could also imagine holding the referent and sign fixed while changing the schema by adding or subtracting schema elements.[*,92] For instance, the solar system input space projects a schema which is applied to the atom in FIGURE 5.4. Bohr's contribution of quantized energy could be added to this schema, still represented by the same sign. This addition produces a new blend, with the atom blend from FIGURE 5.4 becoming an input space and Bohr's quantization of energy[†] providing a second input space. We now say that, unlike planets orbiting the sun, the orbits of electrons implied by FIGURE 5.4 are at fixed radii. In this fashion, the schemata now direct the meaning of the sign, whereas previously, the sign was directing the schemata or elements of the schemata to be used.

Note that while the spatial relationship between the nucleus and electrons is represented explicitly in FIGURE 5.4, this sign does not carry any explicit information about quantization. The spatial relationship can be read out from the sign directly, but the quantization must be learned through a layering process. We represent the result of this layering process by adding intermediate nodes (which represent prior blends) between the sign and schema vertices, shown in FIGURE 5.5. One could imagine any number of intermediate nodes, or blended layers. Each of these layers comprises a blend with an input space wherein a sign is associated with a particular schema for a given referent. For instance, quantization could be represented

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[*] These changes to sign-schema-referent relations may be along the lines of changes to Wittmann's resource graphs.[92]

[†] It might be more accurate to say Plank's quantization of energy, which Bohr blended with Rutherford's model.

by a chart of atomic energy transitions for the hydrogen atom. In the blend, the atom inherits the quantization schema, represented implicitly by the sign in FIGURE 5.5. We note that these intermediate steps may be along the lines of bridging analogies, thereby building on the ideas of Brown and Clement.[25]

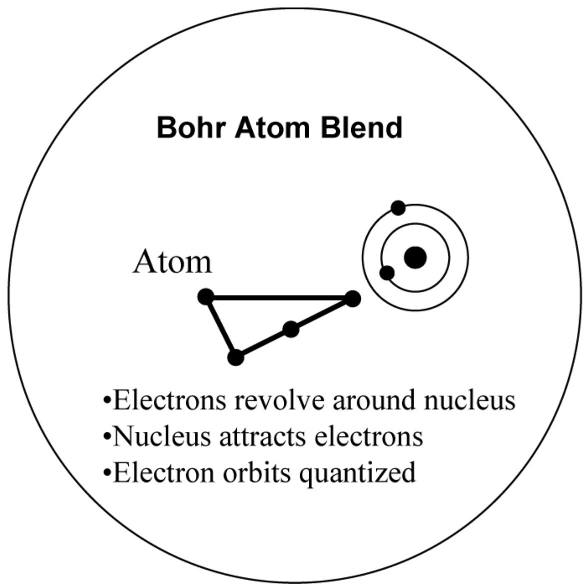

FIGURE 5.5 Adding quantization produces the Bohr atom blend. The additional node between sign and schema represents a blend which incorporates quantization into the schema linked to this sign.

Utility of the concrete and abstract

Physics ideas are often described as concrete or abstract (e.g., an EM wave is considered highly abstract[63]), but the definitions of these descriptors are almost always implicit. Some weigh concrete vs. abstract by the degree to which ideas are tied to particular contexts or objects (e.g., “electron” is more concrete than “particle”, depending upon an individual's prior knowledge).* Our model allows a more precise definition of concrete and abstract to be encoded in a representation such as FIGURE 5.5. Accordingly, concrete is characterized by a sign-schema-referent triangle with few or no intermediate nodes, while abstract is characterized by a sign-schema-referent triangle with many intermediate nodes. Since these nodes correspond to blends, more abstract ideas consist of many blended layers.† We will see that EM waves, a highly abstract idea in physics, can be taught via a series of layered blends from concrete to increasingly abstract input spaces.

The fact that the sign, referent, and schema of concrete ideas are more tightly coupled can be extremely productive. We draw on Elby's use of WYSIWYG (*What You See Is What You Get*).[75] Recall that WYISWYG is one type of readout strategy, or sign-schema connection, along the lines of *x* means *x*. For instance, when viewing a graph which is shaped like a hill, a student applying WYSIWYG would think the graph represented a real hill. Such an interpretation is sometimes productive (e.g., for a graph of height vs. distance), and sometimes not (e.g., for a graph of horizontal velocity vs. time). We suggest that WYSIWYG, applied to concrete ideas, can be productive for cueing schemata that we would like students to apply to more abstract ideas. For instance, suppose we use a sine wave to represent a wave on a string.

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* The delineation of abstract and concrete may depend on the level of expertise. To an expert physicist, an electron is a particular type of particle, and thus the electron is more concrete. To a student, to whom an electron may be an unfamiliar idea, "particle" may be more concrete in the sense of being connected to a real object, like a dust particle. In this sense, students' prior knowledge plays a role in our model to the extent that we can determine which ideas are already concrete for students, and which remain abstract. For a detailed analysis of levels of abstraction, see ref. 93.

† Such a notion of abstraction being a series of blends is consistent with Lakoff and Nunez's[17] notion of layering. The level of perceived abstraction may depend on the student, in that as students become increasingly familiar with abstract sign-schema relations, these relations may become increasingly treated as concrete. In this case, nodes may become so tightly coupled for an expert that the sign-schema link is compressed and the nodes disappear. For instance, to the expert physicist, the notions of "light" and "wave" are not separate ideas – to this expert, light *is* a wave.

Applying WYSIWYG to the sine wave results in a 2D transverse wave schema, which is correct for a wave on a string. The utility of the concrete is that signs are likely to cue schemata that are productive for learning about referents. In a complimentary way, there is utility in using abstract signs such as a sine wave. When representing a wave on a string, this sign (the sine wave) comes to stand for a 2D transverse wave. If this sine wave is then used to describe an EM wave, "transverse" is cued by the sign and inherited by the EM wave schema. Notably, a pitfall of the abstract is that signs are more likely to cue schemata that are not productive for learning about referents. For instance, 2D may also be inherited by the EM wave schema, in which case another intermediate layer becomes necessary, since EM waves are 3D.

The Ladder of Abstraction

We tie the use of signs, blending, and layering together and apply our model to teaching about EM waves. In our model, curricular materials scaffold student learning by layering analogies which progress from concrete to abstract, whence progress from relatively concrete ideas to more abstract ideas is represented by climbing to higher rungs of the "ladder" in FIGURE 5.6.[93] The physics ideas to be learned increase in abstraction as additional blends are layered, compiling more and more elements into a schema connected to a sign, in this case a sine wave. In the next chapter I will describe a specific application of Analogical Scaffolding, a tutorial on EM waves, wherein students learn about increasingly abstract wave phenomena: waves-on-a-string, followed by sound waves and finally EM waves. The lower rungs of the ladder are used as analogical scaffolds, upon which students can climb toward ideas of higher and higher abstraction.

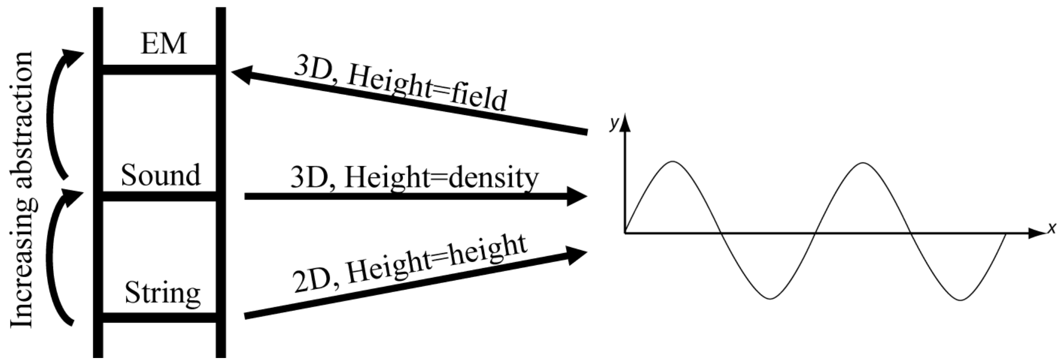


FIGURE 5.6 Scaffolding analogies of increasing abstraction.

Analogical Scaffolding Diagrams

Diagrams (a) and (b) in FIGURE 5.7 show two possible diagrams representing layering of mental spaces. Sign, referent, and schema are denoted S, R, and C respectively. In diagram (a), the same sign (S_1) cues an associated schema (C_1) in each layer, associated with three different referents ($R_{1\text{-}3}$). In diagram (b), several signs associate with several schemata and referents. Here, ideas are compiled into increasingly complex schemata by way of blending. These new schemata come to associate with new referents, followed by further blending. The different blends are connected via a sign that remains constant throughout each blend (in this case, S_1). For example, consider case (a) with S_1 a sine wave, associated with the same schema (C_1) for each referent. This would be initially productive if R_1 were a wave on a string, but could lead to inappropriate projections if R_2 were a sound wave. The EM wave example described in the next chapter is based on diagram (b). We believe this model of representational meaning and analogy can be used to effectively explain others' findings about use of representation, from WYSIWYG[75] to useful abstraction in representation.[86]

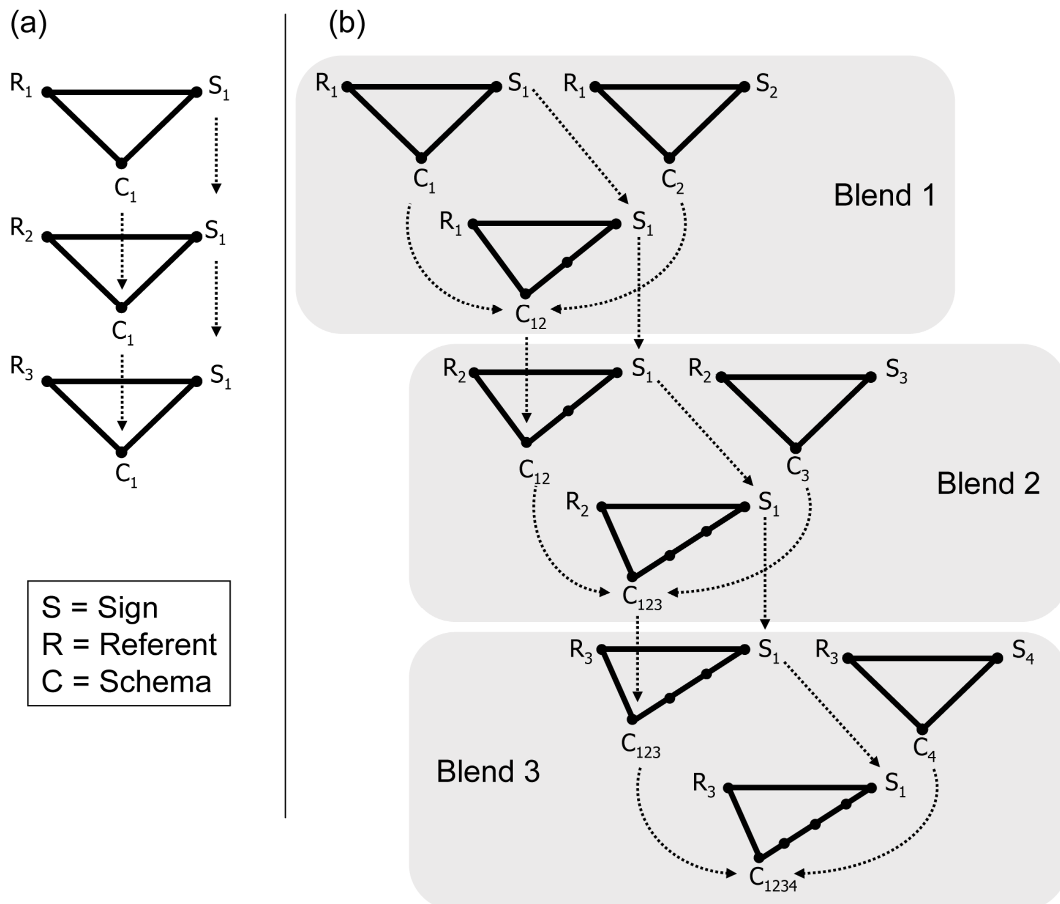


FIGURE 5.7 Analogical Scaffolding diagrams.

Considering Alternative Models of Student Learning

It is important to describe how Analogical Scaffolding lies in contrast to other models of student learning. Here, I will describe how the Analogical Scaffolding model leads to notably different hypotheses, and predicts different experimental results, when compared to other models of students learning. According to one model, students' prior knowledge consists of relatively stable and well formed structures, akin to scientific theories, that are not strongly linked to particular contexts.[94,95] When these ideas are non-canonical they are called *misconceptions*.[*,75] This model predicts that many students will apply these theory-like ideas to conceptual questions about EM waves, often resulting in students answering these questions incorrectly. The way misconceptions are changed is that students are presented with cases that conflict with their prior knowledge, and these students therefore reorganize their knowledge to align with this new case. This model has merit, and in fact the tutorials used in the across-domain study do address common student ideas about EM waves which may be inconsistent with experts' ideas. Consider, however, an experiment in which students are presented with cases that conflict their prior knowledge in a tutorial, but that students in several treatment groups are given versions of the tutorial which differ only in the representations used. This model of student learning alone does not predict nor does it explain how students will reorganize their knowledge under these different conditions. In other words, this model is not sensitive to context – it predicts that students will reorganize their knowledge in all treatment groups (which could differ by representation only), but does not make specific predictions about how students will learn differently in the various groups. Determining which condition is optimal is purely an empirical endeavor. To be sure, curricular materials based on this model can be extremely productive in bringing students ideas closer to experts'. However, we seek

* It may be noted that the PER community recognizes the need for a more refined approach to student thinking than a conceptions/misconceptions approach. However, researchers still argue for the existence of large-scale, stable, consistently activated sets of resources.[49] For an in depth examination of conceptions, see Elby.[75] Note also that the hypothesis that misconceptions are relatively stable across contexts is testable (see ref. 100).

mechanisms which are sensitive to context and can therefore predict and explain how using different representations and analogies impact student learning differently.

Elby[75] proposes one such mechanism of interpreting representations, WYSIWYG. When WYSIWYG is activated, students interpret representations literally, for instance, they may interpret a graph shaped like a hill literally as a hill (even if this graph is of velocity vs. time). We predict that on questions involving sine wave representations, students who apply WYSIWYG will treat the sine wave literally as moving up and down in the *x-y* plane. However, because WYSIWYG is so strongly tied to these representations, it may fail to predict when students will *not* use WYSIWYG. WYSIWYG alone would predict no differences between students who were given identical assessments, but received prior instruction using different representations. One explanation for why students would not use WYSIWYG is simply that student answers have some randomness to them, or alternatively, that those students who answer with a non-WYSIWYG interpretation (but correctly) simply “get it”. Analogical Scaffolding uses WYSIWYG as one mechanism of reasoning from representations, but additional mechanisms are required to explain when and why students will *not* use WYSIWYG, especially when students use other interpretive strategies productively. In other words, Analogical Scaffolding explains *why* some students appear to “get it”, but also why students who do not answer concept questions correctly may nonetheless answer these questions in predictable ways.

Studies of analogy suggest that while potentially powerful, students often fail to use an analogy productively if at all. Therefore, we might expect students to directly apply what they have learned about EM waves during the tutorials to the post-test question, but not use ideas from string and sound waves. That is, students do not apply the analogies provided. In this case, we might expect differences between the abstract, concrete, and blend groups based on the treatment of EM waves in the tutorials. However, WYSIWYG applied to EM wave representations (both sine wave and vectors) does not lead in any obvious way to developing 3D or traveling wave ideas about EM waves, since these ideas were taught only for sound and wave on a string. Again, if students only directly applied their (possibly reorganized) knowledge of EM waves to the post-test question, we would not expect differences between the three groups since these questions specifically test students knowledge of 3D and traveling waves.

These alternative models can be reformulated according to three hypotheses on the role of representations in student learning:

- *The null hypothesis*: Student learning depends mostly on prior knowledge and reorganizing this knowledge to align with a new conflicting case. Representations and student learning are largely independent.
- *The weak hypothesis*: representations do couple to students' prior knowledge along the lines of WYSIWYG, but this coupling is only dependent on the immediate context.
- *The strong hypothesis*: representations not only cue existing prior knowledge, but also lead to the dynamic formation of new knowledge.

This process is strongly dependent on the form and presentation of the representations.

We will return to these hypotheses in later chapters as we validate this model empirically. We will be able to invalidate the null hypothesis, make a significant and compelling case for the weak hypothesis, and also demonstrate the strong hypothesis to be both valid and consistent with a range of experimental results.

* * *

Drawing on empirical results as well as previous theoretical frameworks, we have developed a model of analogy, Analogical Scaffolding. This model is consistent with our previous experimental findings, and, moreover, our model is also consistent with prior framings of analogy such as mapping and bridging. Within the framework of this model, an analogy can be considered as a mapping from a base domain to a target domain, but we adopt the framework of conceptual blending[29] to expand this view of analogy to include bidirectional projections as well as multi-layered analogies. Analogical Scaffolding stands in contrast to both previous models of analogy and broader assumptions about student learning and representations, as encapsulated by the null, weak, and strong hypotheses. Our model does not require stable and coherent knowledge structures that exist *a priori*, but allows for smaller-scale schemata to be cued and blended with other schemata on the fly. Finally, by suggesting how schemata can be cued and blended, our model can be directly applied to curriculum design. Empirical establishment of this model is the subject of the remaining chapters.

Chapter 6 – Testing Analogical Scaffolding in a Teaching with Analogies Experiment

"Experience without theory is blind, but theory without experience is mere intellectual play."

– Immanuel Kant[96]

In this chapter, I present a specific application of Analogical Scaffolding. Wave on a string and sound waves are used as analogical scaffolds for students ultimately learning about EM waves. I will describe how Analogical Scaffolding was used to design tutorials to teach about EM waves with multiple, scaffolded analogies. I will demonstrate that students taught about EM waves using analogies, according to Analogical Scaffolding, outperformed students taught the same EM wave ideas but without analogies on pre-post assessments. This will be one in a series of experimental verifications of Analogical Scaffolding and its utility.[*]

Applying the Model to Teaching EM Waves

Here is the reasoning applied to teaching EM waves with the Analogical Scaffolding approach. We begin with a wave on a string. As demonstrated previously in a large-scale study, a wave on a string, represented by a sine wave, is a concrete

[*] We believe theoretical constructs such as blends, schemata, cueing, etc., to be useful and the present experiments are meant to demonstrate the utility of these theoretical constructs. A philosophical debate about whether these constructs literally exist is interesting, but beyond the scope of this work. We claim only that they are descriptive, predictive, and provide a language and framework for interpreting our prior and present results.

system in the sense that applying WYSIWYG to a sine wave results in an appropriate and productive schema for a wave on a string. In an analogy-based tutorial, students are presented with a sine wave and a picture of an oscillating string, as represented by the top-most input spaces of FIGURE 6.1. Students are asked to compare these two signs. According to the language of our model, we ask students to blend the schemata cued by these two signs. In practice, these two signs are so similar that most students have trouble seeing how they are different, and this works to their advantage. The schema that is cued for the sine wave includes elements such as two-dimensional (2D) and transverse. With appropriate cueing (via another sign for the string), a *traveling wave* schema element is blended into the existing wave on a string schema. That is, a zero-crossing is not a node for a traveling wave since a moment later the string will have moved up or down at that *x*-position. The resulting blend is a 2D, transverse, traveling wave schema represented by a sine wave.

Sound is introduced by having students compare a sine wave to an iconographic representation of compressed and rarefied air particles, shown in the second set of input spaces in FIGURE 6.1. WYSIWYG applied to the sine wave does not result in a 3D schema for sound, but WYISWYG applied to the air particles picture can. Blending these two results in a sine wave that stands for compressed and rarefied air particles spreading in 3D. Carrying the 3D schema further, a peak in the sine wave means high (or low) density everywhere in a plane with the same *x*-coordinate. In other words, students build up the idea of a *plane wave* for sound and associate this idea with a sine wave. In the blend, the sine wave stands for a 3D longitudinal wave that travels in air.

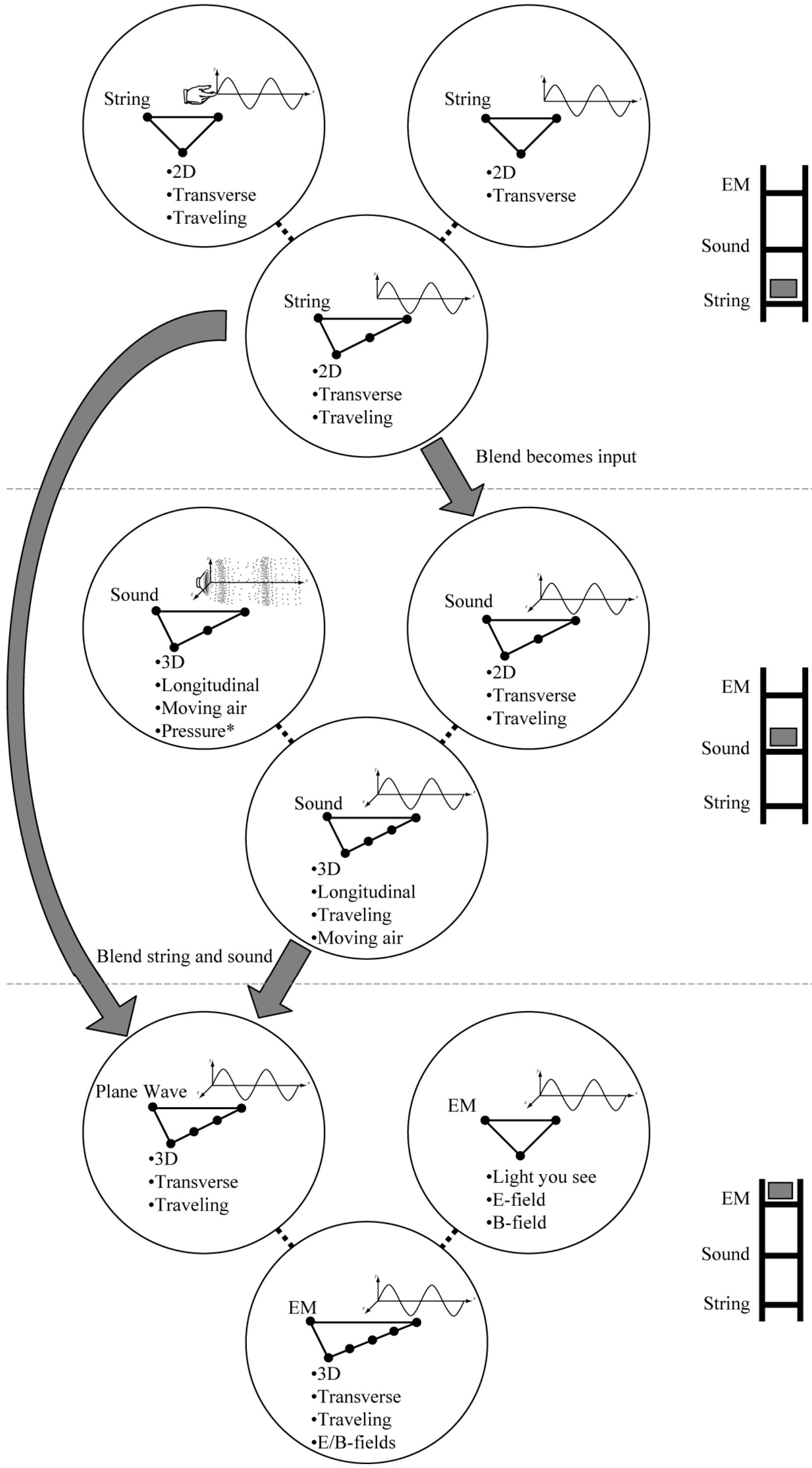


FIGURE 6.1 Layering blends from string (top) to sound (middle) to EM waves (bottom). Each triplet of circles represents two input spaces and a blend space. The string and sound wave blends combine to become an input space for EM waves. Here we show a subset of possible blends. *The association of pressure with air density could potentially come from a previous blend.

At this point, the sine wave is connected to several schemata, and we use particular cues to compile productive schema elements from a wave-on-a-string and sound waves into a new blend. The resulting schema is represented by a sine wave, and includes the elements transverse, traveling, and 3D. This new blend becomes an input space to the plane wave blend. EM waves are introduced, and using a sine wave to represent the EM wave cues the schema just described. Now, the referent is the EM wave, the sign is the sine wave, and the EM wave inherits the schema elements transverse, traveling, and 3D. During this process, more and more layers are blended together, and the result is a highly abstract idea (i.e., an EM plane wave) represented by a sine wave. According to our model, the sign-schema connection has a large number of intermediate nodes, each corresponding to a blend that is cued for students during the tutorial. If these blends are not made, students will not learn that EM waves are 3D plane waves, as such ideas are only taught in the context of waves-on-a-string and sound waves.

Methods

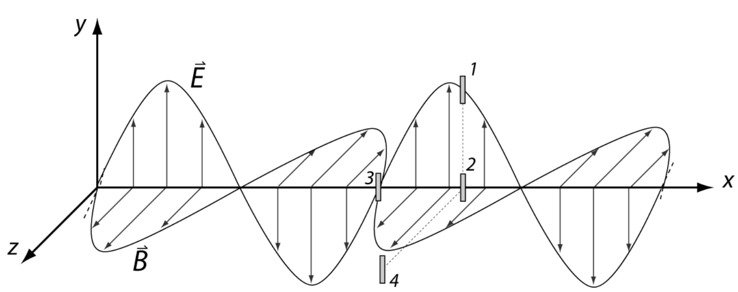
We set out to test the utility of our model in a study in which we applied Analogical Scaffolding to design tutorials to teach EM waves using analogies. This study was conducted in a large-scale, calculus-based introductory physics course – the same course as in the prior study described in Chapter 3, but during a different semester with different students. This course consisted of one lecture section, with three 50 minute lectures per week, and one 50 minute recitation per week with approximately 25 students per recitation section. Students were divided into two groups, denoted the analogy (N=72) and no-analogy (N=74) groups. (These N include only students who completed all stages of the experiment.) Each group completed a different tutorial during recitation, and all students in a given recitation completed the same tutorial.[*] One treatment group was taught about EM waves using multiple analogies (string and sound, as described above), while the other treatment group was taught about EM waves without analogies. Drawing on the original framing of the *Tutorials*,[66] both versions of the tutorial consisted of three sections. For the analogy group, section 1 covered basic wave concepts such as wavelength, frequency, and amplitude as well as traveling vs. standing waves, focusing exclusively on waves-on-a-string. Section 2 covered plane wave concepts, focusing on three dimensional (3D) waves, focusing exclusively on sound waves. Finally, section 3 covered EM wave representations and forces on charges due to the electric and magnetic fields of an EM wave, focusing exclusively on EM waves. The no-analogy group used tutorials with the same sections, but always focusing exclusively on EM waves.

Our goals in using analogies to teach EM waves were the following. First, students should learn that for a traveling wave moving in the +*x*-direction, represented by a sine wave, points in the representation where the sine wave crosses the *x*-axis are only nodes *at one instant in time*. This is a static picture of a dynamic process. For a traveling wave, of which a wave on a string is a concrete and grounded example, points on the wave that are at zero amplitude at one point in time move up and down at later points in time. Second, students should learn that for a plane wave moving in the +*x*-direction, also represented by a sine wave, points in space that have the same *x*-position have the same amplitude regardless of *y*- or *z*-position. In the

[*] These materials are provided in Appendix A.

analogy treatment, we teach the plane wave idea for sound waves. Finally, students should use both of these ideas applied to EM waves to answer a question like the one shown in FIGURE 6.2.

We compared student learning of EM waves in the analogy and no-analogy groups with a challenging concept question given in lecture on the days immediately prior to and after recitation. This concept question is shown in FIGURE 6.2. Since students from both groups attended the same lecture, students were told not to discuss the question with their in-class peers until after the entire class had finished answering. Individual student responses were collected electronically, and only results from students who attended the recitation and answered both the pre and post concept questions were included in the study. All tutorial interventions took place on the same day, thereby isolating experimental effects of the tutorial treatments.



An electromagnetic *plane* wave propagates to the right in the figure above. Four antennas are labeled 1-4. The antennas are oriented vertically. Antennas 1, 2, and 3 lie in the *x-y* plane. Antennas 1, 2, and 4 have the same x-coordinate, but antenna 4 is located further out in the z-direction.

Which choice below is the best ranking of the *time averaged signals* received by each of the antennas. (*Hint*: the time averaged signal is the signal averaged over several cycles of the wave.)

A) 1=2=3>4 B) 3>2>1=4 C) 1=2=4>3 D) 1=2=3=4 E) 3>1=2=4

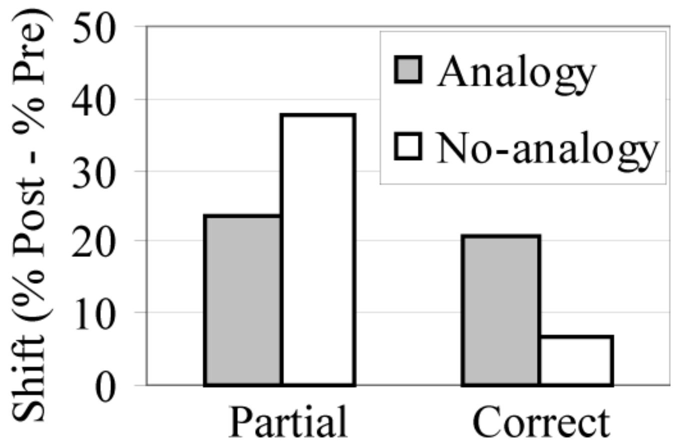
FIGURE 6.2 EM wave concept question. Correct answer is (D) 1=2=3=4.

Answering this question correctly requires students to apply both traveling and plane wave ideas about EM waves. Points 1, 2, and 4 are equal since this is a plane wave, and point 3 is equal to the others since this is a traveling wave and we asked for the *time averaged signal*. Averaged over several cycles, all four points receive the same signal. Therefore, the correct answer is (D) 1=2=3=4.

Results

The results for the pre-post EM wave concept question are shown in FIGURE 6.3. The vertical axis shows the shift in student responses from before recitation (pre) to after recitation (post) for the analogy and no-analogy groups. The analogy and no-analogy groups were matched on the pre-test (χ^2, $p>0.2$). The two answers with positive shifts were the correct answer (1=2=3=4) and the main distracter (1=2=4>3). We consider the main distracter to be *partially* correct, since it contains the plane wave feature (1=2=4) but not the traveling wave feature (3 is equal to 1, 2, and 4). In both groups, more than 86% of students chose one of these two answers on the post concept question. The shift on the correct answer was 21% in the analogy group and 7% in the no-analogy group. The shift for the analogy group is significant (McNemar's test, $p=0.01$), while the shift for the no-analogy group is not significant (McNemar's test, $p=0.47$). The shift on the partially-correct answer was 24% in the analogy group and 38% in the no-analogy group. Both of these shifts are significant (McNemar's test, $p<0.01$).

We also examined how students shifted between the correct and partially-correct answers from pre to post. We observe that students in the analogy group who answered partially-correct on the pre-test took the next step to the correct answer four times as often as students in the no-analogy group, while students in the no-analogy group went backwards from correct to partially-correct at twice the rate of students in the analogy group (χ^2, p=0.01).



	Partial (Analogy)	Partial (No-analogy)	Correct (Analogy)	Correct (No-analogy)
Pre-test	19%	16	22	31
Post-test	43	34	43	38

FIGURE 6.3 Results from study 2. Shifts on partially correct answer (Partial) which includes the 3D feature of EM plane waves. The correct answer includes both 3D and traveling wave features. Numbers below the chart show the raw data – fractions of students answering Partial or Correct on the pre- and post-test (top and bottom), Analogy and No-analogy groups (left and right corresponding to each column of the graph).

Thus, we find that while students in both groups learned some features of EM waves, students taught with analogies learned substantially more about EM waves than students taught without analogies. Note that the smaller shift to the partially-correct answer in the analogy group is due to these students having a greater shift to the correct answer. What these results show is that approximately equal numbers of students in both groups learned the 3D feature of EM plane waves, but significantly more students in the analogy group learned the traveling wave feature. Further, we find the no-analogy treatment fostered more incorrect ideas for students, whereas the analogy treatment helped students learn more correct ideas, as evidenced by shifts between correct and partially-correct.

In the analogy group, students were taught that sound waves are 3D, but they were not taught explicitly that EM waves are 3D. In the no-analogy group, students were taught explicitly that EM waves are 3D. Since both groups learned this characteristic of EM waves, we conclude that the analogies generated inferences about EM waves equally as well as when these characteristics were taught directly for EM waves. Importantly, the analogies also enhanced learning of other characteristics of EM waves and their representations that were not learned as well when taught directly for EM waves (for instance, interpreting a static picture of a sine wave as a traveling wave, taught explicitly about EM waves in the no-analogy group, but taught in the context of a wave on a string in the analogy group).

* * *

This chapter has demonstrated the utility of Analogical Scaffolding to design curricular materials to teach EM waves with analogies. Using our model to design tutorials using analogies to teach about EM waves, students taught with analogies outperformed students taught the same EM wave ideas directly without analogies on a pre-post assessment. Notably, our approach differs from prior efforts to teach physics with analogies. Traditional views of analogy often rely on relatively stable, coherent, large-scale structures that students draw on in using an analogy. Rather than focusing on these large-scale structures, our approach builds a schema for EM waves by blending, piece-by-piece, string and sound wave features. For instance, students apply the 3D characteristic of sound waves to EM waves, but *not* the longitudinal characteristic of sound. Instead, the transverse (and traveling) wave features of a wave-on-a-string are applied to EM waves. While the use of blending and layering may be common implicit practice by instructors, the Analogical Scaffolding model provides a mechanism to be explicit about these processes. By being explicit, we may better understand why particular blends work (in an explanatory sense) and predict which blends will work for students in the future (and ultimately allow for curriculum design).We are interested in how a given target may require multiple base domains, and propose mechanisms that work to layer ideas from multiple analogies. To this end, we build on significant prior work on analogy (e.g., Refs. 15,21, and 25) and draw on several existing cognitive models (i.e., blending,[29] semiotics,[73] and layering[17]) to assemble the Analogical Scaffolding model.

We demonstrate the utility of Analogical Scaffolding by applying this model to design tutorials using analogies. Our model explains how signs associate with schemata, affecting the way students blend and project schemata when learning about waves, leading to observed differences between analogy and no-analogy conditions. Note that these signs carry different meaning depending upon treatment. In both treatment groups, students create blends associated with signs, but in this case blends involving analogies are more productive than blends without analogies. According to this model, students can be cued to make productive blends under certain conditions. For instance, by presenting a sign to students that shares surface level features with a schema to project, in this case a concrete picture of an oscillating string, students are cued to project the dynamic oscillations of a traveling wave-on-a-string to an EM wave. The projection is promoted by associating this schema with a sign that is used consistently across analogical domains, cueing this same schema in the target domain. For instance, our model predicts that students who ground the sine wave in the concrete representation of a wave-on-a-string that is traveling would more likely link traveling as a schema element to this sign, a sine wave, than students who do not ground the sign with concrete representations. And in fact, students simply told that the sine wave represents traveling (as in the case for the no-analogy group) do not link the sign (sine wave) to the schema element (traveling) for abstract ideas (EM) as often as those in the analogy group. To summarize, our model poses one mechanism of analogy, whereby students make meaning of signs by blending signs and schemata in one domain and apply this meaning to another domain.

In order to apply our model, one can employ diagrams similar to FIGURE 6.1. We may consider the constituent parts of these diagrams as modules to be assembled in ways appropriate to certain learning goals. There may be an inexhaustible number of ways to assemble these modules. To determine the form of such a diagram, it may be necessary to work backwards from a desired “target blend”. For instance, it may be necessary to “unpack” or “explode”[97] the blend at the bottom of FIGURE 6.1, working upward in the diagram to determine the preceding blends. In this way, our model can provide a specific and productive approach to teaching highly abstract ideas in physics using analogies.

Chapter 7 – Digging Deeper: Applying Analogical Scaffolding to the Role of Representations in Student Analogy Use*

"Un bon croquis vaut mieux qu'un long discourse."

Translation: "A good sketch is better than a long speech."

- Napoleon Bonaparte[96]

The previous chapter described a study supporting the utility of Analogical Scaffolding for teaching abstract physics ideas, in this case EM waves. We found that students taught with analogies out-performed students taught the same ideas about EM wave, but without analogies. Analogies and representation were found to be coupled, in the sense that students' interpretations of an EM wave diagram depended on the analogies used to teach (vs. no analogies used). This chapter will describe a series of experiments designed to further examine the interplay between these two essential components of scientific reasoning, use of representation and analogy.

Scientists use multiple representations (including verbal, graphical, and gestural) and easily shift among these representations.[59,98] Scientists also frequently generate and use analogies to reason and communicate in day-to-day activities.[99] Representation and analogy are often considered convenient ways of communicating concepts, but with the implication that concepts transcend these forms of discourse. This view is controversial.[100] In Chapter 4 we proposed a model of student reasoning

* Chapters 7 is largely drawn from published work, Podolefsky, N.S. & Finkelstein, N.D. (2007) Analogical Scaffolding and the learning of abstract ideas in physics: Empirical studies. *Phys. Rev. ST - Phys. Educ. Res.* 3, 020104

which combines the roles of representation, analogy, and layering of meaning – Analogical Scaffolding.[101] The present empirical studies build on this model to examine its utility. In this chapter we present a series of results demonstrating the vital intertwining of representation, analogy, and conceptual learning in physics.

Recently, some progress has been made by other researchers toward identifying possible mechanisms of student analogy use. Several lines of research have suggested a tradeoff between within-domain and across-domain learning of abstract principles (e.g., modulo-3 arithmetic[86]). This tradeoff appears to be coupled to the concreteness of the representations used to teach students. [85,86] Here, the concreteness of a representation is gauged by the degree to which the representation contains salient, information-rich features (e.g., a picture of a soccer ball is considered more concrete than a black dot meant to represent a generic rolling object). While researchers find that concrete representations are more productive for students learning within a single domain, the use of abstract representations better facilitates students productively using those ideas in a second domain. Along these lines, Van Heuvelen and Zou[102] successfully used concrete representations to scaffold students interpretations of abstract (i.e., mathematical) representations when solving work-energy problems. Interestingly, Goldstone and Sakamoto[85] report a tradeoff in learning for "low-achieving" students, but they find little or no such effect for "high-achieving" students. Sloutsky et al[86] find that irrelevant concreteness (e.g., pictures of insects used to represent mathematical entities) can hinder across-domain learning of mathematical principles. Surprisingly, Goldstone and Sakamoto find that even *relevant* concreteness can hinder across-domain learning.

These recent results parallel our own findings that representations can play a key role in teaching students with analogies.[80] Based on these findings, we proposed a model of analogy use, Analogical Scaffolding,[101] which describes mechanisms by which multiple analogies may be layered in order to learn abstract ideas. According to this model, concrete and abstract representations play key, complementary roles in this layering process. Using this model, we have modified curricular materials aimed to teach college physics students about electromagnetic (EM) waves by using analogies. Ambrose et al[63] have identified a number of student conceptual difficulties with EM waves in order to develop curricular interventions. These prior findings call for further study of how students can learn this challenging topic, particularly with regard to the use of wave representations.

In the last chapter we found that students taught EM waves concepts using materials based on Analogical Scaffolding outperformed students taught the same EM waves concepts without analogies on a pre-post assessment. In this chapter, we describe two follow-up studies, the first examining student learning across multiple conceptual domains and the second examining student reasoning within a single domain. Primarily, we consider Analogical Scaffolding to be a cognitive model and these studies seek to examine the utility of this model to explain student reasoning and responses in educational environments.

In the first study across-domains, we taught students about EM waves using analogies from multiple domains (wave-on-a-string and sound waves). We explore the implications of varying the concreteness of the representations used to teach undergraduates in an algebra-based introductory physics course. In this study we ask, how does the model explain (and predict) student reasoning under different conditions, i.e., using different representational forms to teach? As a secondary goal, we investigate Analogical Scaffolding as a teaching intervention. To this end, we ask how closely the ideas of students taught with materials designed according to an Analogical Scaffolding framework align with canonical physics ideas.*

In the within-domain study, conducted in another algebra-based introductory physics course, we explore the implications for student reasoning and use of analogy by varying the concreteness of the representations used on a quiz within a single conceptual domain, sound waves. We pose the following research questions for this second study. (1) Previously, we found students associated different representations of a sound wave with various conceptions of sound waves (e.g., though sound is a longitudinal wave, students associate a sine wave representation of sound with transverse wave motion).[80] In the present within-domain study we explore the directionality of this association and ask: do representations drive student use of analogy and, by proxy, conceptions of sound waves? (2) How does varying the concreteness of representations affect students' reasoning about sound waves? The findings of this second study give insight into student learning of sound waves, one of three analogical scaffolds we pose as productive for layered student learning of EM waves in the preliminary[101] and across-domain studies.

In both the across- and within-domain studies, we find students demonstrate markedly different response patterns to concept questions depending on the form of representations used to teach. In the across-domain study, the model makes accurate

* Developing well honed curricular materials would require an iterative cycle of design, testing with students, and modification. Analogical Scaffolding serves as a cognitive model on which to base curricular materials and may be considered a guide in this iterative development process.

predictions about students' responses to EM waves concept questions and, furthermore, predicts which form of Analogical Scaffolding (of three investigated) is optimal for student learning of EM waves. In the within-domain study, we find representation can drive student reasoning about analogies. Furthermore, the Analogical Scaffolding model predicts how representational forms (and combinations of these) will affect students' ability to make productive use of an abstract representation of a sound wave, such as a sine wave.

Recall that the Analogical Scaffolding model draws on theories of representation,[73] conceptual blending,[29] and layering of ideas.[17] We draw on the work of Roth and Bowen[73] to describe the relationship between a signifier, *sign*, the thing the sign refers to, *referent*, and a knowledge structure mediating the sign-referent relationship, *schema*. For example, consider a picture of compressed and rarefied air particles, a sign representing the referent *sound wave* (FIGURE 7.1a). The sign and referent are coupled to a schema containing the elements "longitudinal" and "disturbance spreading through space" or "3D". Now, consider a sound wave represented by a sine wave (FIGURE 7.1b). The surface-level features of a sine wave are more tightly coupled to schema elements such as "transverse" and "2D" (a sine wave is generally drawn in a single plane).[80] In this case, the sine wave can cue a schema that is unproductive for sound. However, these two sign-referent-schema systems may be *blended*, whereby the sine wave comes to be coupled to a schema containing "longitudinal" and "3D". This process constitutes one *layer* within a conceptual domain. If, in a subsequent layer, the sine wave is coupled to the referent

EM wave, the 3D longitudinal schema may be inherited by the EM wave mental space via another blend.

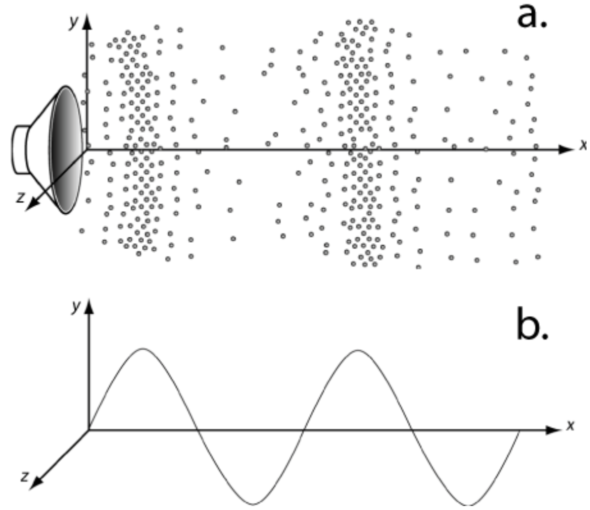


FIGURE 7.1 A sound wave represented by a picture of compressed and rarefied air particles (a) and a sine wave (b).

Blends combine mental spaces, linked by some connection between these spaces (e.g., the same sign or sometimes the same referent with different signs), and project selected schema elements (e.g., "3D") from these mental spaces to generate a blended space. Increasingly complex and abstract ideas can be built up by a series of blended layers.[17] For instance, a wave-on-a-string blends with sound waves, building up to EM waves. As a concrete example, in the next section we will describe a detailed application of this model to predict the outcomes of two empirical studies of student learning of E/M waves by layered analogies of string and sound waves.

Part A. Student Learning Across Multiple Domains

Methods

The participants in the across-domain study were 151 college students enrolled in the second-semester of an algebra-based introductory physics course, focusing largely on electromagnetism. The first semester of this course included instruction on waves-on-a-string and sound waves as well as general wave properties. Prior to the tutorial activities described here, lectures and homework had covered electric and magnetic fields, but had not yet covered EM waves. This typical introductory course consists of three 50-minute lectures, used the Touger text[103], an online HW system[104], and included one 2-hour recitation each week. Recitations generally included laboratory activities, but on occasion students worked on pencil and paper tutorials in lieu of hands-on experiments. Students generally worked in groups of three to five. During these tutorial activities, the teaching assistant roamed the classroom answering students' questions and probing students' understanding of the materials to be learned. In the across-domain study, groups of students within a given recitation section were assigned to one of three treatment groups, denoted *abstract*, *concrete*, and *blend*. Table 7.1 lists the number of students (N) for each group, summed over all recitation sections, and the average course grade for students in each group. We found no statistically significant difference between the average grades for the three groups ($p>0.3$, 2-tailed z-test[*]).

[*] Unless otherwise stated, all statistics are based on a 2-tailed z-test.

Table 7.1 Across-domain Study Experimental Groups

Group	N	Average Course Grade
Abstract	49	77.0%
Concrete	51	76.5%
Blend	51	78.6%

Assessment Before and After Tutorials

In recitation, students were issued pre- and post-tests on EM waves immediately before and after tutorials. These assessments were identical in all three treatment groups. The pre-test was administered at the beginning of recitation. These were collected, and students were then divided into three groups, each receiving a different version of a tutorial on EM waves, described below. After completing the tutorial, students were issued a post-test, identical to the pre-test. The only difference in treatment between the three groups was the type of representation used in the tutorial.

The assessments consisted of two open response questions, shown in FIGURE 7.2. These questions are based on the materials used by Ambrose et al[63] to evaluate the *Tutorial*[66] on EM waves, but were modified based on student interviews.* Since these questions were open response, there was a large range of possible answers, and the likelihood of students guessing the correct answer was extremely low. Students were asked to explain their reasoning on each question. Note that these two questions require that students interpret the pictures in FIGURE 7.2 as representing a *snapshot*

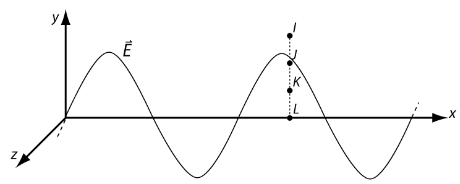
* These interviews were conducted as part of the modifications of the materials used in these studies. The standard EM-wave representations often used in textbooks includes crossed E and B fields represented by superimposed vectors and sine waves. This standard representation is problematic for several reasons which were revealed in student interviews. In these interviews, we found that students often did not distinguish between the electric and magnetic fields, resulting in false positives on question 2. This is because with the B-field shown, students might answer P=R since both of these points lie near a wave peak (the E-field for P and the B-field for R). We also removed the vectors from these representations in Figure 2 in order to examine how students make sense of this "stripped down" and more abstract representation.

in time of EM plane wave traveling to the right. Thus, the correct answer to question 1 is I=J=K=L, since for a plane wave the magnitude of the electric field depends only on the *x*-coordinate. The correct answer to question 2 is P=Q=R=S, since, for a plane wave, the magnitude of the electric field depends on *x*, but since this is also a traveling wave, the *time average signal* is independent of *x*. Note that if question 2 had asked for the magnitude of the electric field *at this instant in time*, the correct answer would be P=Q=R>S, with S having magnitude zero.*

Question 1 The figure on the right shows an electromagnetic plane wave at one instant in time. The wave travels in the +x-direction. Four points in space are labeled I,J,K,L.

For the instant shown, rank the points I, J, K, and L according to the magnitude of the electric field at these points, from largest to smallest. If the electric field is zero at any of these points, state that explicitly. For example, if you think K is the largest, and the rest are the same, you should answer K > I = J = L.

Explain your answer.



Question 2 The figure on the right shows an electromagnetic plane wave at one instant in time. The wave travels in the +x-direction. Four antennas are labeled P,Q,R, and S. Antennas P, Q, and S lie in the x-y plane. Antennas P, Q, and R have the same x-coordinate, but R is located out of the page in the x-z plane. All four antennas are oriented parallel to the y-axis.

Rank the time-averaged signals received by the antennas P, Q, R, and S, from largest to smallest. If the time-averaged signal is zero at any of these points, state that explicitly. (Hint: the "time averaged signal" is the signal averaged over several cycles of the wave.)

Explain your answer.

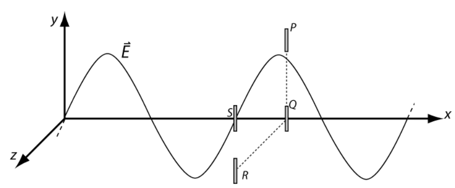


FIGURE 7.2 Questions given on the pre- and post-tests in the across-domain study.

* The idea that, for a traveling wave, a point where the wave crosses the *x* axis at one instant in time will be non-zero as the wave propagates was covered explicitly for a wave on a string. We expected that, combined with the hint on question 2 in Figure 2, students would be able to reason productively about the "time averaged" signal of an EM wave. Our classroom observations indicate that students make sense of traveling waves in the context of a wave on a string, but do not, in general, apply this idea when answering question 2 in Figure 2. We have some evidence that addressing the term "time averaged signal" explicitly in helps students make this connection.

Tutorials

The tutorials used in the across-domain study were based in part on *The Tutorials in Introductory Physics*,[66] but were modified to teach about EM waves using wave-on-a-string and sound wave analogies. The tutorials consisted of three main parts. Part 1 used a wave on a string to introduce transverse and traveling wave ideas. Part 2 used sound waves to introduce three-dimensional (3D) waves (close approximations to plane waves). Part 3 covered properties of EM plane waves, including basic wave properties such as frequency, wavelength and amplitude, the interpretation of EM wave diagrams, and ways of detecting EM waves with an antenna.

The tutorials in each experimental group were nearly identical in content and wording, but differed in the representations used. FIGURE 7.3 shows a subset of the representations of string, sound, and EM waves used in the abstract, concrete, and blend groups. The complete set of tutorials and surveys can be found in Appendix A. The canonical wave representation is a sine wave, which we consider an abstract representation.[101] In the abstract group, a sine wave is used consistently to represent string, sound, and EM waves.* The representations used in the concrete group include more salient features, for instance, showing compressed and rarefied air particles in a sound wave spread throughout space. In the blend group, students were presented

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* We note that since a surface-level interpretation of a sine wave results in productive ideas for a wave-on-a-string, but not for sound or EM waves, a sine wave can be considered an abstract representation of a sound or EM wave, but relatively concrete for a wave-on-a-string. We purposefully designed this particular "abstract" sound wave representation in order to complement the EM wave representation (e.g., Figure 2) and promote students use of the sound wave analogy in a manner that will facilitate understanding of this EM representation. Additionally, what is labeled as abstract or concrete will depend upon the individual using these representations. See discussion of abstraction in Chapter 5.

with both abstract and concrete representations simultaneously.[*] The tutorials included significant framing for students to make sense of these representations and learn about EM waves, generally in the form of Socratic questioning written into the tutorials. This framing (and wording) was nearly identical for the three treatment groups.

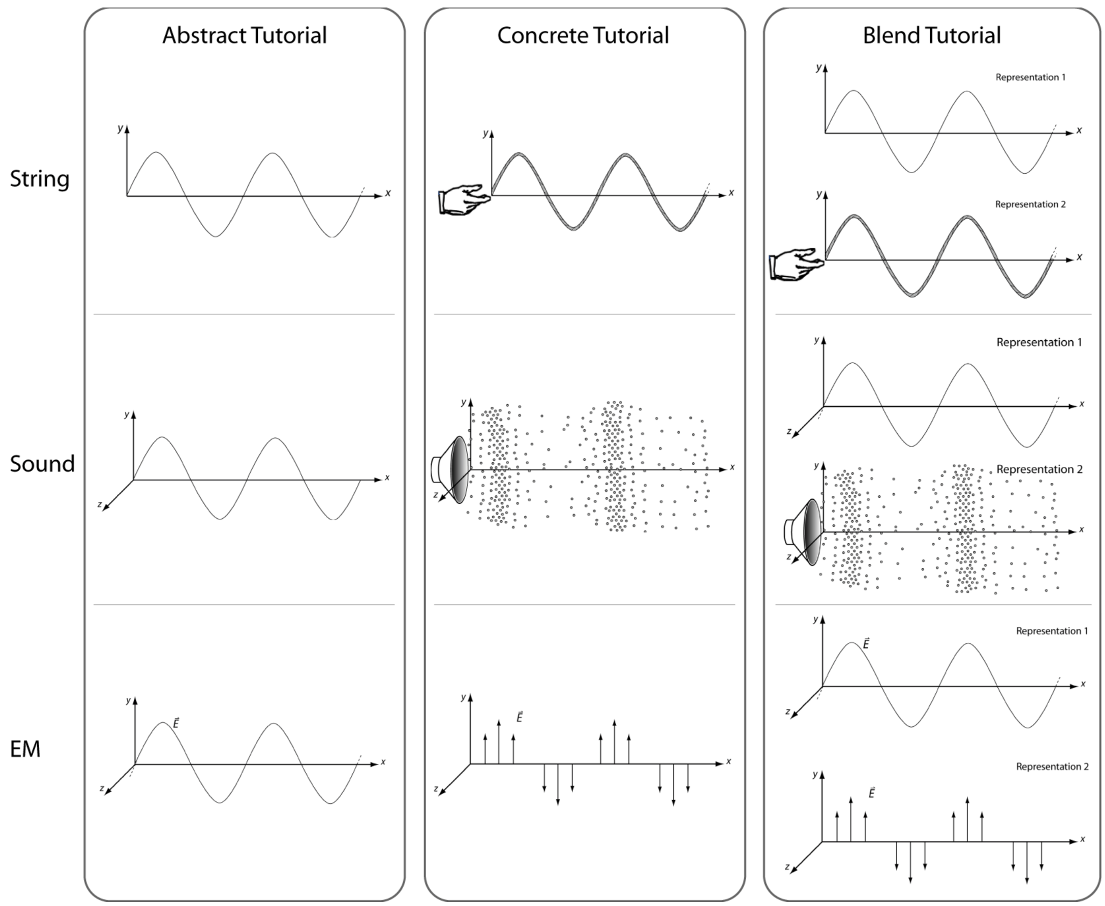

FIGURE 7.3 Examples of representations of a wave on a string, sound wave, and EM wave used in the abstract, concrete, and blend tutorials in the across-domain study. The complete set of representations can be found in the supplementary materials, Appendix A.

[*] We note that in the *blend* tutorial, two separate representations are presented to students. We use the *blend* label to indicate that we predict students will make blends when these representations are provided side by side. This prediction is based on the Analogical Scaffolding model.

Predictions

We claim the Analogical Scaffolding model can make correct predictions of student answers (and associated reasoning) to explain the results of the across- and within-domain studies. In this section, we apply Analogical Scaffolding theory to predict the outcomes of the across-domain study. First, we will frame these predictions in terms of the three hypotheses described in Chapter 3. These hypotheses are restated below, accompanied by the implications for the present studies.

- *The null hypothesis*: Student learning depends mostly on prior knowledge and reorganizing this knowledge to align with a new conflicting case. Representations and student learning are largely independent.

  Implication: Both within- and across-domains, we should therefore expect no differences between the three groups in both the across- and within-domain studies since the only variation between conditions was the representations used.

- *The weak hypothesis*: representations do couple to students' prior knowledge along the lines of WYSIWYG, but this coupling is only dependent on the immediate context.

Implication: Observed differences between the treatment groups in the *within-domain study* would be sufficient to confirm this hypothesis, since students in this study received different representations on the assessment. However, this hypothesis would also predict no measurable differences between treatment groups in the *across-domain study*, since all students received the same representations on the assessments.

- *The strong hypothesis*: representations not only cue existing prior knowledge, but also lead to the dynamic formation of new knowledge. This process is strongly dependent on the form and presentation of the representations.

  Implication: To confirm this hypothesis, we would need to observe differential response patterns between the treatment groups on assessments in both the within- and the across-domain study. Differential response patterns would show that the representations used to teach had different effects on how students learn new interpretations of the representations in FIGURE 7.2.

Three possible sequences of representational cueing, blending, and projection are shown in FIGURE 7.4 and FIGURE 7.5. The sign (representation) is shown at the upper right vertex of each triangle, referent at the upper left vertex, and schema at the bottom vertex. FIGURE 7.4 represents the abstract tutorial, with only sine wave representations used, and the concrete tutorial, with concrete representations used. In the abstract tutorial, the surface level interpretations of the sine wave would lead students to use a schema including the features 2D and "up means up".[80] This schema is projected through to EM waves, cued in each layer by the same sine wave. Alternatively, in the concrete tutorial, surface level interpretations lead students to apply different schemata to string, sound, and EM waves. However, students are predicted not to project these schematic elements (e.g., 3D) from one domain to the next as often as in the blend group since there is no corresponding sign (e.g., sine wave) to cue blended schemata in subsequent layers. This predicted lack of projection, one possible approach students may take, is represented by dashed arrows in FIGURE 7.4.

Note that in FIGURE 7.4, schemata are presented as separate, unblended pieces. In these cases, WYSIWYG operates within each piece, cueing schemata that are tightly coupled to signs. In the blend tutorial (FIGURE 7.5), these schemata are cued by signs in a similar fashion to the abstract and concrete tutorials, but in this case schemata blend. Blended schemata then project through each layer and subsequently re-blend. Each blend corresponds to an additional node between the sign (upper right) and schema (bottom) vertices of the resulting triangles. The final blend for EM waves has three nodes, corresponding to three prior blends. Note that in the blend treatment group, the schemata resulting from each blend are *non*-WYSIWYG.

This model predicts that students in the abstract group will be most likely of the groups to apply 2D, "up means up" object-like schema elements when answering the post-test questions. This reasoning would be consistent with the answer I>J>K>L on question 1. (This reasoning would also be consistent with a number of incorrect answers to question 2 based on "up means up" reasoning, for instance P>Q=S>R, P>Q>S>R, P>Q=S=R, etc.). Students in the blend group will more likely apply 3D, time-varying schema elements, and treat the sine wave as representing an abstract quantity (e.g., field) rather than treating the sine wave as an object (i.e., an object that goes up and down in space like a string). This reasoning would be consistent with I=J=K=L on question 1 and P=Q=R=S (both traveling and 3D) or P=Q=R>S (only 3D) on question 2. Students in the concrete group will fall somewhere in between, having been exposed to the essential schema elements, but not led to create blends of these schemata. Pre-test results for all groups would be most similar to the post-test predictions for the abstract group, since students are asked to answer questions about a sine wave representation of an EM wave before instruction. Note that we do not expect these coarse categorizations to describe individual students, as individual student resources and reasoning are sure to vary. We therefore note that these predictions are probabilistic and we predict trends in students reasoning for statistically robust numbers of students. (Our studies use N>100 subjects.)

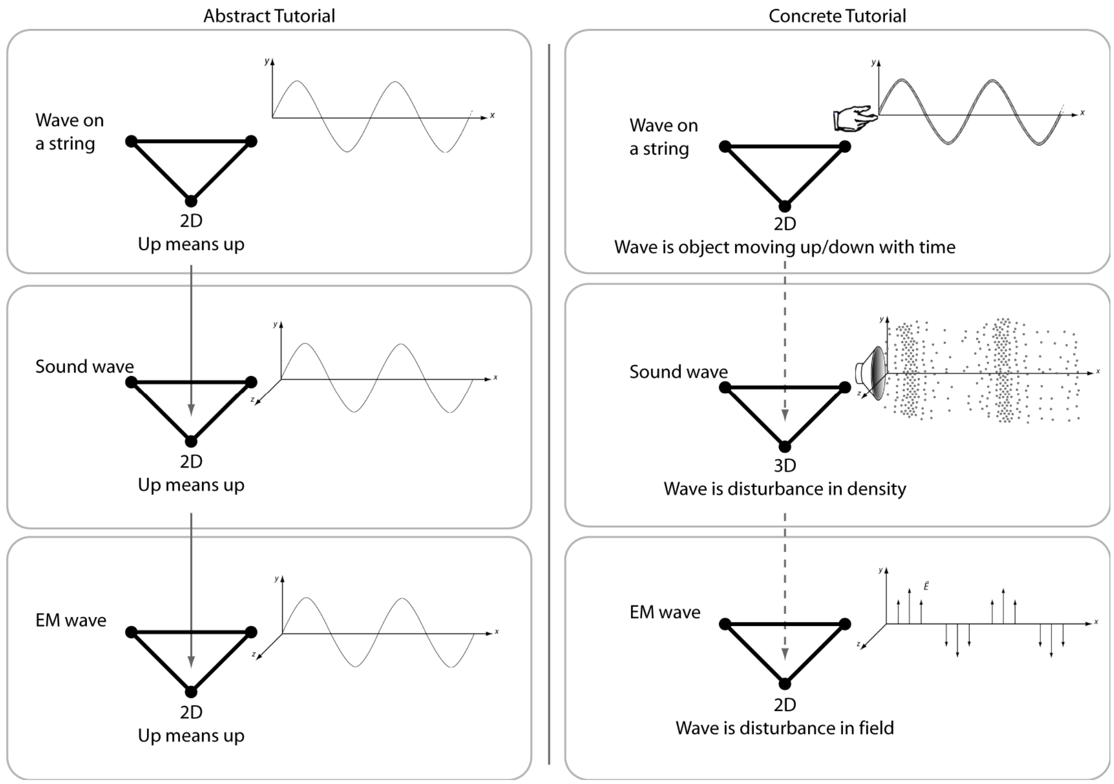

FIGURE 7.4 Analogical Scaffolding schematics for the abstract (left) and concrete (right) tutorials.

Results

Question 1

Key results from the pre- and post-tests in the across-domain study are shown in FIGURE 7.6 and FIGURE 7.7. On pre-test question 1 (FIGURE 7.6), less than 10% of students answered correctly (I=J=K=L), with no statistically significant difference between groups ($p>0.7$). On the post-test, the scores for all groups increased, but, compared to the abstract group, students in the concrete and blend groups were more likely to produce the correct answer by factors of more than two and three, respectively ($p<0.01$). As predicted, the concrete group lies in between, with the blend group marginally more likely to produce the correct answer (a difference of 16%, $p<0.1$).

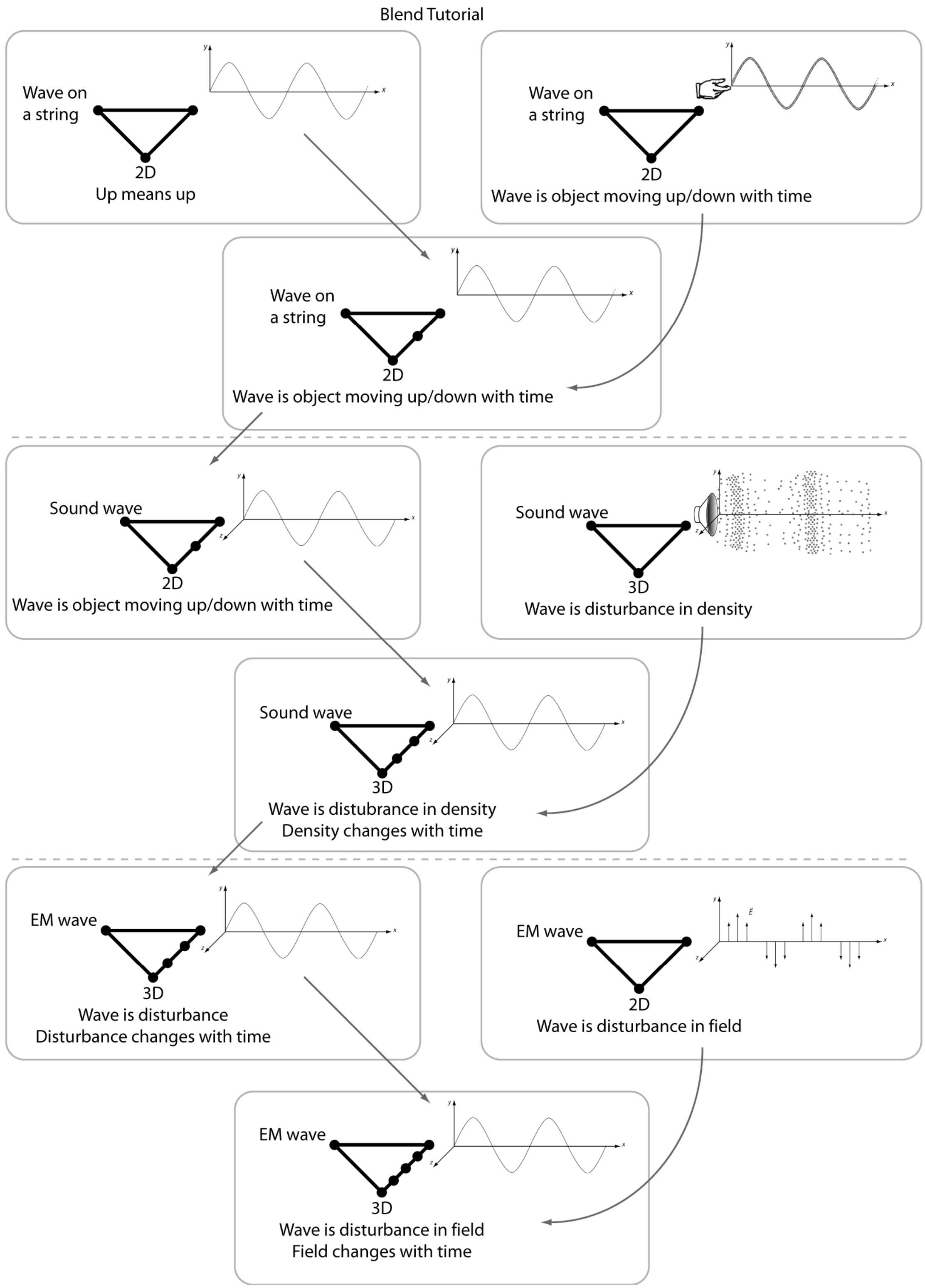

FIGURE 7.5 Analogical Scaffolding schematic for the blend tutorial. Dashed lines delineate string, sound, and EM wave domains.

The most popular (incorrect) answer on pre-test question 1 was I>J>K>L, answered by approximately 24% of students. This pre-test result was the same, statistically, in all three groups(p>0.3). On the post-test, less than 10% of student in the concrete and blend groups answered I>J>K>L, while 18% of students in the abstract group wrote this answer. This result is statistically significantly different

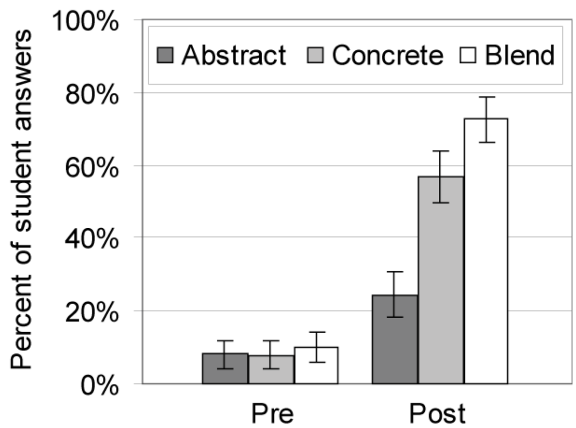

FIGURE 7.6 Fraction of correct answers on pre-post question 1 from the across-domain study. Error bars represent ± the standard error on the mean.

between abstract and blend groups (p=0.055), but not between abstract and concrete groups (p=0.22). Another somewhat popular answer, J>I=K>L, was produced by 18% of students on the pre-test. On the post-test, 22% of students in the abstract group answered J>I=K>L, while less than 5% of students in the concrete and blend groups wrote this answer, significantly less than the abstract group (p<0.01). We note that these incorrect answers are similar to those observed by Ambrose et al.[63]

We coded student explanations of reasoning on question 1 according to 4 categories, shown in Table 7.2. We used an emergent coding scheme based on students' answers. Examples of student explanations are provided in Table 7.2.

Proximity to Line corresponds to primitive reasoning* such as "closer is more", i.e., interpreting the sine wave as an object and reasoning that a closer proximity to this object means stronger field. *Read as Graph* corresponds to primitive reasoning such as "higher is higher", i.e., interpreting the positions of the points as heights on a graph of amplitude.† *Same X Position* corresponds to reasoning that the magnitude of the E-field depends only on the *x*-coordinate. *Sound Words* indicates usage of words related to sound, such as pressure and density. We group all students together on the pre-test, since these responses came before any differential instruction.

In Table 7.3, the number of students (N) producing each answer is shown above the corresponding answer, with the percentage of students producing that answer binned into each reasoning category as described in Table 7.2. In Table 7.3, we include a fifth category, *Other, which* corresponds to explanations that were left blank, rare (less than 10%), or unintelligible. On the pre-test, and across all three groups on the post-test, we find similar patterns in Table 7.3. The results are generally diagonalized, suggesting a strong association between answer and reasoning.‡ Students answering correctly (I=J=K=L) used *Same X Position* reasoning, students answering I>J>K>L used *Read as Graph* reasoning, and students answering J>I=K>L used *Proximity to Line* reasoning. Zero students in the blend group answered J>I=K>L on the similar reasoning across all three groups. We note that very few students in the concrete and blend groups answered I>J>K>L or J>I=K>L, but that for these few students, their reasoning patterns match those of students in the

* Along the lines of diSessa's p-prims.[71]

† This might be an example of WYSIWYG type reasoning.[75]

‡ Note that nearly all of the off-diagonal elements, 35 of 36 cells are zero (not including the category Other). In this case a χ^2 test is invalid. However, because of the nearly perfect diagonalization, we may conclude a strong association between answer choice and stated reasoning.

abstract group. However, combining these results with FIGURE 7.6, we find that students in the concrete and blend groups were significantly more likely to answer correctly and, thus, more likely to use *Same X Position* reasoning on the post-test compared to students in the abstract group.

Table 7.2 Long answer coding for explanations on question 1 from the across-domain study.

Category	**Sample Statements**
Proximity to line	"The closer the point is to the electromagnetic plane, the larger the electric field will be." "The closer you are to the wave the stronger the E will be."
Read as graph	"Ranked in order from top of peak to bottom." "The greater the amplitude, the greater the magnitude of the electric field."
Same X Position	"This is because the magnitude of the E-field will be the same, since they are at the same point on the *x*-axis." "They are all along the same *x*-coordinate, and therefore have the same E-field."
Sound Words	"They are all in the same band of density." "Because if you compare it to air particles then the electric field is never at one point or on a straight line, it's spacious."

Table 7.3 Coded student explanations on question 1 from the across-domain study pre- and post-tests.

	All Groups Pre			Abstract Post			Concrete Post			Blend Post		
N =	13	36	35	12	10	13	28	5	2	36	3	0
Answer:	I=J=K=L	I>J>K>L	J>I=K>L	I=J=K=L	I>J>K>L	J>I=K>L	I=J=K=L	I>J>K>L	J>I=K>L	I=J=K=L	I>J>K>L	J>I=K>L
Proximity to Line	0%	0	100	0%	0	100	0%	0	100	0%	0	-
Read as Graph	0	81	0	0	60	0	0	80	0	0	100	-
Same X Position	54	0	0	67	0	0	50	0	0	69	0	-
Sound Words	0	0	0	33	10	0	36	0	0	47	0	-
Other	46	19	0	17	20	0	21	20	0	6	0	-

Question 2

FIGURE 7.7 shows the results for question 2. On the pre-test, less than 10% of students answered correctly (P=Q=R=S), with no statistically significant difference between groups ($p>0.6$). This question proved challenging for students, and less than 18% answered correctly on the post-test (no difference between groups, $p>0.3$). We did, however, find significant results on another popular answer, P=Q=R>S, which would be correct if the question had asked for the magnitude of the E-field *at the instant shown*. We consider this answer *partially correct*, since it includes the plane wave feature of EM waves, but not the time average feature. On the pre-test, students in the abstract group produced the partially correct answer more often than the other two groups ($p<0.06$). This trend reversed on the post-test – the blend group produced the partially correct answer significantly more often than both the concrete group and abstract group ($p<0.02$). The concrete group was marginally more likely to produce the partially correct answer than the abstract group ($p=0.09$). Interestingly, the fraction of students in the abstract group answering partially correct was unchanged from pre- to post-test, but the majority of these students answering partially correct on the post-test were not the same students that selected the partially correct answer on the pre-test.[*] The only other answer produced by more than 10% of students on the post-test was S>P>Q>R. 14% of students in the abstract group produced this answer, while almost none of the concrete or blend group did (2% and 0% respectively, significantly less than the abstract group, $p<0.05$).

[*] We do not have a compelling explanation for the unexpectedly large number of students in the abstract group answering partially correct on the pre-test. Since the majority of these students answered differently, and incorrectly, on the post-test, we consider this result curious, but insignificant to our broader findings.

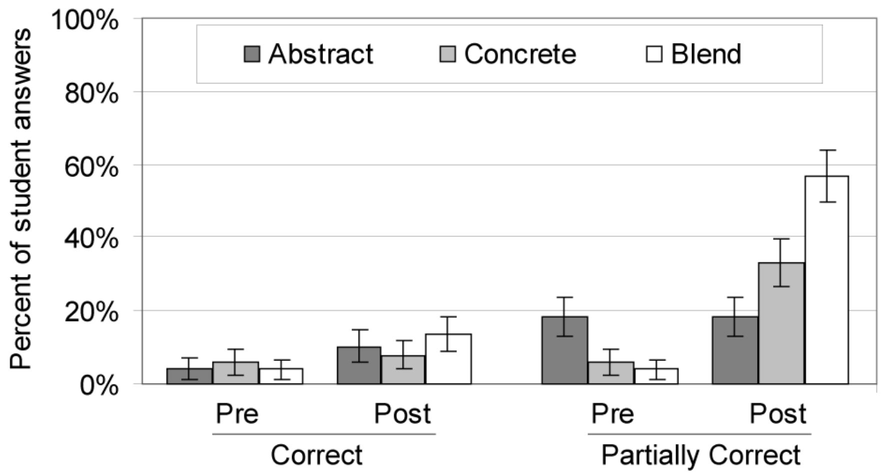

FIGURE 7.7 Fraction of correct (P=Q=R=S) and partially correct (P=Q=R>S) answers on pre-post question 2 from the across domain study.

Further Analysis & Follow-up studies

As a direct measure of changes in student reasoning, we found that 40% of students in the abstract group did not change their answers to question 1 from pre to post, while less than 20% of students in the other two groups did not change their answers from pre to post. On question 2, 15% of the abstract group did not change their answers from pre to post, while less than 8% of students in the other groups answered the same way from pre to post.

Two weeks after the EM waves tutorial, students were issued an online quiz with one question directly targeting the EM wave concepts in the tutorial. Lectures and homework during this two week interval included material covering EM waves. On this follow-up question, while the fractions of correct responses from students in the blend and concrete groups were not significantly different (p=0.2), both of these groups chose the correct answer significantly more often than students in the abstract group (p<0.05). Evidently, the instruction during this two week gap, the same for all students, did not help students taught only with abstract representations catch up with students who were taught with concrete representations (or both abstract and concrete in the blend group).

Goldstone and Sakamoto[85] found that varying the concreteness of representations affected the learning of low-performing students, but that high-performing students were relatively unaffected by this variation. We explored the possibility of finding a similar result by analyzing the preceding results for the upper and lower halves of the class (high- and low-performers, respectively) based on overall course grade. Overall, we found no significant differences between the post-test results of high- and low-performers across all three treatment groups (X^2, p = 0.19).[*,†] Thus, we find that teaching with multiple representations (as in the blend group) may benefit low-performing students compared to teaching with single abstract or concrete representations. At the same time, teaching with single abstract representations appears to limit learning even for typically high-performing students in this population studied.

Discussion of the Across-Domain Study

In the across-domain study, three versions of a tutorial on EM waves used varying representations according to a model of Analogical Scaffolding, and the model was employed to predict trends in student learning with these tutorials. We demonstrated applications of the model to construct schematic representations of

[*] One possible exception was found in these results. The response patterns of high-performers in the concrete group were statistically indistinguishable students in the blend group ($p>0.1$), but low-performers in the concrete group produced responses that were marginally different from the blend group ($p<0.1$).

[†] We observe 25% differences between concrete and blend groups on both questions 1 (correct answer) and 2 (partially correct answer) with $p<0.1$. In this case, since we expect high-performers to be less susceptible to representational effects than low-performers, a 1-tailed z-test may be appropriate which would result in $p<0.05$.

these tutorials (FIGURE 7.4 and FIGURE 7.5) and to make specific predictions about student response patterns under these three conditions. The across-domain study demonstrated two key findings on student learning with analogies. (1) Across several conceptual questions on EM waves, students taught with abstract, concrete, or blend representations produced substantially different response patterns on these questions. (2) We found that students' reasoning about and answers to these questions were associated in similar ways for the abstract, concrete, and blend groups. Thus, while students produced consistent associations between reasoning and response patterns, the representations used to teach appear to drive different types of reasoning (and thus responses). These results bolster the utility of the Analogical Scaffolding model to predict differences in across-domain student learning under different conditions.

Analogical Scaffolding may be employed to analyze cases where treatments led students to produce responses associated with non-canonical ideas about EM waves. Abstract representations may not provide students with the useful schemata for string and sound waves to apply to EM waves. Rather, we found students used surface-level reasoning to interpret the meanings of these representations (see Table 7.3), leading these students to apply unproductive schemata to EM waves. For instance, a surface-level interpretation of a sine wave leads students to read string, sound, and EM wave diagrams as "higher means higher" or "closer means stronger". Concrete representations do provide productive string and sound wave schemata for students, and we observe students applying these schemata to EM waves sometimes. However, without an abstract representation to blend, these schemata are less likely to be applied to EM waves compared to when students are presented with both concrete and abstract (i.e., blend) representations together. Notably, the concrete group responses are closer to the blend group than the abstract, especially for the high-performing students. This is consistent with the hypothesis that high-performing students in the concrete group are using blends on their own.

Part B. Student Reasoning Within a Single Domain

Methods

The participants in the within-domain study were 353 college students enrolled in the first-semester of an algebra-based introductory physics course, focusing on Newtonian mechanics. This is the same course sequence as the across-domain study and has a similar structure to the second-semester course described above. Since both studies took place during the same semester, the two studies involved different students. Students were again assigned to one of three treatment groups, denoted as abstract, concrete, and blend groups. All students in a given recitation were assigned to the same group and issued a quiz on sound waves. In recitation the week prior to this study, students had completed a laboratory activity on sound. This activity involved using a microphone to take measurements of sound waves inside a long tube.[*] Lectures prior to this study had covered mechanical waves, but students had received no explicit instruction on plane (3D) waves. Differences among teaching assistants (TA's) were mitigated by distributing the treatment group assignments evenly among the TA's. Table 7.4 lists the number of students (N) for

[*] In this lab activity, sound was consistently represented by a sine-wave, with one pictorial representation of air-particles along the lines of the concrete representation used. No blended representations were used.

each group, combined over all recitation sections, and the average course grade for students in each group. The average grade for the concrete group was not statistically different from the other two groups ($p>0.27$). The blend group's grades were higher than the abstract group, with weak significance ($p=0.064$). This weak difference does not account for the variance we find in our results, and all following significant results remain so when normalized to account for this small variation in student grade.

Table 7.4 Within-domain study Experimental Groups

Group	N	Average Course Grade
Abstract	120	74.3%
Concrete	114	76.1%
Blend	119	78.7%

Sound Waves Quiz

Students were issued a quiz on sound waves at the beginning of recitation. (See Appendix A.) The quizzes for the abstract, concrete, and blend groups were nearly identical in content and wording, but differed in the representations used. The quiz contained three multiple-choice questions. Questions 1 presented an abstract, concrete, or blend representation of a sound wave, corresponding to each treatment group, directly to the right of the question statement as shown in FIGURE 7.8. The text of question 1 was the same for all treatment groups. The analogy choices in question 1 draw on students' conceptions of a sound wave described by Hrepic.[70] Note the representations in FIGURE 7.8 are the same as those shown in the middle of FIGURE 7.3 for the sound part of the tutorial in the across-domain study.

Questions 2 and 3 are shown in FIGURE 7.9 and FIGURE 7.10, respectively. For the abstract and blend groups, the representations used on question 2 were the

Consider the following four analogies for a sound wave:

a. A crowd in a stadium doing "the wave".
b. A wave on a string.
c. A long row of people passing footballs from person to person.
d. A wave made with a stretched slinky.
e. Something else.

Which analogy or analogies (you may use more than one) seem the best for describing a sound wave? Explain your reasoning. Note there is no "correct answer" - it is up to your interpretation.

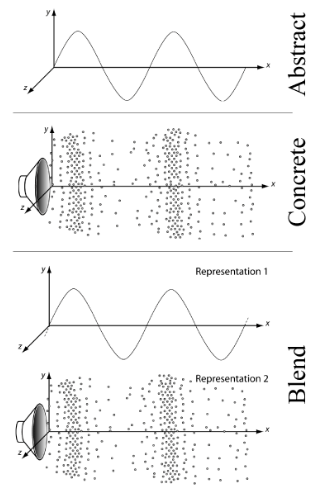

FIGURE 7.8 Question 1 from the sound waves quiz. Each group (abstract, concrete, and blend) received a different representation shown on the right.

The diagram on the right shows four points (labeled 1-4) in space in front of a speaker. The points are separated by a small distance (less than the size of the speaker.) Points 1, 2, and 4 lie in the x-plane. Point 3 has the same x-coordinate as 1 and 2, but lies out of the page (in the z-direction).

Which of the following is the best ranking of magnitude of the pressure at the four points? Note the pressure is proportional to the density of the air particles.

a. 1>2=4>3 b. 1=2=3>4 c. 4>1=2=3
d. 1=2=4>3 e. 1=2>4>3 f. 1=2>4=3

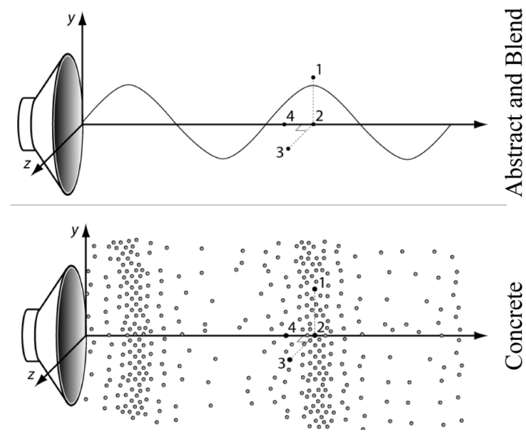

FIGURE 7.9 Question 2 sound waves quiz. The same representation (top) was used for the abstract and blend groups. A different representation (bottom) was used for the concrete group.

In the diagram on the right, a dust particle sits directly in front of a speaker. The speaker plays a sound of constant frequency. Which choice below best describes the motion of the dust particle?

a. Oscillating up and down
b. Moving to the right away from the speaker
c. Oscillating left and right
d. The dust particle will not move

FIGURE 7.10 Question 3 from the sound waves quiz, identical for all three experimental groups.

same (a sine wave). For the concrete group, question 2 used a picture showing air particles. The wording of question 2 was the same for all three treatment groups. Question 3 was identical for all three groups in both wording and representation used. Questions 1 and 2 both appeared on the first page of the quiz, and question 3 appeared on a separate second page.

Predictions

Following our discussion of alternate models in the across-domain study, we briefly outline what these alternate models might predict for the within-domain study. The analysis is similar. Note that the only differential conditions for students occurred during the assessment. If we assume students' ideas are relatively stable and theory-like, a misconceptions model would predict no differences between the groups. WYSIWYG would predict differences between the groups on question 1 – students in the abstract group will differentially choose transverse wave analogies, while students in the concrete group will more likely choose longitudinal wave analogies. However, in the blend group, students are presented with two representations, and WYSIWYG does not provide a mechanism for why students would apply WYSIWYG to one representation over another. Therefore, using WYSIWYG alone, we would predict an even distribution of transverse and longitudinal wave analogies in the blend group. On question 2, WYSIWYG predicts that the concrete group will be likely to answer correctly, since this information can be read directly from the diagram, but does not distinguish between the abstract and blend groups, both of which had the same representation on question 2. WYSIWYG predicts that all three groups will answer similarly (or with similar distributions of answers) on question 3, which was identical in all three groups.

Applying the Analogical Scaffolding model to question 1, surface-level interpretations of signs couple to associated analogies, and we predict students will respond differently as follows: Students in the abstract group, presented with a sine wave, will be more likely than students in the concrete and blend groups to select analogies that involve vertical motion (e.g., crowd and string analogies). Students in the concrete and blend groups, presented with a picture of air particles, will be more likely than students in the abstract group to select analogies that involve horizontal motion (e.g., slinky and football).[*] Students' surface-level interpretations of these signs will play key roles for questions 2. Students in the abstract group will be more likely than other groups to use "up means up" reasoning, answering 1>2=4>3 on question 2. Students in the concrete and blend groups will also use surface-level reasoning, but in this case students will interpret the sign (air particles) as meaning the pressure is the same where the air particle density is the same (and therefore are likely to answer correctly, 1=2=3>4, or possibly take a more literal reading of the picture and answer 1=2>4=3). Importantly, students in the concrete group can map this information directly from the picture on question 2, while students in the blend group must interpret the sine wave in question 2 as standing for a 3D sound wave. On question 3, absent an overt sign, students' choice of analogy on question 1 will play a key role. Students in the abstract group will more likely answer vertical motion ("up

[*] According to the model, the concrete (air particles) sign is privileged over the abstract (sine wave) sign for making meaning of sound. Thus, the sine wave inherits the 3D schema from the air particles picture (and not the other way around).

and down") while students in the concrete and blend groups will more likely answer horizontal motion ("to the right" or "left and right").

Results

Question 1

FIGURE 7.11 shows the six most popular single (or combination of) analogies selected by students according to their assigned treatment group, accounting for more than 92% of student responses. Overall, the slinky analogy was the most popular choice, accounting for 43% of all student answers.* If we include students who selected the slinky analogy in combination with others, we find 66% of students selected the slinky analogy. We find significant differences, however, between treatment groups in FIGURE 7.11. Students in the concrete and blend groups were significantly more likely to select the slinky analogy than students in the abstract group ($p<0.01$). The concrete group was significantly more likely than the abstract

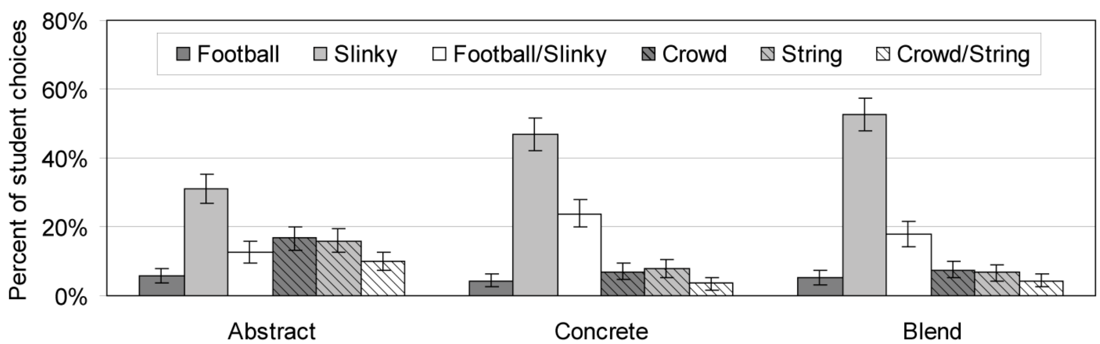

FIGURE 7.11 Question 1 results from the sound waves quiz. Percent of students in the abstract, concrete, and blend groups choosing single analogies (football, slinky, crowd, string) or two analogies (football/slinky, crowd/string). Other combinations accounted for less than 8% of student responses.

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* Note that both transverse and longitudinal waves can be generated on a stretched slinky. Most of the students who chose the slinky analogy indicated in their open response that their choice was associated with a longitudinal wave.

group to select both football and slinky in combination (*p*=0.03), and the abstract group was significantly more likely to select the crowd and string analogies than the other two groups (p<0.05). Thus, we observe the predicted association between the representation presented to students and students' choices of analogy.

Question 2

FIGURE 7.12 shows student answers to question 2 according to treatment group, with substantial differences between the three groups. Here, we show the four main answers, accounting for 86% of student responses. The abstract group was less likely to choose the correct answer (1=2=3>4) than both the blend (p=0.002) and concrete (p=0.024) groups, with the blend group most likely of the three groups to choose the correct answer. Turning to the distracters, students in the abstract group were most likely to select 1>2=4>3 (p<0.002), followed second by students in the blend group (p<0.002), with students in the concrete group least likely to select this distracter. Students in the concrete group were most likely to select two other distracters, 1=2>4>3 (p<0.002) or 1=2>4=3 (p<0.002).

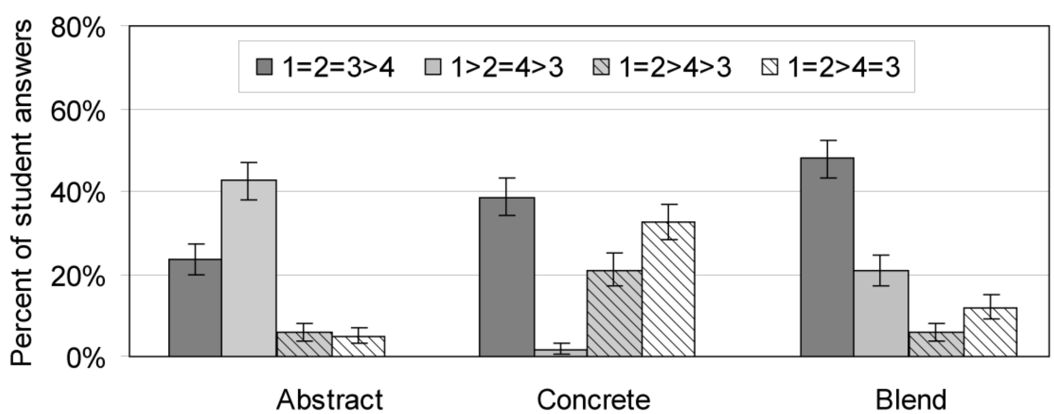

FIGURE 7.12 Question 2 results from the sound waves quiz. Student answers to the sound waves quiz question 2 according to experimental group. The correct answer is the left-most dark gray bar (1=2=3>4).

Question 3

FIGURE 7.13 shows student answers to question 3 according to experimental group. Here, we present the three main answers, accounting for 93% of student responses. The concrete group was marginally more likely to chose the correct answer ("left/right") than the abstract (p=0.07) and blend (p=0.1) groups. On the distracters, the abstract group was more likely than the concrete (p=0.005) and blend (p=0.064) groups to select "up/down". While we see differences, the effects of changes in representation on student answers to question 3 are limited, (generally p>0.05).

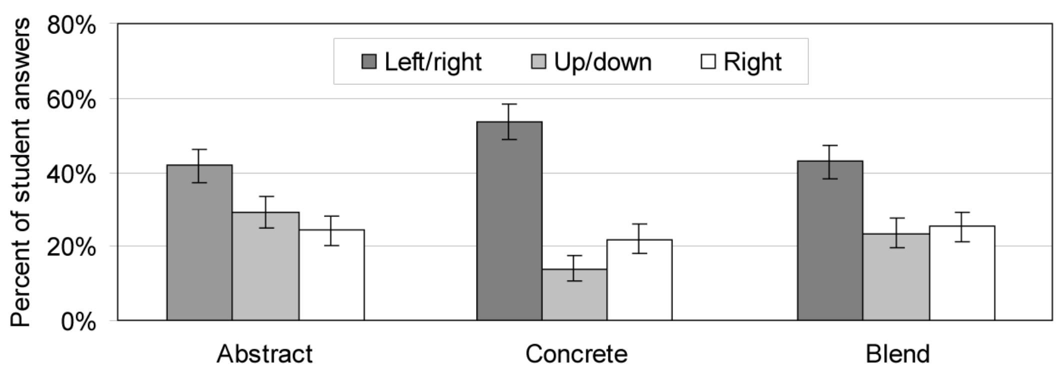

FIGURE 7.13 Question 3 results from the sound waves quiz. Students answers to the sound waves quiz question 3 according to experimental group. The correct answer is the left-most dark gray bar (left/right).

Given the marked effects of representation on question 2, it is noteworthy that we found student responses to question 3 depending only weakly on representation. Note that the representations presented to students on the first two questions are absent on question 3. To gain some insight, we look within each treatment group to examine how students' analogies (as they selected in question 1) affected their reasoning. Table 7.5 shows the number of students (N) selecting a given analogy (single or multiple) on question 1 above the corresponding analogy, and the fraction of student answers to question 3 below. These associations between analogy and answer are all statistically significant (χ^2, p<0.001). In the concrete and blend groups, the majority of students selected slinky and/or football analogies, and more than half of these students answered question 3 correctly. Conversely, students in the abstract group tended to select string and/or crowd analogies to a greater degree than student in the other groups. We found that among these students in the abstract group who selected string and/or crowd analogies, 51% answered "up/down" on question 3. However, a substantial number of students in the abstract group did select slinky and/or football analogies, and, within this select group, 63% answered question 3 correctly. Interestingly, we found that among the few students who selected only the football analogy, 55% of these students answered "to the right" on question 3 – far more than for any other analogy, and the only group choosing "right" to a considerable degree.

Table 7.5 Student answers to question 3 from the sound quiz, split by treatment and analogies selected.

	Abstract		Concrete		Blend	
N =	59	51	85	21	89	22
Analogy:	Slinky and/or Football	String and/or Crowd	Slinky and/or Football	String and/or Crowd	Slinky and/or Football	String and/or Crowd
Up/down	10%	51	11%	29	15%	50
Right	25	22	21	29	27	23
Left/right	63	24	59	38	49	18

Discussion of the Within-Domain Study

The purpose of the within-domain study was to examine student reasoning within a single layer, sound waves. According to the Analogical Scaffolding model, signs can cue productive schemata for students to apply across multiple layers, and in the across-domain study one of these layers involved sound waves. The within-domain study demonstrated three key findings: (1) Signs can drive students' choice of analogy (question 1). (2) Students in the blend group productively applied the 3D idea to an abstract (sine wave) representation of sound (question 2), while students in the other treatments did not. (3) Absent an overt sign (question 3), there is only weak association between students' answers and the representations presented on earlier questions. However, we do find a stronger association between students' answers and the analogies they bear in mind.

Thus, representation can drive analogy and, therefore, schemata. We could represent this as an arrow pointing from sign to schema in FIGURE 7.5, indicating the direction of cueing.[*] (Note that the slinky was the most popular choice in all three treatment groups, and, therefore, representation is one of several mechanisms driving analogy.) Within the sound waves layer, schemata preferentially cued by abstract and concrete signs were consistent with the predictions of the Analogical Scaffolding model. For instance, students presented with only an abstract sine wave on the quiz select answers to question 2 reflecting "up means up" reasoning about this representation of a sound wave (i.e., these students tended to select 1>2=4>3 in

[*] This finding is consistent with an interpretation of our model where signs drive the forms (or formation) of schemata. Similarly, schemata may also drive the meaning (or creation) of signs. We might consider this latter directionality an indicator of expert reasoning.

FIGURE 7.12). Conversely, students presented with a concrete picture of air particles selected answers reflecting 3D conceptions of a sound wave. However, note that in the concrete treatment, sound was represented by a concrete picture of air particles on question 2. We might therefore argue that students in the concrete group were able to map this information directly from the diagram shown in FIGURE 7.9. Only students in the blend group interpreted a sine wave as representing a *3D* sound wave. According to the Analogical Scaffolding model, for the blend group, this sine wave took this 3D meaning by way of a prior blend (in question 1) with a concrete picture of air particles. This result confirms the models prediction that schemata that are tightly coupled to concrete signs preferentially project to blends over schemata which are not (or are only weakly) coupled to abstract signs.[101] If concrete signs were not privileged in this way, we would expect many more students in the blend group to answer similarly to students in the abstract group (on both questions 2 and 3).

Without explicit signs, students' mechanistic reasoning about sound (i.e., motion of air particles) remains strongly coupled to the analogies they bear in mind, as evidenced by Table 7.5. Interestingly, though students in the abstract group were more likely than other students to use "up means up" reasoning on question 3 (answering "up and down"), the three groups were not significantly different on the correct answer. We may describe this as "weak cueing", whereby the schemata coupled to questions 1 and 2 of the quiz were not strongly cued by the representation (or lack thereof) on question 3. In this situation, students may rely on the analogies they bear in mind (Table 7.5), or on other prior knowledge of sound. In summary, we note that one indicator of difficulty for students on questions 2 and 3 may be the use of representations. We find students in the abstract group relatively unprepared to interpret abstract representations on question 2, while students in the blend group demonstrated the highest level of ability to productively interpret these abstract representations.

* * *

As part of ongoing studies of student learning with analogy, we have conducted two sets of empirical studies to examine the utility of the Analogical Scaffolding model. In the first of these studies, we found that Analogical Scaffolding constitutes a productive tool for analyzing student learning with analogies. In this across-domain study, students taught about EM waves with a tutorial incorporating blends (appropriately presented according to the Analogical Scaffolding model) produced response patterns markedly different from students taught the same material without blends. Though our aim in these studies was to observe differential response patterns independent of overall student performance, one cannot escape the conclusion that, of the three treatments examined, the blend treatment is generally more productive of correct student reasoning. Students taught with blends achieved post-test scores three times those of students taught with canonical (abstract) representations alone. In addition, the model explains why tutorials that did not use blends were less beneficial for student learning. Abstract signs (e.g., sine wave) do not always couple to productive schemata, while concrete signs (e.g., air particles) that are coupled to productive schemata do not readily cue these schemata across layers. However, abstract signs do cue productive schemata across layers when blended with a concrete sign in previous layers. Without abstract signs presented explicitly, we might hypothesize that students in the concrete group may be making their own blends.

In a second complementary study, we examined student reasoning about sound waves, demonstrating how blends occur in particular instances. We find that signs can cue particular schemata and associated analogies that appear to drive student reasoning about sound waves. Consistent with the across-domain study, we find abstract signs can couple to unproductive schemata when used alone, but these abstract signs can cue productive schemata when blended previously with a concrete sign. Again, performance differences stand out. On a quiz focusing on sound waves, students presented with blends outperformed (by a factor of two) students presented with abstract representations alone in their ability to productively interpret these canonical representations.

These across- and within-domain studies provide consistent evidence in support of the weak hypothesis that signs can cue associated, but pre-existing, schemata. This cueing leads to significant variations in student reasoning about waves as measured by the assessments in both studies. Further, the across-domain study provides evidence in support of the strong hypothesis that signs and blending can lead to the formation of new schemata. The various ways these new schemata are formed may depend strongly on the signs used to teach. The across- and within-domain studies support the model of Analogical Scaffolding and provide a prototype for future studies of this kind.

Chapter 8 – Revisiting Abstraction: Towards a Functional Definition, Dynamics, and the Community Map*

"People only see what they are prepared to see."

– Ralph Waldo Emerson[105]

"If we knew what it was we were doing, it would not be called research, would it?"

– Albert Einstein[105]

Thus far, we have described the Analogical Scaffolding model and a series of empirical studies that confirm the utility of this model for explaining and predicting student learning with analogies in physics. While the Analogical Scaffolding model develops a new way of understanding student reasoning with analogy, we attempted to be fairly conservative as to the claims made and conclusions drawn in previous chapters. This chapter is meant to be somewhat more speculative. We will consider some novel and challenging ideas that flow from an Analogical Scaffolding perspective on learning. It should go without saying that, as with any theoretical framework, the model described in Chapter 5 is not yet complete, and we anticipate identifying additional structures and mechanisms to augment the model. Here, we develop one such thread, focusing on the notion of abstraction as framed by

* Chapter 8 is partially based on published work, Podolefsky, N.S. & Finkelstein, N.D. (2007) Salience of Representations and Analogies in Physics. *Proceedings of the 2007 Physics Education Research Conference*. AIP Press

Analogical Scaffolding. We explore the interaction of individuals within communities and how a modified Analogical Scaffolding model can be used to explain certain aspects of learning that are culturally bound. This chapter begins with theoretical arguments which are grounded in analyses of student reasoning as evaluated through a student interview where the student uses string and sound waves to ground his understanding of EM waves.

Previously, the Analogical Scaffolding model tracked the number of blends required for a particular sign-schema relation, and demarcated these blends by additional nodes between sign and schema vertices in a mental space triangle. I suggested that the number of nodes could be used to represent the level of abstraction of a sign-schema relationship. In this chapter, I will revisit this idea in order to suggest that this prior notion of abstraction had compressed two distinct dimensions that may characterize mental spaces (as represented by sign-schema-referent triangles). I will propose that these two dimensions, which I call *abstraction* and *salience*, were tacitly fused in our prior treatment of abstraction.* Prior discussions of abstraction had essentially collapsed these two dimensions into one.

Recall that according to the Analogical Scaffolding view, the level of abstraction may depend on the individual, whereby some ideas, such as a sine wave standing for a longitudinal sound wave, might be treated as more abstract by students than by seasoned physicists. To the physicist, the idea that a sine wave moving up and

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* The reader may be troubled that I am using the same term, abstraction, to label the original collapsed dimension and also as one of the new distinct dimensions. I am retaining the word abstraction because of the way the physics community tends to use the term. The dimension I will now call abstraction is, I believe, more along the lines of this common usage in physics, while salience describes a dimension that is not so commonly recognized or discussed. I will further elucidate these choices as we proceed.

down stands for air moving back and forth might be considered fairly concrete – it is not a difficult connection for this expert to make. Simultaneously, however, experts can recognize and articulate the idea that sound waves are more abstract than waves on a string, and that EM waves are “highly abstract”.[63] How can we account for the experts’ recognition (perhaps categorization or ontological classification) of certain sign-schema-referent systems as highly abstract, while we, at the same time, observe people of different expertise using these systems in substantially concrete, albeit diverse, ways? Rephrasing, how can a physicist, on the one hand, declare an EM wave highly abstract, and, on the other hand, use the sine wave representation of an EM wave with relatively little difficulty or confusion? I do not believe such a case need be viewed as a contradiction. What I will suggest is that *abstraction* can be considered to describe a sort of sign-schema relation in a second triangle representation, a counterpart to the individual’s mental space triangle we have used previously. While the original mental space triangle characterized an individual’s cognitive processes (e.g., student), this second triangle is meant to characterize knowledge generated within a collective, or *community of practice* (e.g., the physics community).[106,107] I will shift the notion of abstraction to describe this second, community-based triangle and introduce a separate dimension, *salience*, to now characterize sign-schema relations as used by the individual reasoner. The process of learning will involve the coordination, cross-mapping, and alignment of vertices and nodes between these two triangles. The process I propose draws on some mechanisms described by classical analogy theory (e.g., mapping), but employs these mechanisms in substantially different ways.

Recall the standard view which posits an analogy as a mapping from a familiar conceptual domain, *base*, to an unfamiliar conceptual domain, *target*. I have repeatedly pointed out that analogies in this sense can be productive when generated by the user, but students may not generate and/or use analogies productively. The explanations proposed for these findings on student use of analogy include insufficient student understanding of the base domain, incorrect completion of the analogical mapping by students, or a failure by students to recognize an analogy relation at all. Researchers have, nonetheless, found cases where students can use analogies productively when prompted, say with a hint.[44,45,47] Furthermore, in my own work, I have been able to repeatedly create circumstances in which students used analogical reasoning on a range of physics questions. For instance, I showed that students prepared with wave-on-a-string analogies preferentially chose ”2D” distracters on a post-test about EM waves, while students who were prepared with a sound wave analogy preferentially chose ”3D” distracters on the same post-test. Notably, these students may not have been fully aware of the cognitive process they were engaged in. One question that leads from these general findings is this: how do students know what to map and, importantly, when to map? I have partially answered this question, proposing that representations, blending, and cueing of blends are the mechanisms of analogical reasoning. Presently, I will seek ways in which we can describe student use of analogy in particular academic contexts, and move towards operationalizing the learning goals of physics instruction with analogy. Augmenting the individuals’ mental space triangle with a community triangle may be a significant step towards these goals.

This chapter extends the prior theoretical structure, and will ultimately evaluate this broader perspective by focusing both on characterizing the "community map"[49] of the domain of sound waves and conducting a fine-grained analysis of a student reasoning about E/M waves with a sound-wave analog. I will employ the augmented Analogical Scaffolding model as an analytic toolset, with the three following outcomes. (1) I anticipate this analysis will bolster the case for distinguishing abstraction from salience. I will apply this framing to describe the dynamics of student reasoning about EM waves using wave on a string and sound wave analogies. (2) This fine-grained analysis will confirm prior large scale findings, that (external) representations play a key role in student use of analogy.[101] (3) Finally, I will argue that an approach that treats representations as *part of* concepts can be extremely productive for understanding the dynamics of student reasoning. Though controversial, this view is articulated and supported elsewhere.[108,100]

Distinguishing Salience and Abstraction

According to the Analogical Scaffolding view of analogy, students presented with an unfamiliar (and perhaps challenging) problem can draw on existing mental structures to solve that problem by analogy. We have seen that mental structures may be cued by the student recognizing some similarity between the problem and prior experience (which may include another problem the student has solved previously). Two possible mechanisms for recognizing similarity, and hence making productive use of an analogy, are the following. If the two domains contain surface features (e.g., signs) that are similar, this can cue the student to make an analogical comparison.

This mechanism explains why students may consistently solve problems that include inclined planes using kinematics equations (even for problems where the optimal solution method uses conservation of energy).[48] Note that in this case, students *do* use an analogy, albeit the base domain may be inappropriate. A second possible mechanism is that similarity is based on mental structures that transcend the surface features of a problem representation. (See ref. 109 on similarity and analogy.) This mechanism may explain how physics experts can productively apply different solution methods to different problems which, say, all involve blocks on inclined planes. Still, *something* contained in the problem representation cues physicists to use one method or another, so the representation and mental structure cannot be completely separated.

The expert framing[13] of a problem including an inclined plane as a conservation of energy problem can be characterized as relatively abstract, at least compared to other possible framings of this problem as described above. However, in practice experts may treat the conservation of energy approach in a fairly concrete way. That is, this more abstract approach is quickly recognized and readily accessible to the expert. In order to better understand this situation, I suggest that the notion of abstraction, in terms of physics concepts, has a sort of split-structure that we had earlier glossed over.[*] This splitting becomes a necessary hypothesis in light of the arguments made above, in particular the notion that individuals can recognize something as abstract and yet treat that something in a concrete fashion. One of these

[*] This does not mean that a wholesale reworking of the previously developed theory is necessary. What I am attempting here is a foray into *potentially* useful mechanisms, abstraction and salience, which are somewhat akin to a *fine-structure* in the model. The prior notion of abstraction, collapsed over these possible new dimensions, still has utility within the bounds that it was applied in earlier chapters.

productive dimensions for analyzing analogy use is *salience*, which I will define as the strength of associations between sign features and schemata.[110] Note that what I now call the salience dimension replaces a subset of the scale labeled previously as abstraction. We retain the representation employing nodes between sign and schema, but now use these nodes to represent the number of salient connections (i.e., blends) relating sign to schema. I propose that *abstraction* is more proximally a measure of the *community consensus* on the sign-schema relationship. I will represent abstraction in the following way. *Schema* will be reserved for the individual mental space. In FIGURE 8.1, I introduce a second triangle (top). This triangle has sign and referent vertices that are analogous to the same vertices in the lower triangle. Schema is replaced with *community map.*[49] The community map may be structurally similar to the schema, but instead of being held by an individual, the community map is a resource that is generated over time and shared by a community, for instance the physics community. The community map is historically rooted and developed, but can be considered relatively static and stable over short time scales. For example, the physics community's map for EM waves is not likely to change much over the course of a few years or even decades. Abstraction belongs along the sign-community map leg of the upper triangle in FIGURE 8.1. In short, a community of practice such as the physics community has certain agreed upon interpretations signs and how these signs relate to referents (i.e., how representations describe phenomena) that are generated, refined, and made canonical over time. What physicists often consider the "correct" interpretation of signs is determined by the community map.

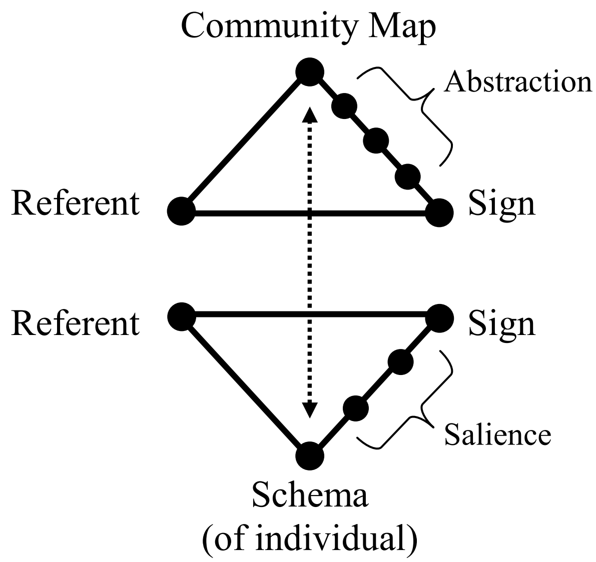


FIGURE 8.1 Sign-referent-schema triangle representing a mental space (bottom) has been augmented by a sign-referent-community map triangle (top). Dashed arrows represent coordination of vertices and nodes from the upper and lower triangles.

Potentially, this teasing apart of abstraction and salience has considerable explanatory power. A physicist may, for example, describe an EM wave (represented by a sine wave) as highly abstract based on the tacit recognition that the relationship between the sine wave and an EM wave is richly layered and complex. One may also simply say that the relation between the sine wave and an EM wave is not at all straightforward. We can operationalize the more informal phrase "not straightforward" by ascribing multiple layers of meaning for the sine wave with each layer represented by a node along the sign-community map axis. The layering process depends on a series of blends, such as the series depicted in FIGURE 6.1. We may differentiate abstraction in this (more-or-less) absolute sense from how a sign is interpreted by an individual. The functional element of the triangle in FIGURE 8.1, for the individual, is the schema. The schema that is coupled to a particular sign depends on a reflexive relationship whereby additional blends (nodes) lead to new schemata. Salience is demarcated along the sign-schema leg of the lower triangle in FIGURE 8.1. One interpretation of the goal of instruction, in the sense of enculturation or adoption of cultural norms and beliefs, can be considered the coordination, cross-mapping, and alignment of the schema with the community map, where the latter has been generated and agreed upon by a surrounding culture. We represent this process with the vertical dashed line in FIGURE 8.1 Note that this does not assume a complete mapping, or that this mapping will always occur. Importantly, the nodes of salience need not be *the same as* the nodes of abstraction. In our view, a constructivist model of learning will be satisfied by a correspondence between the two sets of nodes, recognizing that each individual may hold their own unique interpretations and still legitimately participate in a community (e.g., physics).*

Representations, salience, and abstraction

Let us now focus on a specific application. We can employ our new notion of salience to argue that it is not the surface features of a sign that are salient *per se*, but the associations made with those surface features. Salience depends on the individual and context. A student presented with a sine wave may associate that sign with a material object (e.g., a wave on a string or a water wave), while a physicist may associate the same sine wave with a graph (e.g., of electric field strength oscillating in

* I note that the student nodes that that represent salience (nodes in the sign – schema leg) should be a subset of the nodes that represent abstraction (the community map- sign leg). Notably, individuals may not have a complete understanding of a domain, or may have compiled layered blends (thereby fusing nodes together), and hence are expected to have fewer nodes than the community.

time). Both the student and physicist cue on surface features – where the sine wave goes up, *something* goes up – but *what goes up* is very different.

This new framing leads to an important question for instructional practice. Namely, how can the salient associations that students already use (often quite readily) lead to these students making associations that are salient to physicists? The extended Analogical Scaffolding model suggests productive ways of scaffolding students' use of analogy by capitalizing on associations that are already salient for students and layering these associations toward more expert-like ideas.

In order to delve into this new framing of abstraction, let us return to the Analogical Scaffolding treatment of EM waves, represented in FIGURE 6.1. Notably, this diagram employed a sort of hybrid representation of expert and novice interpretations of representations (i.e., signs). The lowermost mental space in FIGURE 6.1, which included an EM wave diagram, could be considered highly abstract. According to our extended framing we should move the 4 inter-vertex nodes from the sign-schema leg in the lower triangle leg to the sign-community map leg in the upper triangle of FIGURE 8.1. Those particular nodes are those arrived at by the community and built into the curricula used to teach students. The interpretation of the EM wave sign (sine wave and vectors) by the individual, or student, can now be treated separately. The highly abstract and multiply-layered sign-community map relation can be considered a goal of instruction. Clearly, students will not begin in this expert-like interpretive state. Students also need not recapitulate the community's development of ideas. Student learning may be considered to involve a reflexive relationship between the community map and the student schema, but we may not be able to completely specify how this happens *a priori.* Nonetheless, I have demonstrated Analogical Scaffolding to have both descriptive and predictive power, and my intention here is to enhance the resolution of the model. Here is an example. A student may initially use a WYSIWYG interpretation of the sine wave, a 2-D, substance like interpretation of this representation – perhaps a *wave on a string*-like interpretation. We could represent this student interpretation with one inter-vertex node between sign and schema, indicating that the salient interpretation of the sine wave utilizes few apparent blends – up means up, for example. Note that the number of nodes is inversely related to how salient the sign is to the schema. The number of nodes is an indicator of the number of layered blends that are simultaneously salient to an individual at a particular time. An important point here is that the number of salient nodes in generally less than or equal to the number of abstraction nodes. The learner, new to a community, will always be several steps behind the community map during the learning process until that individual has internalized the community map.*

We distinguish between *compiling* ideas and *building associations*. Compiling is the fusing of separate nodes via a blending process, and it takes conscious effort to break apart a compiled set of ideas.† Building associations is the explicit linking of ideas to representations in a layered fashion. We suggest that expert reasoning can be characterized by the use of compiled ideas, such that the expert is not explicitly aware

* This representation of mental spaces leads us to explore the meaning, if any, of a state in which the number of salience nodes exceeds the number of abstraction nodes. This particular state may be a necessary condition for scientific progress, whereby the insight of an individual outpaces the status quo of the community.

† Compiling in this sense is analogous to compiling a computer program. The programmer can read the original code and see the individual pieces. Once compiled into an executable file, however, the individual pieces of written code cannot be extracted from the resulting compiled file. Given a particular output from the executable, however, the original pieces of written code may be reconstructed, sometimes with significant effort.

of the individual associations that have been compiled. Experts can reason efficiently with these compiled ideas, whereas novices reason less efficiently since they must consciously attend to several ideas at once (or an individual may not yet have the necessary ideas to reason productively in a particular domain). Consider, for example, the idea of a *plane wave*. This compiled idea can be broken into several constituent ideas, such as propagation, vector space, 3D, and relation of these ideas to a representation such as a sine wave. Individuals build associations in a layered fashion and these associations can be compiled via a blending process (or associations may remain un-blended). In a representation like FIGURE 8.1, the number of salience nodes represents the number of blends that have been *compiled* by the individual and that serve as an interpretive mechanism.

This framework allows us to reframe particular flavors of constructivist learning.[75] So-called *misconceptions* views may be reinterpreted in several ways. We might view a misconception as the condition in which the set of abstraction layers (call these set A) and the set of salience layers (call these set B) are disjoint, or at least where the union of A and B is only a subset of either set A or set B.[*] A fine-grained approach to constructivism takes a somewhat similar relativist stance, wherein student ideas are "correct" only insofar as these student ideas align with expert views. The distinction of abstraction from salience described here adds the following corollary to a fine-grained resources approach – there exist collectively agreed upon interpretations (so-called expert views) which can be distinctly non-WYSIWYG in some cases. A fine-grained analysis of student representations use

[*] We might also consider the connection between sets A and B as a mapping, where the student set of layered blends are a distinct from the community set, but an isomorphism from A to B exists.

should be able to gauge student interpretations on a similar, but distinct, scale of WYSIWYG to non-WYSIWYG interpretations and also should gauge these student conceptions relative to expert views.

This new framing attends explicitly to the dynamic nature of learning. Student ideas are not static structures, even on time scales of minutes or seconds under certain conditions. Our model allows for the dynamic representation of student reasoning and learning of abstract ideas in physics. In the remainder of this chapter, we apply our newly modified model of analogy to explore the community representation of the material used to teach students EM waves, and the dynamics of student reasoning around the curricula.

Applying Analogical Scaffolding: unpacking an example

We have interviewed numerous introductory physics students in our studies of analogy. These think-aloud interviews generally involve students using curricular materials. Here, we focus on an interview with student "S". We describe several segments of this interview briefly and then apply Analogical Scaffolding in detail to a segment involving sound waves. At the time of the interview, S was enrolled in the first semester of a calculus-based introductory physics course. At this point in the semester, S told us the class was "doing oscillations and starting pressure." The interview was clinical in the style of diSessa,[76] and the student was prompted to solve a challenging EM wave question and given various representational analogs (string and sound) along the way.

Before analyzing S's use of analogy in the interview, we present an example of the community map representation of a sound wave blend which shows up in this clinical interview. Shown top left of FIGURE 8.2 is a community interpretation of a sine wave representation. Each node between sign (lower right vertex) and community map (top vertex) is labeled with a particular interpretative phrase. For instance, this sine wave is a physical representation drawn in the *x-y* plane. Shown at the upper right is a similar representation for a pictorial representation of sound. These two spaces blend, and at the bottom of FIGURE 8.2 is a blended space which uses a sine wave to represent sound (which is a 3D, longitudnal movement of air particles). In this representation, the top vertex of each triangle stands for a community map, while the nodes along the right leg demarcate individual layers of meaning that relate a given sign to an associated community map. Note that the triangle at the bottom of FIGURE 8.2 uses five nodes between sign and community map, whereas the two upper (input space) triangles each use four nodes. When an expert uses a representation, the ideas represented by the individual nodes on the right leg may be activated but not explicitly recognized as distinct ideas. The representation in FIGURE 8.2 makes explicit constituent parts of a community map that may be used as a unitary object by an expert.

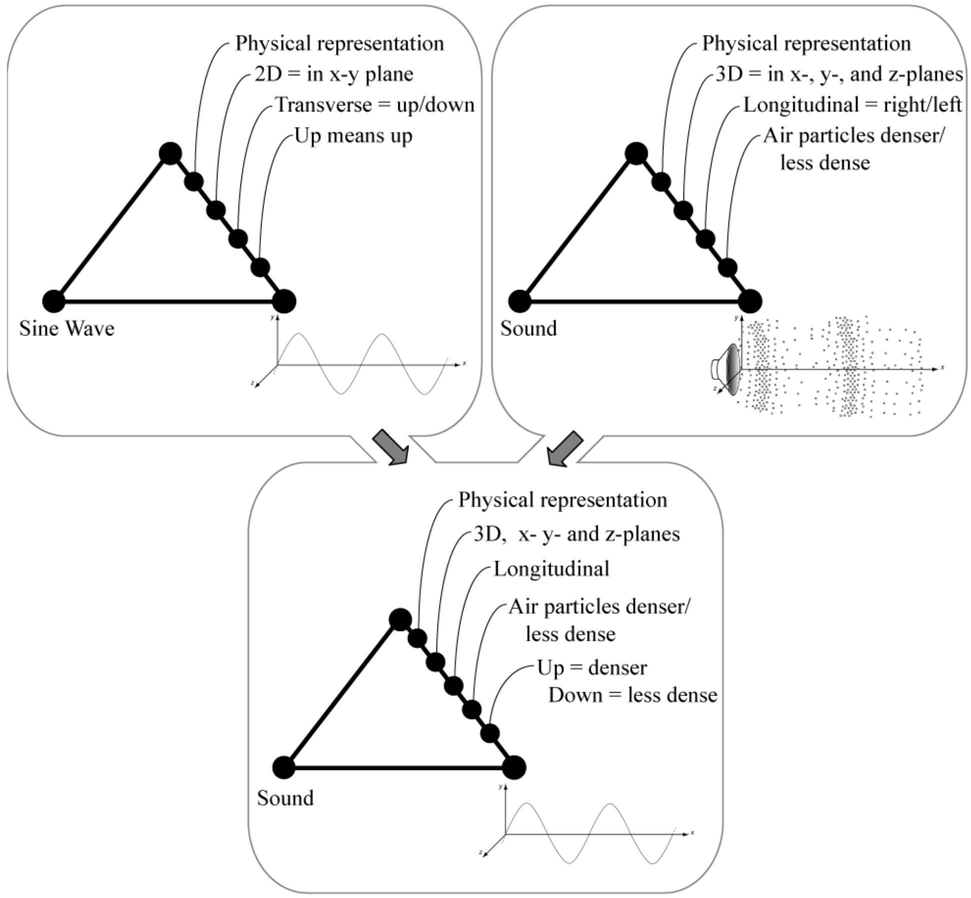
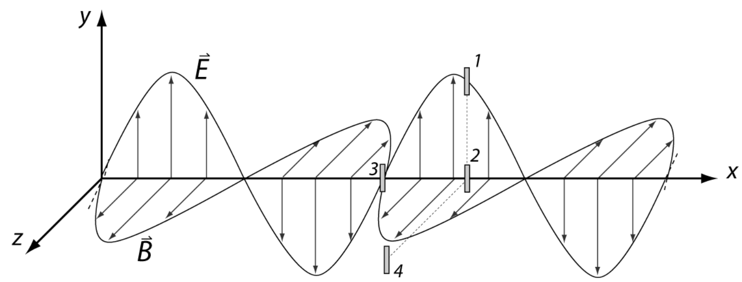


FIGURE 8.2 Blend of community interpretations of a sine wave (upper left) and sound (upper right), resulting in a blend with new meaning for the sine wave used to represent sound (bottom).

In the interview, S was presented with the EM wave concept question in FIGURE 8.3 and asked to give his best answer.* S said he had "never seen anything like this before" and cycled through several answers over the next five minutes, finally settling on answer choice B (3>2>1=4). His reasoning was that "3 would be the first" and that "1 and 4 are on the same position on the two different waves." At one point, S stated that "x would…be the time."

* Answering this question correctly requires a blend in which the EM wave sign is coupled to a schema including *3D* and *traveling (moving in time)* schema elements.



An electromagnetic *plane* wave propagates to the right in the figure above. Four antennas are labeled 1-4. The antennas are oriented vertically. Antennas 1, 2, and 3 lie in the *x-y* plane. Antennas 1, 2, and 4 have the same x-coordinate, but antenna 4 is located further out in the z-direction.

Which choice below is the best ranking of the *time averaged signals* received by each of the antennas. (*Hint*: the time averaged signal is the signal averaged over several cycles of the wave.)

A) 1=2=3>4 B) 3>2>1=4 C) 1=2=4>3 D) 1=2=3=4 E) 3>1=2=4

FIGURE 8.3 EM wave concept question. D is correct.

We can already identify blends in S's reasoning. S appears to blend this picture, where *x* is position, with a graph in which the horizontal axis is *time* – hence 3 is "first". This idea may have been cued by the word "time" in the problem statement. In this case, a *position is time* blend is salient for S. However, the peaks in the wave also cue a salient blend, along the lines of *a peak is a position* (S does not distinguish between the *E*- and *B-field* waves). Note that these salient blends are coupled to signs – the *x* axis, the two sine waves, and the word "time".

Next, the interviewer suggested analogies *verbally* and asked S if this helped with the EM wave concept question. First, the interviewer stated that an EM wave was like a wave on a string, but with no elaboration on how to use the analogy. S said that the string would "follow a similar pattern" to the EM wave, but that this would not help him answer the concept question. The interviewer then suggested that an EM wave was like a sound wave, again with no elaboration. S said immediately that, for making sense of the concept question, the sound wave analogy "wouldn't change too much." Note that in both cases, the sign is the verbal statement of the analogical comparison, but in these cases productive blends are not salient for S. We emphasize the distinction between not *salient* and not *existing*. As we will see, S does use several productive ideas about strings and sound, but these ideas become salient only under different conditions.

S was next presented with two new signs printed on a sheet of paper: one a sine wave and the other pictorial, depicting a hand at one end of a realistic representation of a string. He was told both represented a wave on a string. S stated that one representation was "a physical form of the other," implying a blend *sine wave is physical object*. S applied this blend to the EM wave, stating "2 is not really on…I guess none of them, other than 3, are on it." S also stated "as the hand moves it would follow the up and down with the hand," implying a blend *sine wave is moving object*. Note that the *moving object* element is not explicitly contained in the sign (a static picture), but becomes salient for S after he sees the two signs of a string.

At this point, it is worth applying Analogical Scaffolding to sound waves in a detailed manner. Selected transcript segments accompany a schematic representation of Analogical Scaffolding in FIGURE 8.4. Following the string discussion, S was presented with two different signs, a sine wave and a picture of a loud speaker with the "arrangement of air particles", and told that these both represented a sound wave. These signs are shown in the topmost boxes of FIGURE 8.4. S first stated that the particles sign is a "physical representation of the sine", implying the blend *sine wave* *height is particle density*. S related quantities of the sine wave (e.g., “negative”) with arrangements of particles (e.g., “grouping”) and descriptive terms like “strong” and “weak” signal. S then applied these ideas to the EM wave at time 17:37. Previously, S had said antennas 1 and 4 are less than 3 (answer B), but with the sound wave he said 1 and 4 are greater than 3 because they are where there are “more particles”.

Now S has a new idea about antenna 2. Using the sound wave blend, S suggests that 2 is the same as 1 and 4, since the particles are “all the way down” (pointing at the dense region of particles in the pictorial sound representation, 17:59). He has applied ideas from the sound wave, now part of the *sine wave height is particle density* blend, to the EM wave.

S next tried to decide whether the particles extend in the *z* direction (in the picture, they are drawn only in the *x-y* plane). He created a new blend, drawing on experiential knowledge of sound – i.e., it is *3D* (18:37). Finally, S used a blend – *sine wave is 3D particle density* – to reason that 1, 2, and 4 are equal.

At this point, S proceeded to reason about point 3 by using the *sine wave is moving object* blend from the string. He could not deduce from the materials in front of him whether the wave at antenna 3 is stationary or moves as the wave propagates, (essentially whether the EM wave is a standing wave or a traveling wave) and he wavered between answers C and D in FIGURE 8.4. Nonetheless, S explicitly voiced the idea that a static picture of a string represents a moving object, and that an EM wave exhibits a similar property.

Interviewer (I) directs student (S) to sound representations

16:48	S	This is saying, like, the signal almost, the signal is strong, well, I mean they're once again its another physical representaiton of the sine. So just looks like it's strongest here, [points to dense area of particles] so at the high point it'd be strongest. [points to peak of sine wave]
17:05	I	Mm hmm.
17:10	S	And, when you come down negative there's the least particles. [points to trough of sine wave] So it'd be weakest.

I directs S to EM wave concept question (Figure 2)

17:37	S	That, um, 1 and 4 are actually gonna be greater because this is gonna be a stronger signal. [points near dense particles on sound rep] Essentially like the sound wave. There's more particles into the peak. Greater signal.
17:52	I	OK. Greater signal than...
17:54	S	Than, like, 3 which is out here. [points near rarefied particles on sound rep] There'd be less particles.
17:59	S	2 I'm still kind of confused on. It seems like, well maybe it'd be the same 'cause if you look, if you look, um, at this high point [points at dense area in sound rep], there's a lot of particles all the way down. [sweeps pencil up/down over dense area of pictorial representation] Seems to follow it.

S self-directed to sound representations

18:24	S	Yeah. Because, like if you put them all here and, I don't know, is this saying that the particles come out on the z-axis as well or not?
18:35	I	Um, well what do you think?
18:37	S	I think they would. It'd be three dimensional.
18:39	I	What makes you think that?
18:40	S	Just have the idea of sound, so that it'd be a big circle [makes circle with both hands] of particles essentially.
18:47	I	Mm hmm.

S self-directed to EM wave concept question

18:48	S	But that would make me think since these are all along the same path, [sweeps pencil up/down over 1,2,4] they would all, and 4 and, yeah, 2's gonna be the same place as 1 and 4 as well, so they're all gonnafit in essentially into this grouping of particles in that case. [sweeps pencil up/down over dense particles]

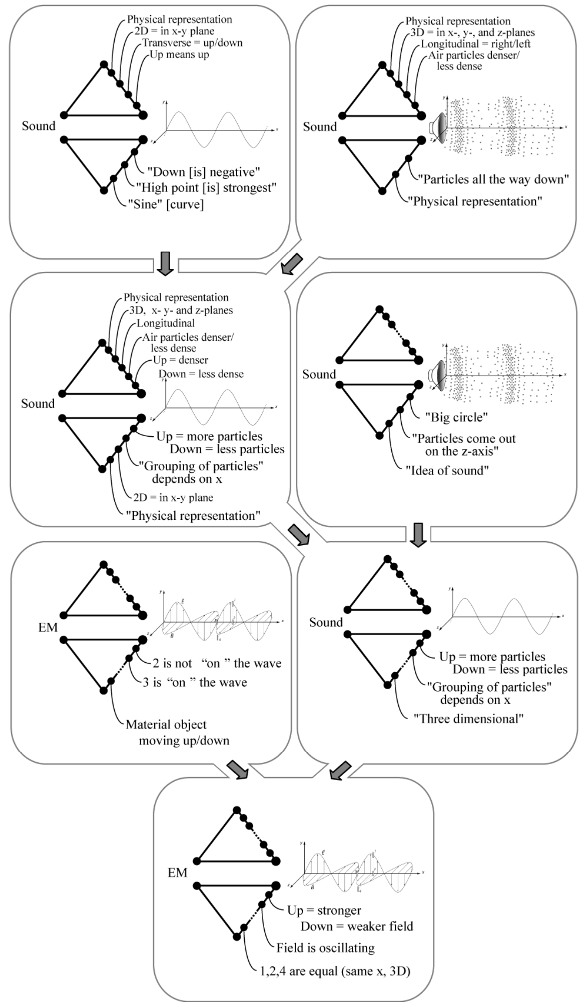


FIGURE 8.4 Transcript (left) and Analogical Scaffolding analysis (right) for selected portions of transcript. In left table, timestamps in column 1; interviewer (I) and student (S) in column 2. Italics in column 3 indicate quotations used to code schema elements (repeated in quotations in the diagram on the right for the lower triangles). For instance, the two sound spaces on the top (with sine wave and air particles signs) blend as shown by the arrows. A blend space can then become one of two inputs for another layered blend. The EM wave input space (3rd row from top, left) comes from a blend with a wave on a string earlier in the interview. Ellipses have been used between nodes to simplify the diagrams, and indicate additional nodes that may be present from prior blends. The community maps in the initial blend (top) are the same as those in FIGURE 8.2

* * *

What does it mean to *know* a concept? S did not articulate significant knowledge of string or sound waves nor did he apply theses analogs productively to EM waves when these analogies were cued verbally. Presented with pictorial signs he did both. Did these signs cue schemata that S already had but simply did not articulate at first, or did S create new schemata during the blending process? We take a pragmatic position, framing concepts as observable through the reification of students' talk, gesture, and interactions with the environment. Our observations suggest that S's string, sound, and EM wave concepts changed dramatically under different conditions. The Analogical Scaffolding model captures this observed coupling between sign, schema, and referent. For S, concepts appear to depend strongly on the salience of sign-schema associations – for instance, he did not know to associate a sine wave with a 3D sound wave, nor did he know to apply a sound wave blend to EM waves, without the signs in FIGURE 8.4. These associations became salient as S layered blend upon blend. We therefore argue that including signs as *part of* concepts can be extremely productive for understanding the dynamics of student learning. Furthermore, Analogical Scaffolding can be a useful tool for studying these dynamics.

At the same time we can establish the target goal of instruction as well as the community consensus on particular concepts (i.e., the community map), and how these are associated with particular representations. Applying the extended Analogical Scaffolding moves research on analogy in the following directions. (1)

We can now better understand how to unpack the blends of the community map. To be sure, unpacking and describing the community maps for a range of physics topics will require some effort on the part of researchers. Analogical Scaffolding specifies the structures and signals to look for in aggregate student responses within a theoretical framework. We use this framework to describe community maps, student schemata, and the relationship between these in the learning process. (2) We can establish a consensus on "abstraction" in the physics education research community. This consensus on abstraction will benefit researchers by operationalizing abstraction, moving towards a better understanding of why some ideas are difficult for students, and how these difficult ideas may be taught via a series of ideas that are more concrete to students. The extended Analogical Scaffolding model specifies what it means for ideas to be abstract vs. concrete to physicists, and how these physics ideas may couple to more or less salient ideas for students. (3) Ultimately, unpacking richly blended physics ideas will help the physics education research community understand and build curricula to help students develop their own understandings of abstract ideas in physics. Physics education research appears to be moving towards curriculum design that is not only empirically grounded, but also informed by high resolution cognitive models of how students learn specific ideas in physics. Analogical Scaffolding is an effort to develop one such high resolution model, specifying how particular representations can be used to coordinate student schemata with the community map of physics.

Chapter 9 – Conclusion

"The important thing is not to stop questioning."

-Albert Einstein[96]

One of the things that models do for scientists is to frame research questions, to guide inquiry, and to specify the patterns that scientists try to observe. The standard model of analogy, a mapping from a base domain to a target domain, serves these purposes to some extent, and, with such a model in mind, the range of analogies used in physics is considerable. Analogical Scaffolding extends prior models, specifying mechanisms, such as blending and layering meaning of representations, in order to point scientists to new types of research questions. How do students use analogies and how are the tools (e.g., representations and analogies) that scientists use interpreted and used by students? What sorts of environmental factors should we look for to understand student analogy use? How do students know which analogies to use, and, moreover, how do students know if and what they know?

These are not easy questions, and some may seem paradoxical: how does the student know what mappings to make in an analogy if they do not have sufficient knowledge of the target *a priori*? (Not to mention knowledge of the base domain.) Recall the quote by Socrates mid-way through Chapter 2, but note that Socrates did *not* think this paradox should be ultimately intractable (lest people would never learn anything). There must be ways around this circular logic trap, but the solutions may not be obvious, and, in fact, many of my observations of student reasoning with analogies seem to substantiate the paradox. Student interviews, such as the example analyzed in the last chapter, reveal that simply suggesting an analogy to students (e.g., stating "an EM wave is like a wave on a string") does not generally enhance these students' reasoning in any apparent way about the target domain. I have, however, shown that Analogical Scaffolding can lead to effective ways of promoting the productive use of analogy by students.

The empirical studies described in previous chapters demonstrated the need for and utility of Analogical Scaffolding. Students demonstrated significantly greater learning gains when taught about EM waves with multiple (vs. no) analogies and with multiple (vs. single) representations. These findings called for an explanatory model, and based on student interviews, classroom observations, and large-scale (N>100) studies, I developed the Analogical Scaffolding model. This model was validated through a series of further experimental studies. The representational forms used to teach students played significant roles in students' learning with analogies, and I have shown that Analogical Scaffolding had substantial explanatory and predictive power in these studies, which focused on EM waves. Future studies of this kind could use Analogical Scaffolding to unpack and re-blend other abstract ideas in physics. I foresee applications in advanced topics such as wave functions and potential well diagrams in quantum mechanics, or introductory topics such as acceleration vs. time graphs, or free-body diagrams. There are, of course, many others.

Earlier, I said that I would focus on students' analogical competence rather than *meta*-analogical competence. However, meta-analogical competence is a very interesting topic, and one that I would like to see explored. My research has shown that which analogies students use can be influenced by representations, but these students may or may not have been consciously aware of this influence. A *meta*-question is this: can curricular materials be designed that enhance students' awareness of how, when, and why they are using analogies? After all, if one of the key skills of practicing scientists is the productive use of analogy, shouldn't this be considered as one (of many) important goals of instruction? Analogical reasoning may be part of the "hidden curriculum"[49] of physics courses, and it may prove necessary to make the teaching of this skill explicit. Analogical Scaffolding may be a useful model for developing such curricular materials for teaching analogical reasoning skills, the ability to recognize analogies, and students' abilities to develop and critique their own use of analogies. Future efforts of this kind would certainly contribute to the larger effort on the part of physics education researchers to better understand what students know, what students learn, and how students learn in physics courses. In carrying out my research, I hope to have contributed in some significant way to this larger effort.

Appendix A. Materials Used in Studies*

I. Chapter 3 Studies
   a. Spring 2005 – Wave on a String Analogy Tutorial
   b. Spring 2005 – Sound Waves Analogy Tutorial
   c. Spring 2005 – No Analogy Tutorial
   d. Fall 2005 – Wave on a String Analogy Tutorial
   e. Fall 2005 – Sound Waves Analogy Tutorial
   f. Fall 2005 – No Analogy Tutorial
   g. Spring/Fall 2005 – String Representation Assessment
   h. Spring/Fall 2005 – Sound Representation Assessment

II. Chapter 6 Studies
   a. Analogy Tutorial
   b. No-analogy Tutorial

III. Chapter 7
   a. Abstract Tutorial
   b. Concrete Tutorial
   c. Blend Tutorial
   d. Abstract Sound Waves Quiz
   e. Concrete Sound Waves Quiz
   f. Blend Sound Waves Quiz

IV. Chapter 8 – Wave on a String and Sound Wave Representations from Student Interview

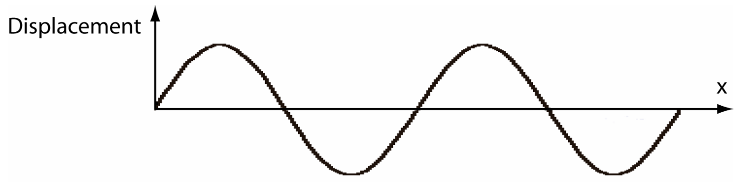
*The materials presented here have been rescaled to retain their original formatting.

Spring 2005 – Wave on a String Analogy Tutorial

Name: ___________________________ **Student ID:** _____________________________

Strings and Radio Waves

1. Open the *Wave on a String* application by double clicking the icon. Play around with the

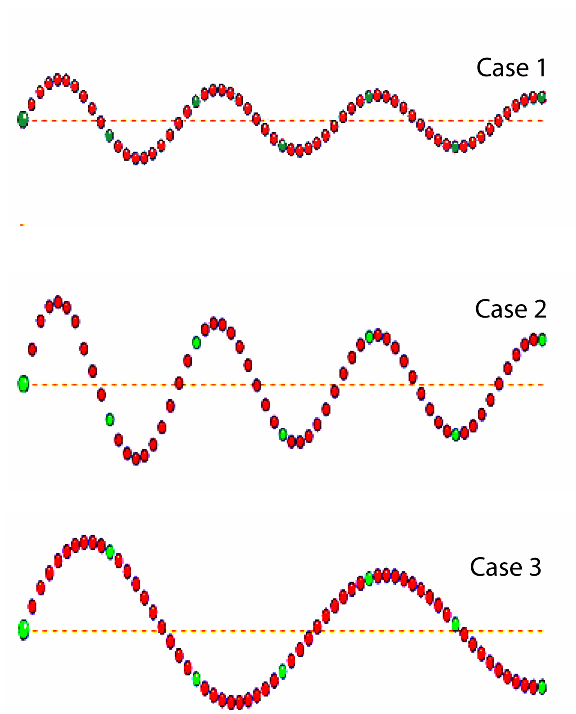


simulation for a minute. Try moving the sliders and see what changes in the simulation.

   a. Physicists often describe a vibrating string as a *displacement wave*, as in the diagram above. Describe how this wave corresponds to what you see in the simulation.

   b. Justify the term *displacement wave* for this vibrating string. Explain in terms of the motion of the string.

2. Three vibrating strings are represented at right; all have the same tension. The diagrams are drawn to the same scale.



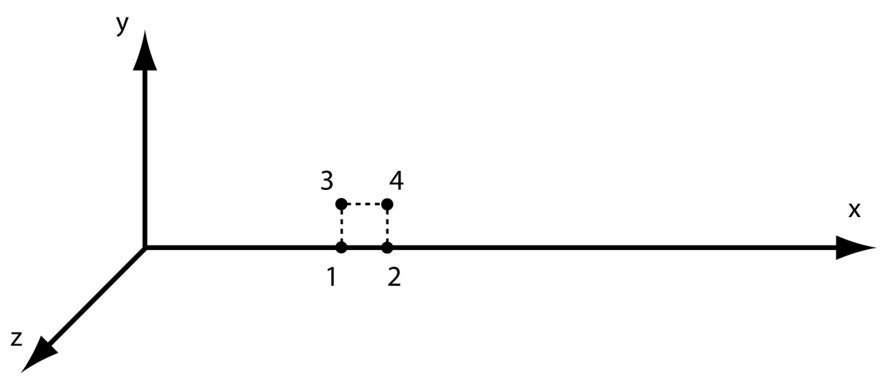
   a. How is the wave in case 1 different from the wave in case 2? Explain how you can tell from the diagrams.

   b. If the wave in case 2 was 50 Hz, could the wave in case 3 be 30 Hz or 70 Hz? Explain

Electromagnetic Waves

In this tutorial you will use the *Radio Waves* simulation. Before moving on, get familiar with the simulation by playing around with the simulation for a minute. Try viewing the wave as a *curve with vectors* or as a *full field* (use the buttons on the right panel). Note that the simulation only shows the *electric field* part of the radio wave.

I. Representations of electromagnetic waves



Imagine an electromagnetic wave is traveling *towards the right* along the x-axis on the graph above.

1. Draw the *electric field* part of the electromagnetic wave on the graph above. Draw the *magnetic field* part of the wave on the same graph.

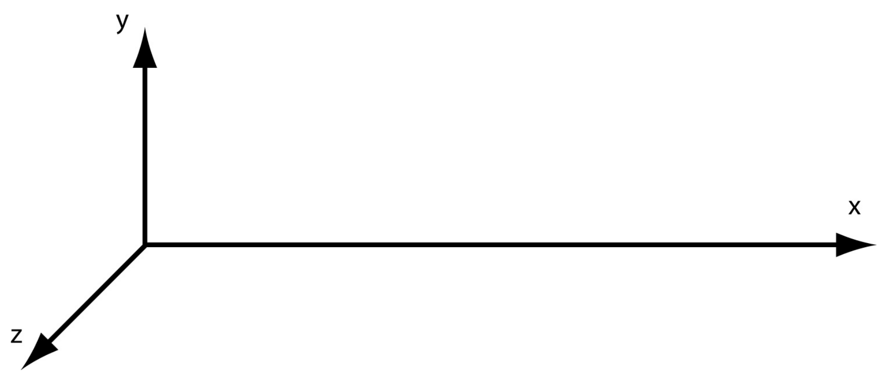
   Is the direction of the magnetic field that you drew consistent with the direction of propagation of the wave? Explain.

2. The points 1-4 in the diagram above lie in the x-y plane.

   For the instant shown, rank these points according to the magnitude of the *electric field*.

   For the instant shown, rank these points according to the magnitude of the *magnetic field*.

3. The axes of the graph below have the same scale as the graph in part I. On the graph below, draw an electromagnetic wave with a *smaller amplitude* than the wave you drew in part I. Explain.



4. On the graph to the right, draw the electric and magnetic fields of an electromagnetic wave that is propagating *to the left*. Explain.

5. Suppose the electromagnetic wave you drew in #4 was green light. On the graph to the right, draw a wave that could be *blue light*. Explain.

II. Detecting Electromagnetic Waves

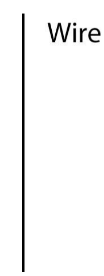
1. Write an expression for the force exerted on an electron (with charge *e*) by (1) an electric field, ***E***, and (2) a magnetic field, ***B***.

   If an electric field and a magnetic field were both present, would a force be exerted on the charge even if the charge were initially not moving? Explain.

2. Suppose a long, thin conducting wire (see figure at right) is placed in the path of a radio wave as shown in the *Radio Waves* simulation (the electric field is in the x-y plane).

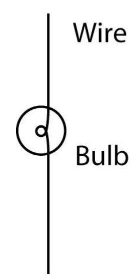
   a. Suppose that the wire were oriented parallel to the y-axis.

   As the wave propagates past the wire, would the *electric field* due to the radio wave cause the charges in the wire to move? If so, would the charges move in a direction along the length of the wire? Explain.



   As the wave propagates past the wire, would the *magnetic field* due to the wave cause the charges in the wire to move in a direction along the length of the wire? Explain.

   b. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)



   If the wire were placed in the path of the radio wave and oriented parallel to the y-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

   How, if at all, would you answer change if the wire were oriented:

   - parallel to the *z*-axis? Explain.

   - parallel to the *x*-axis? Explain.

III. Wave Representations

Both vibrating strings and radio signals can be described as waves.

1. Why do you think physicists use the word *wave* to describe these phenomena?

2. How is a radio wave like a wave on a string?

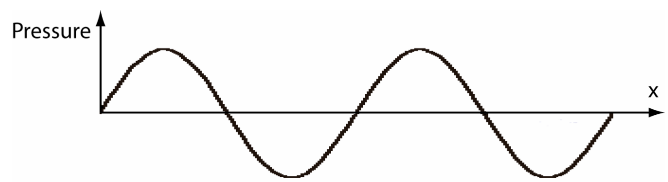
3. How are the two waves different?

<u>Spring 2005 – Sound Waves Analogy Tutorial</u>

Name: ___________________________ Student ID: _____________________________

Sound Waves

3. Open the *SoundWaves* application by double clicking the icon. Play around with the simulation for

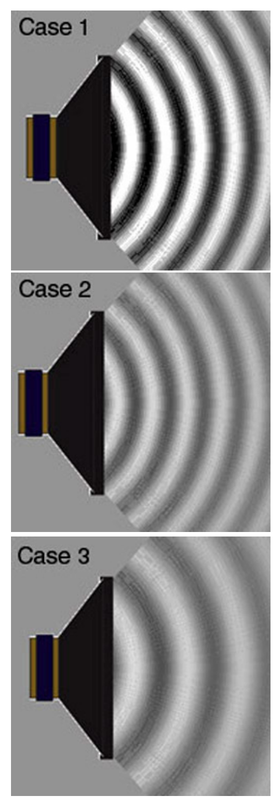


a minute. Try moving the sliders and see what changes in the simulation.

   a. Physicists often describe sound as a *pressure wave*, as in the diagram above. Describe how this wave corresponds to what you see in the simulation.

   b. Justify the term *pressure wave* for this sound wave. Explain in terms of the motion of the air.

4. Three sound waves are represented at right. The diagrams are drawn to the same scale.



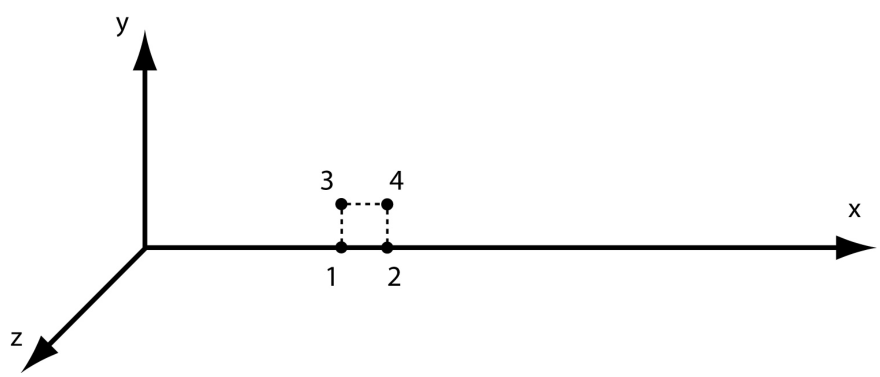
   a. How is the wave in case 1 different from the wave in case 2? Explain how you can tell from the diagrams.

   b. If the wave in case 2 was 500 Hz, could the wave in case 3 be 400 Hz or 600 Hz? Explain.

Electromagnetic Waves

In this tutorial you will use the *Radio Waves* simulation. Before moving on, get familiar with the simulation by playing around with the simulation for a minute. Try viewing the wave as a *curve with vectors* or as a *full field* (use the buttons on the right panel). Note that the simulation only shows the *electric field* part of the radio wave.

I. Representations of electromagnetic waves



Imagine an electromagnetic wave is traveling *towards the right* along the x-axis on the graph above.

6. Draw the *electric field* part of the electromagnetic wave on the graph above. Draw the *magnetic field* part of the wave on the same graph.

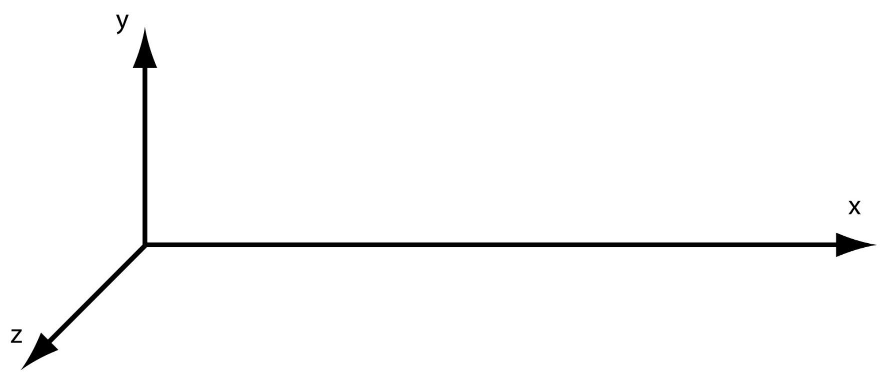
   Is the direction of the magnetic field that you drew consistent with the direction of propagation of the wave? Explain.

7. The points 1-4 in the diagram above lie in the x-y plane.

   For the instant shown, rank these points according to the magnitude of the *electric field*.

   For the instant shown, rank these points according to the magnitude of the *magnetic field*.

8. The axes of the graph below have the same scale as the graph in part I. On the graph below, draw an electromagnetic wave with a *smaller amplitude* than the wave you drew in part I. Explain.



9. On the graph to the right, draw the electric and magnetic fields of an electromagnetic wave that is propagating *to the left*. Explain.

10. Suppose the electromagnetic wave you drew in #4 was green light. On the graph to the right, draw a wave that could be *blue light*. Explain.

II. Detecting Electromagnetic Waves

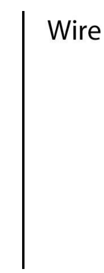
3. Write an expression for the force exerted on an electron (with charge *e*) by (1) an electric field, ***E***, and (2) a magnetic field, ***B***.

   If an electric field and a magnetic field were both present, would a force be exerted on the charge even if the charge were initially not moving? Explain.

4. Suppose a long, thin conducting wire (see figure at right) is placed in the path of a radio wave as shown in the *Radio Waves* simulation (the electric field is in the x-y plane).

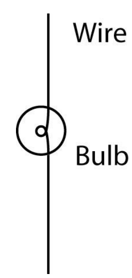
   a. Suppose that the wire were oriented parallel to the y-axis.

   As the wave propagates past the wire, would the *electric field* due to the radio wave cause the charges in the wire to move? If so, would the charges move in a direction along the length of the wire? Explain.



   As the wave propagates past the wire, would the *magnetic field* due to the wave cause the charges in the wire to move in a direction along the length of the wire? Explain.

   b. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)



   If the wire were placed in the path of the radio wave and oriented parallel to the y-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

   How, if at all, would you answer change if the wire were oriented:

   - parallel to the *z*-axis? Explain.

   - parallel to the *x*-axis? Explain.

III. Wave Representations

Both sound and radio signals can be described as waves.

4. Why do you think physicists use the word *wave* to describe these phenomena?

5. How is a radio wave like a sound wave?

6. How are the two waves different?

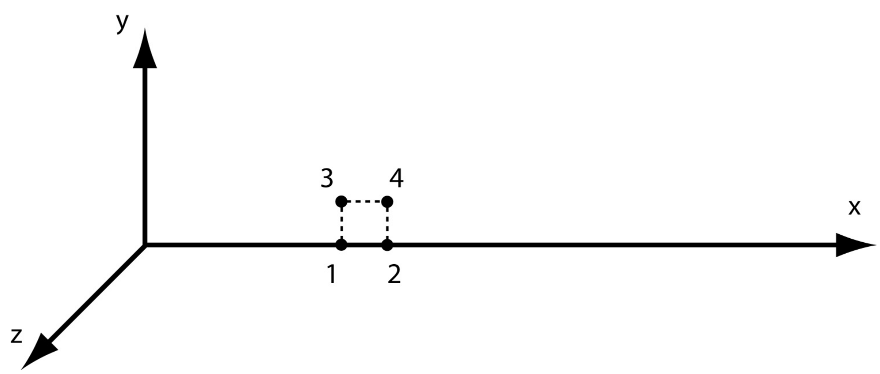
Spring 2005 – No-analogy Tutorial

Name: ___________________________ Student ID: _____________________________

<u>Chapter 10</u> Electromagnetic Waves

In this tutorial you will use the *Radio Waves* simulation. Before moving on, get familiar with the simulation by playing around with the simulation for a minute. Try viewing the wave as a *curve with vectors* or as a *full field* (use the buttons on the right panel). Note that the simulation only shows the *electric field* part of the radio wave.

I. Representations of electromagnetic waves



Imagine an electromagnetic wave is traveling *towards the right* along the x-axis on the graph above.

11. Draw the *electric field* part of the electromagnetic wave on the graph above. Draw the *magnetic field* part of the wave on the same graph.

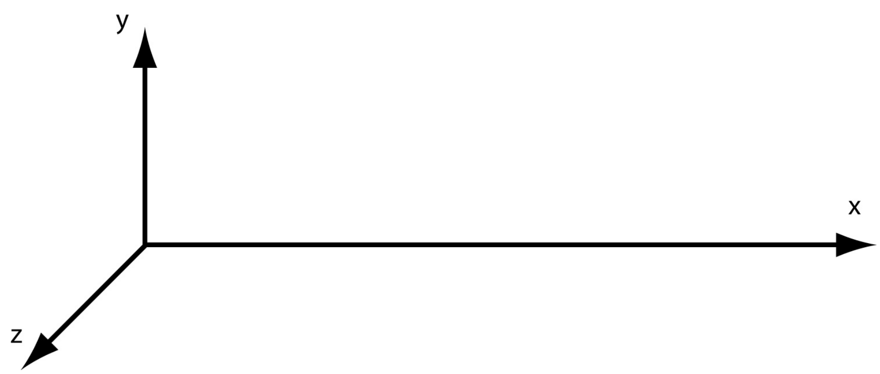
    Is the direction of the magnetic field that you drew consistent with the direction of propagation of the wave? Explain.

12. The points 1-4 in the diagram above lie in the x-y plane.

    For the instant shown, rank these points according to the magnitude of the *electric field*.

    For the instant shown, rank these points according to the magnitude of the *magnetic field*.

13. The axes of the graph below have the same scale as the graph in part I. On the graph below, draw an electromagnetic wave with a *smaller amplitude* than the wave you drew in part I. Explain.



14. On the graph to the right, draw the electric and magnetic fields of an electromagnetic wave that is propagating *to the left*. Explain.

15. Suppose the electromagnetic wave you drew in #4 was green light. On the graph to the right, draw a wave that could be *blue light*. Explain.

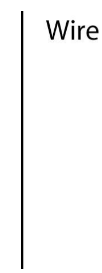
II. Detecting Electromagnetic Waves

5. Write an expression for the force exerted on an electron (with charge *e*) by (1) an electric field, ***E***, and (2) a magnetic field, ***B***.

   If an electric field and a magnetic field were both present, would a force be exerted on the charge even if the charge were initially not moving? Explain.

6. Suppose a long, thin conducting wire (see figure at right) is placed in the path of a radio wave as shown in the *Radio Waves* simulation (the electric field is in the x-y plane).

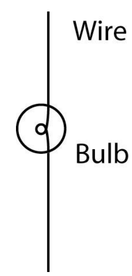
   a. Suppose that the wire were oriented parallel to the y-axis.

      As the wave propagates past the wire, would the *electric field* due to the radio wave cause the charges in the wire to move? If so, would the charges move in a direction along the length of the wire? Explain.

      As the wave propagates past the wire, would the *magnetic field* due to the wave cause the charges in the wire to move in a direction along the length of the wire? Explain.

   b. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)

      If the wire were placed in the path of the radio wave and oriented parallel to the y-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

      How, if at all, would you answer change if the wire were oriented:

      - parallel to the *z*-axis? Explain.

      - parallel to the *x*-axis? Explain.

III. Wave Representations

7. Why do you think physicists use the word *wave* to describe radio signals?

8. Is a radio wave like any other wave you are familiar with? Explain.

   How are the waves similar?

   How are the waves different?

Fall 2005 – Wave on a String Analogy Tutorial

Name ___________________________ SID___________________________

Please turn this tutorial in to your TA. You will get it back at the next recitation.

Introduction

In this activity, you will learn about two types of waves: waves on a string and electromagnetic waves. As you work though the activity, think about how the word "wave" can be applied to each phenomenon.

Electromagnetic waves are often approximated as *plane waves*. In a plane wave, the *wavefronts* propagate in *planes* and the *amplitude* of the wave does not change as the wave propagates forward. Light from the sun and radio waves are examples of electromagnetic waves. You probably know that light from the sun or radio waves from a broadcast antenna gets weaker as you move far away from the source. However, if you only move a small distance (a few meters) the strength of the wave does not change very much. That is *why* we use the approximation of a plane wave.

What is the range of wavelengths for *visible light*?

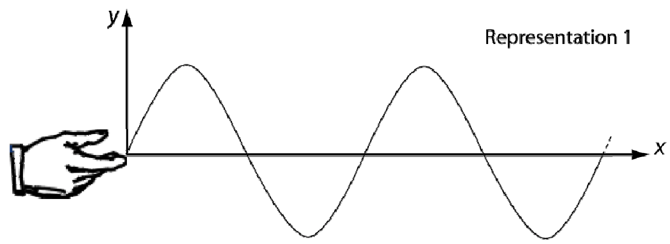
If you moved around over a distance of a few wavelengths of visible light, would you notice any difference in the *brightness* of the light?

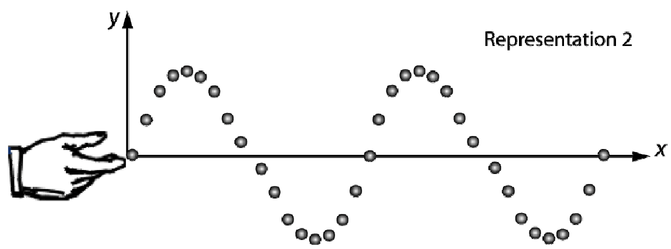
In the space below, draw a picture showing why a *plane wave* is a good way to approximate the light waves coming from the sun.

Waves on a String

A. Shown below are mathematical and pictorial representations of a wave on a string. Representation 1 uses a *sine wave* to represent the string at one instant in time. Representation 2 shows the arrangement of string segments (beads) at this instant in time. The letter *D* stands for *displacement*.

$$D(x,t) = D_o \sin(kx - \omega t)$$





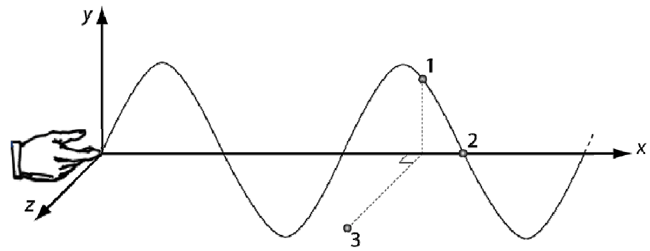
1. On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2. How does representation 1 relate to representation 2? What do the *peaks* and *troughs* of the wave in representation 1 indicate?

2. Is this wave transverse or longitudinal? How can you tell from the diagrams above?

3. A wave on a string is sometimes called a *displacement* wave. Use the representations above to explain why this term makes sense.

B. In the figure below, a wave propagates along a string in the +x direction. There are beads at locations 1, 2, and 3. Beads 1 and 2 are part of the string, while bead 3 sits unattached in the *x-z* plane (out of the page).

Compare the *displacement* of each bead as the wave propagates. (*Hint:* if you were wiggling the string, how would you see each bead move?)



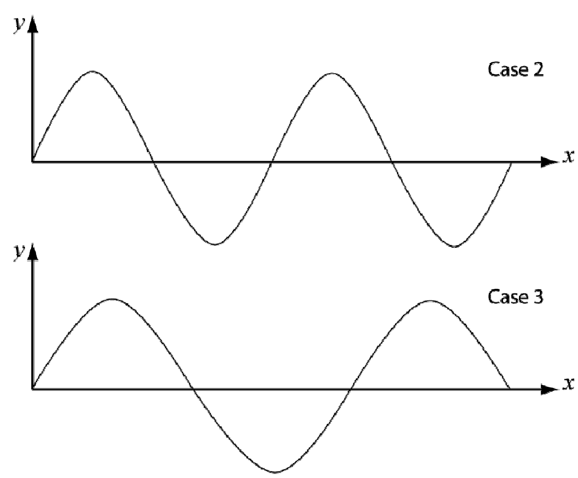
Based on your answer, what do you think the *sine wave* representation of the wave on the string is intended to show?

C. Three waves on strings are represented below. The diagrams are drawn to the same scale.

1. Compare the *amplitude* of the wave in Case 1 to that of Case 2. Explain how you can tell from the diagrams.

2. Compare the *wavelength* of the wave in Case 2 to that of Case 3. Explain how you can tell from the diagrams.

3. Compare the *frequency* of the wave in Case 2 to that of Case 3. Explain how you figured this out.

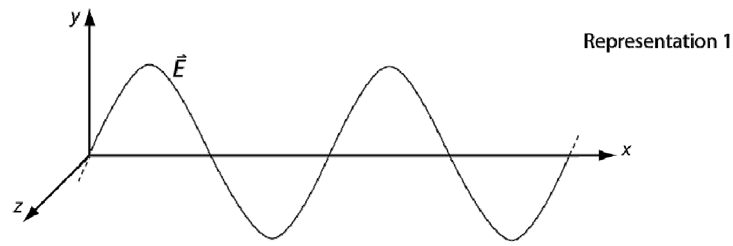


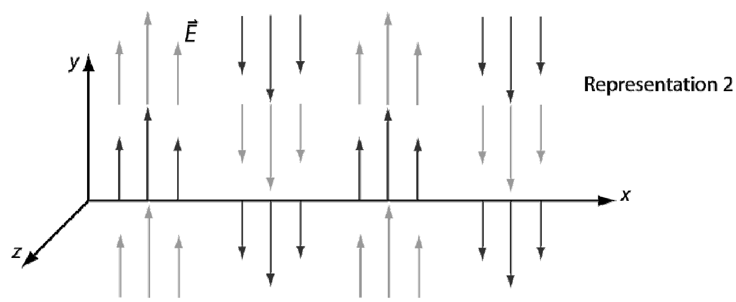
<u>Electromagnetic Waves</u>

I. Representations of electromagnetic waves

1. Shown below are mathematical and pictorial representations of an electromagnetic plane wave propagating through empty space. Representation 1 uses a *sine wave* to represent the electromagnetic wave at one instant in time; representation 2 uses a *vector field* to represent the wave at that instant.

$$\vec{E}(x,y,z,t) = E_o \sin(kx - \omega t)\hat{y}$$





1. On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2. How does representation 1 relate to representation 2? What do the *peaks* and *troughs* of the wave in representation 1 indicate?

2. Is this wave transverse or longitudinal? How can you tell from the diagrams above?

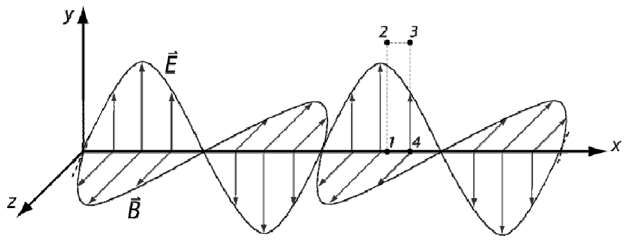
3. The electromagnetic wave shown above is called a *plane wave*. Using the representations above, explain why this term makes sense.

4. How is this electromagnetic wave similar to a wave on a string? How are the two types of wave different? It may help to refer to your answers to questions 1 and 2 above.

2. An electromagnetic wave is often represented as in the figure below. This figure is a combination of representations 1 and 2 from part A. The electromagnetic plane wave propagates to the right. The electric field, $\vec{E}$, is parallel to the *y*-axis; the magnetic field, $\vec{B}$, is parallel to the *z*-axis.

Four points in space (labeled 1,2, 3 and 4) lie in the *x-y* plane.

Compare the magnitude of the *electric field* at each of the four points. (*Hint:* use an analogy to wave on a string.)

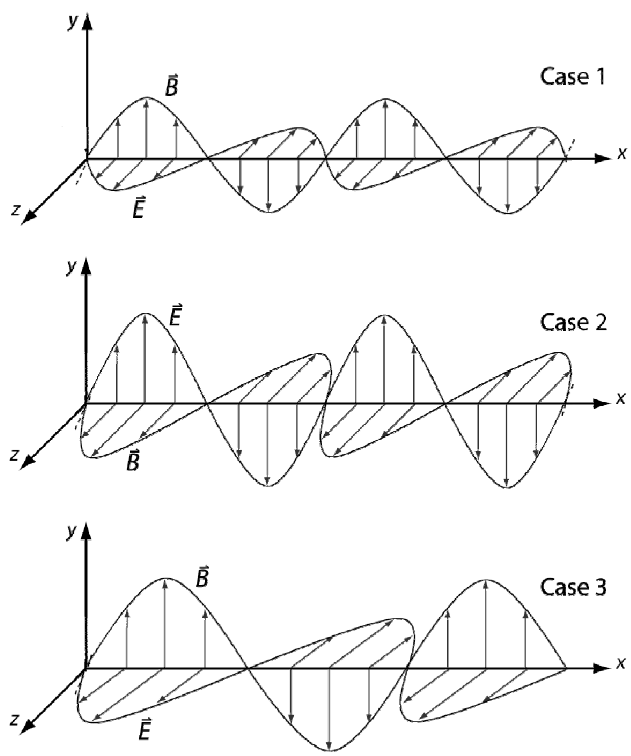


Compare the magnitude of the *magnetic field* at each of the four points.

3. Three light waves are represented at right. The diagrams are drawn to the same scale.

    1. How is the wave in case 1 different from the wave in case 2? Explain how you can tell from the diagrams.

    2. If the wave in case 2 were green light, could the wave in case 3 be *red light* or *blue light*? Explain.

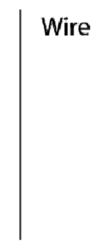


II. Detecting electromagnetic waves

A. Write an expression for the force exerted on a charge, q, by (1) and electric field, $\vec{E}$, and (2) a magnetic field, $\vec{B}$.

If an electric field and a magnetic field were both present, would a force be exerted on the charge even if the charge were initially not moving? Explain.

B. Imagine that the electromagnetic wave *in section I, part A,* is a radio wave. A long, thin conducting wire (see figure at right) is placed in the path of the wave.

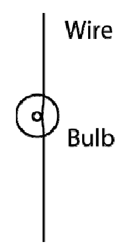


1. Suppose that the wire were oriented parallel to the y-axis.

   As the wave propagates past the wire, would the *electric field* due to the radio wave cause the charges in the wire to move? If so, would the charges move in a direction along the length of the wire? Explain.

   As the wave propagates past the wire, would the *magnetic field* due to the wave cause the charges in the wire to move in a direction along the length of the wire? Explain.

2. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)



   If the wire were placed in the path of the radio wave and oriented parallel to the y-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

How, if at all, would your answer change if the wire were oriented:

- parallel to the x-axis? Explain.

- parallel to the z-axis? Explain.

Fall 2005 – Sound Waves Analogy Tutorial

Name ___________________________ SID___________________________

Please turn this tutorial in to your TA. You will get it back at the next recitation.

Introduction

In this activity, you will learn about two types of waves: sound waves and electromagnetic waves. As you work though the activity, think about how the word "wave" can be applied to each phenomenon.

Electromagnetic waves are often approximated as *plane waves*. In a plane wave, the *wavefronts* propagate in *planes* and the *amplitude* of the wave does not change as the wave propagates forward. Light from the sun and radio waves are examples of electromagnetic waves. You probably know that light from the sun or radio waves from a broadcast antenna gets weaker as you move far away from the source. However, if you only move a small distance (a few meters) the strength of the wave does not change very much. That is *why* we use the approximation of a plane wave.

What is the range of wavelengths for *visible light*?

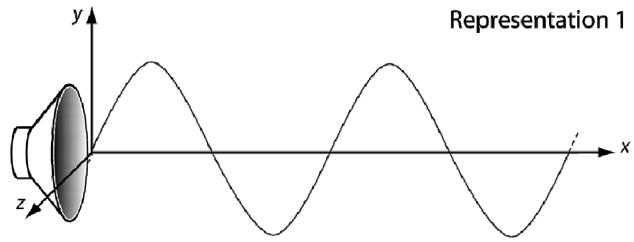
If you moved around over a distance of a few wavelengths of visible light, would you notice any difference in the *brightness* of the light?

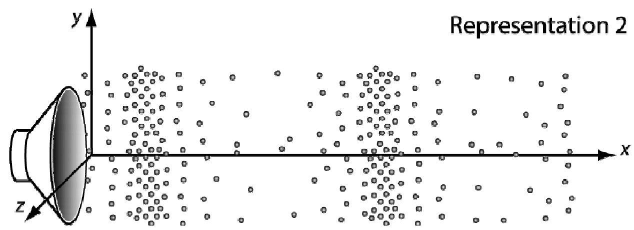
In the space below, draw a picture showing why a *plane wave* is a good way to approximate the light waves coming from the sun.

Sound Waves

A. Shown below are mathematical and pictorial representations of a sound wave. Representation 1 uses a *sine wave* to represent the sound at one instant in time. Representation 2 shows the arrangement of air particles at this instant in time. The letter *P* stands for *pressure*.

$$P(x,y,z,t) = P_o \sin(kx - \omega t)$$





1. On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2. How does representation 1 relate to representation 2? What do the *peaks* and *troughs* of the wave in representation 1 indicate?

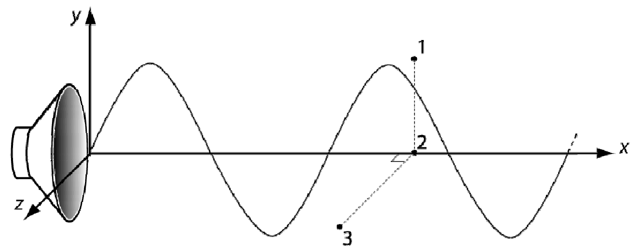
2. Is this wave transverse or longitudinal? How can you tell from the diagrams above?

3. A sound wave is sometimes called a *pressure* wave. Use the representations above to explain why this term makes sense.

4. Suppose this sheet of paper is the *x-y* plane. If you were standing somewhere out of the page (off of the *x-y* plane in the *+z* direction), would you be able to hear the sound?

B. In the figure below, a sound wave propagates in the +*x*-direction. Three points in space are labeled 1,2, and 3. Point 1 and 2 lie in the *x*-*y* plane; point 3 lies in the *x*-*z* plane (coming out of the page).

Compare the sound *pressure* at each of the three points. (*Hint:* if you were standing near each point, what would you hear?)



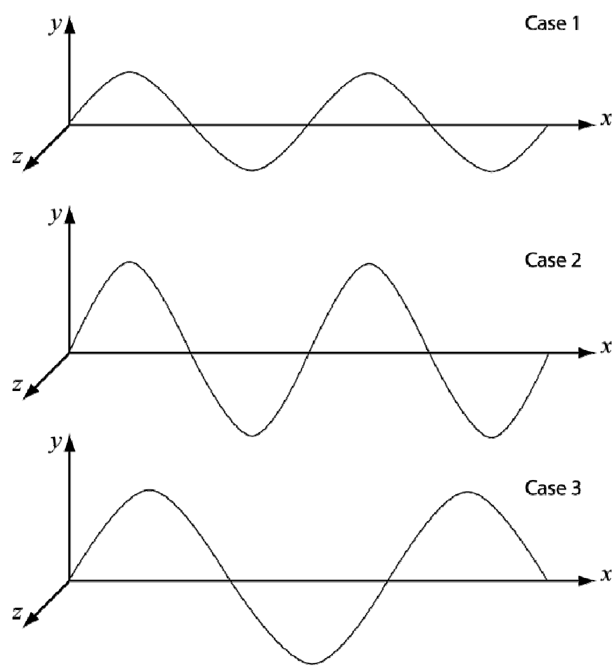
Based on your answer, what do you think the *sine wave* representation of the sound is intended to show?

C. Three sound waves are represented below. The diagrams are drawn to the same scale.

1. Compare the *amplitude* of the wave in Case 1 to that of Case 2. Explain how you can tell from the diagrams.

2. Compare the *wavelength* of the wave in Case 2 to that of Case 3. Explain how you can tell from the diagrams.

3. Compare the *frequency* of the wave in Case 2 to that of Case 3. Explain how you figured this out.

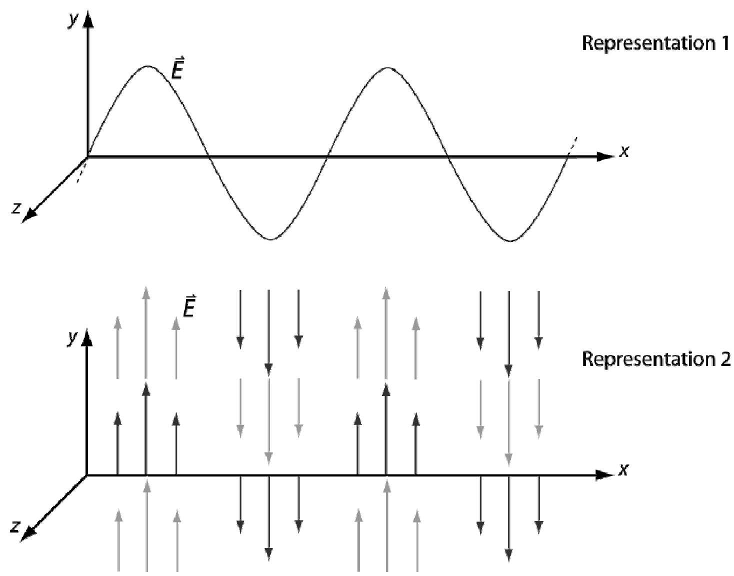


Electromagnetic Waves

I. Representations of electromagnetic waves

A. Shown below are mathematical and pictorial representations of an electromagnetic plane wave propagating through empty space. Representation 1 uses a *sine wave* to represent the electromagnetic wave at one instant in time; representation 2 uses a *vector field* to represent the wave at that instant.

$$\vec{E}(x,y,z,t) = E_o \sin(kx - \omega t)\,\hat{y}$$



1. On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2. How does representation 1 relate to representation 2? What do the *peaks* and *troughs* of the wave in representation 1 indicate?

2. Is this wave transverse or longitudinal? How can you tell from the diagrams above?

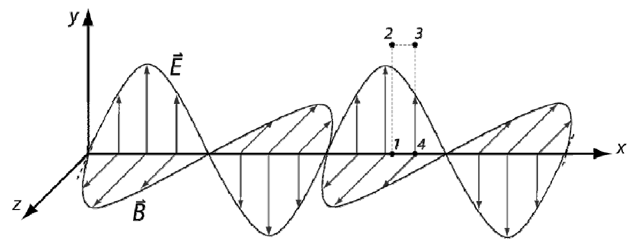
3. The electromagnetic wave shown above is called a *plane wave*. Using the representations above, explain why this term makes sense.

4. How is this electromagnetic wave similar to a sound wave? How are the two types of wave different? It may help to refer to your answers to questions 1 and 2 above.

B. An electromagnetic wave is often represented as in the figure below. This figure is a combination of representations 1 and 2 from part A. The electromagnetic plane wave propagates to the right. The electric field, $\vec{E}$, is parallel to the *y*-axis; the magnetic field, $\vec{B}$, is parallel to the *z*-axis.

Four points in space (labeled 1,2, 3and 4) lie in the *x*-*y* plane.

Compare the magnitude of the *electric field* at each of the four points. (*Hint:* use an analogy to a sound wave.)

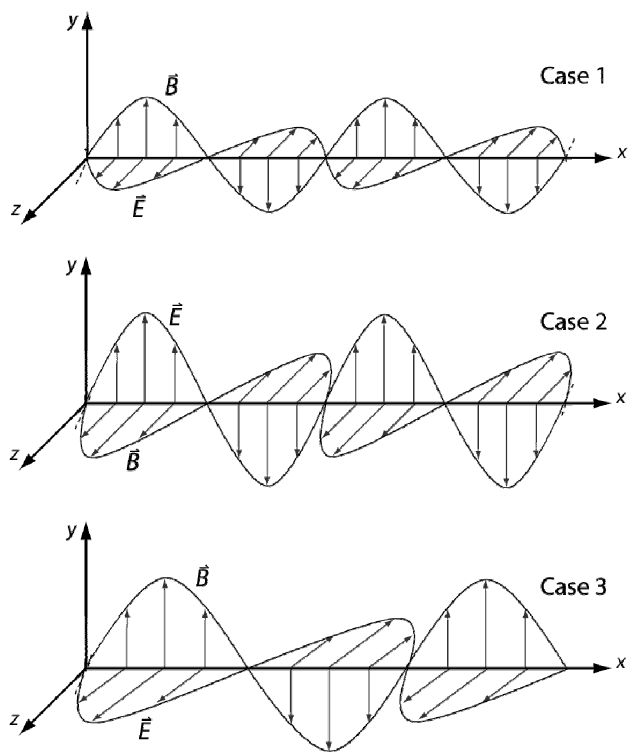


Compare the magnitude of the *magnetic field* at each of the four points.

C. Three light waves are represented at right. The diagrams are drawn to the same scale.

1. How is the wave in case 1 different from the wave in case 2? Explain how you can tell from the diagrams.

2. If the wave in case 2 were green light, could the wave in case 3 be *red light* or *blue light*? Explain.

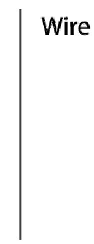


II. Detecting electromagnetic waves

A. Write an expression for the force exerted on a charge, q, by (1) and electric field, $\vec{E}$, and (2) a magnetic field, $\vec{B}$.

If an electric field and a magnetic field were both present, would a force be exerted on the charge even if the charge were initially not moving? Explain.

B. Imagine that the electromagnetic wave *in section I, part A,* is a radio wave. A long, thin conducting wire (see figure at right) is placed in the path of the wave.

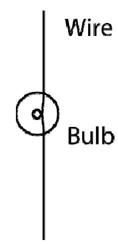


1. Suppose that the wire were oriented parallel to the y-axis.

   As the wave propagates past the wire, would the *electric field* due to the radio wave cause the charges in the wire to move? If so, would the charges move in a direction along the length of the wire? Explain.

   As the wave propagates past the wire, would the *magnetic field* due to the wave cause the charges in the wire to move in a direction along the length of the wire? Explain.

2. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)



   If the wire were placed in the path of the radio wave and oriented parallel to the y-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

How, if at all, would your answer change if the wire were oriented:

- parallel to the x-axis? Explain.

- parallel to the z-axis? Explain.

Fall 2005 – No-analogy Tutorial

Name ____________________________ SID____________________________

Please turn this tutorial in to your TA. You will get it back at the next recitation.

Introduction
In this activity, you will learn about electromagnetic waves. As you work though the activity, think about how the word "wave" can be applied to this phenomenon.

Electromagnetic waves are often approximated as *plane waves*. In a plane wave, the *wavefronts* propagate in *planes* and the *amplitude* of the wave does not change as the wave propagates forward. Light from the sun and radio waves are examples of electromagnetic waves. You probably know that light from the sun or radio waves from a broadcast antenna gets weaker as you move far away from the source. However, if you only move a small distance (a few meters) the strength of the wave does not change very much. That is *why* we use the approximation of a plane wave.

What is the range of wavelengths for *visible light*?

If you moved around over a distance of a few wavelengths of visible light, would you notice any difference in the *brightness* of the light?

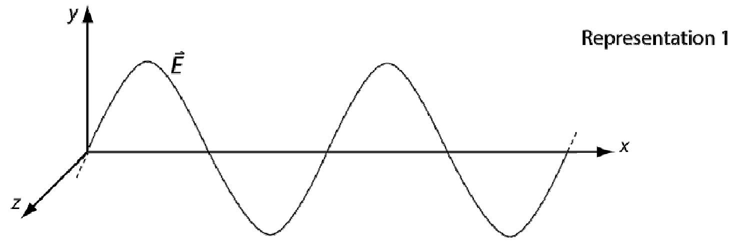
In the space below, draw a picture showing why a *plane wave* is a good way to approximate the light waves coming from the sun.

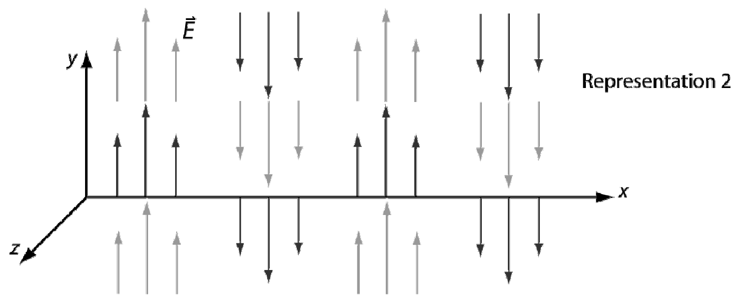
Electromagnetic Waves

III. Representations of electromagnetic waves

A. Shown below are mathematical and pictorial representations of an electromagnetic plane wave propagating through empty space. Representation 1 uses a *sine wave* to represent the electromagnetic wave at one instant in time; representation 2 uses a *vector field* to represent the wave at that instant.

$$\vec{E}(x, y, z, t) = E_o \sin(kx - \omega t)\,\hat{y}$$





1. On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2. How does representation 1 relate to representation 2? What do the *peaks* and *troughs* of the wave in representation 1 indicate?

2. Is this wave transverse or longitudinal? How can you tell from the diagrams above?

3. The electromagnetic wave shown above is called a *plane wave*. Using the representations above, explain why this term makes sense.

B. In the figure below, an electromagnetic wave propagates in the +*x*-direction. Three points in space are labeled 1,2, and 3. Point 1 and 2 lie in the *x*-*y* plane; point 3 lies in the *x*-*z* plane (coming out of the page).

Compare the magnitude of the *electric field* at each of the three points.

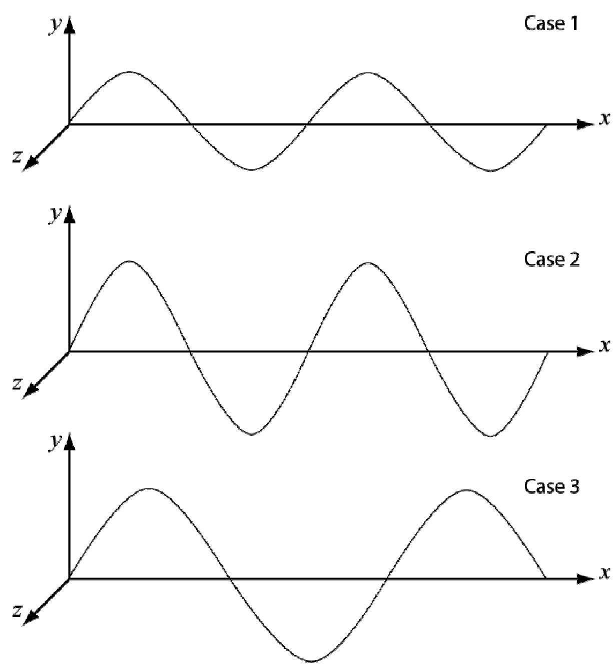
Based on your answer, what do you think the *sine wave* representation of the electromagnetic wave is intended to show?

C. Three electromagnetic waves are represented below. The diagrams are drawn to the same scale.

1. Compare the *amplitude* of the wave in Case 1 to that of Case 2. Explain how you can tell from the diagrams.

2. Compare the *wavelength* of the wave in Case 2 to that of Case 3. Explain how you can tell from the diagrams.

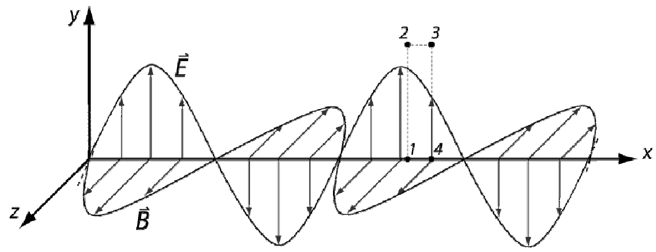
3. Compare the *frequency* of the wave in Case 2 to that of Case 3. Explain how you figured this out.



D. An electromagnetic wave is often represented as in the figure below. This figure is a combination of representations 1 and 2 from part A. The electromagnetic plane wave propagates to the right. The electric field, $\vec{E}$, is parallel to the *y*-axis; the magnetic field, $\vec{B}$, is parallel to the *z*-axis.

Four points in space (labeled 1,2, 3 and 4) lie in the *x*-*y* plane.

Compare the magnitude of the *electric field* at each of the four points.

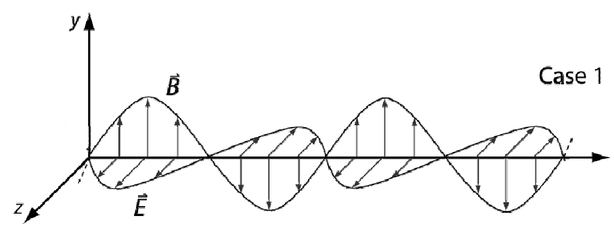


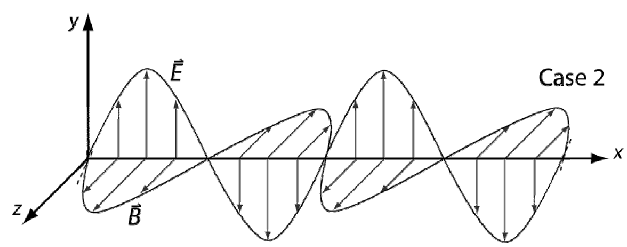
Compare the magnitude of the *magnetic field* at each of the four points.

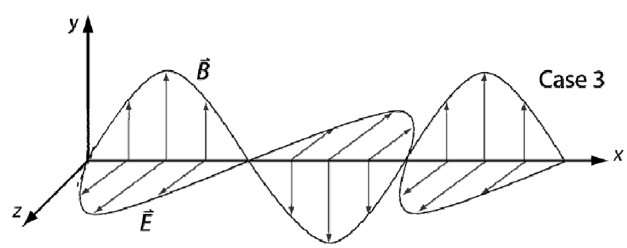
E. Three light waves are represented at right. The diagrams are drawn to the same scale.

1. How is the wave in case 1 different from the wave in case 2? Explain how you can tell from the diagrams.

2. If the wave in case 2 were green light, could the wave in case 3 be *red light* or *blue light*? Explain.





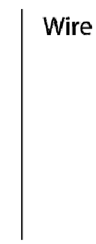


IV. Detecting electromagnetic waves

A. Write an expression for the force exerted on a charge, q, by (1) and electric field, $\vec{E}$, and (2) a magnetic field, $\vec{B}$.

If an electric field and a magnetic field were both present, would a force be exerted on the charge even if the charge were initially not moving? Explain.

B. Imagine that the electromagnetic wave *in section I, part A,* is a radio wave. A long, thin conducting wire (see figure at right) is placed in the path of the wave.

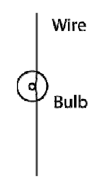


1. Suppose that the wire were oriented parallel to the y-axis.

   As the wave propagates past the wire, would the *electric field* due to the radio wave cause the charges in the wire to move? If so, would the charges move in a direction along the length of the wire? Explain.

   As the wave propagates past the wire, would the *magnetic field* due to the wave cause the charges in the wire to move in a direction along the length of the wire? Explain.

2. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)



   If the wire were placed in the path of the radio wave and oriented parallel to the y-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

How, if at all, would your answer change if the wire were oriented:

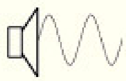
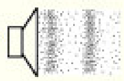
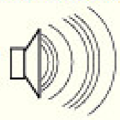
- parallel to the x-axis? Explain.

- parallel to the z-axis? Explain.

Spring/Fall 2005 – Sound Representation Assessment

A dust particle is located in front of a speaker. The speaker moves back and forth at a constant rate, creating a sound wave.

1. Shown below are several ways to represent the sound wave.
   Which one makes the most sense to you? (*Note: the choice is up to you - there is no "wrong" answer.*)

   ○  ○  ○  ◉ (not answered)

   Explain your choice in the space below.

2. Which choice below will describe the motion of the dust particle?

   ○ ↕ Up and down.

   ○ → Pushed away from the speaker.

   ○ ↔ Side to side.

   ○ The dust particle will not move at all.

   ○ ⟳ Circular path.

   ○ None of these.

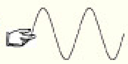
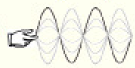
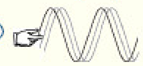
   ◉ (not answered)

   Explain your reasoning in the space below.

Spring/Fall 2005 – String Representation Assessment

A long taut string is held at the left by a hand and extends far off to the right (indicated by dashes). A yellow dot is painted on the string. A demonstrator moves her hand up and down at a constant frequency, creating a wave on the string.

1. Several ways of drawing the string wave are shown below.
   Which one makes the most sense to you? (*Note: the choice is up to you - there is no "wrong" answer.*)

   ○  ○  ○  ⦿ (not answered)

   Explain your choice in the space below.

2. Which choice below will describe the motion of the yellow dot?

   ○ ↕ Up and down.

   ○ → Pushed away from the hand.

   ○ ↔ Side to side.

   ○ The grey dot will not move at all.

   ○ Circular path.

   ○ None of these.

   ⦿ (not answered)

   Explain your reasoning in the space below.

Chapter 6 – Analogy Tutorial

Name ____________________________ SID____________________________

Introduction
In this activity, you will learn about different types of waves. As you work though the activity, think about how the word "wave" can be applied to these different phenomena.

Electromagnetic waves are often approximated as *plane waves*. In a plane wave, the *wavefronts* propagate in *planes* and the *amplitude* of the wave does not change as the wave propagates forward. Light from the sun and radio waves are examples of electromagnetic waves. You probably know that light from the sun or radio waves from a broadcast antenna gets weaker as you move far away from the source. However, if you only move a small distance compared to your distance from the source, the strength of the wave does not change very much. That is *why* we use the approximation of a plane wave.

The range of wavelengths for *visible light* is about 400-750 nm. If you moved around over a distance of a few wavelengths of visible light, would you notice any difference in the *brightness* of the light?

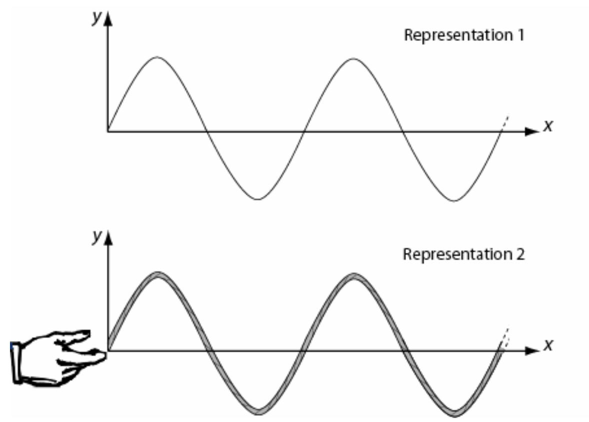
In the space below, draw a picture showing why a *plane wave* is a good way to approximate the light waves traveling from the sun to the earth. (*Hint:* What would the wavefronts look like?)

Waves on a String

A. Shown below are two representations of a wave on a string. Representation 1 uses a *sine wave* to represent the string at one instant in time. Representation 2 shows a picture of the string at this instant in time.

1. On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2.

2. How does representation 1 relate to representation 2? What do the *peaks* and *troughs* of the wave in representation 1 indicate?

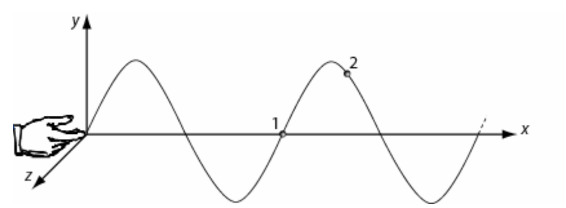


3. As the hand wiggles the string, a *traveling* wave is created on the string. The wave travels to the right. Which direction do *segments* of the string move?

4. Is this wave transverse or longitudinal? How can you tell?

B. The figure below shows a wave on a string at one instant in time. The wave is *traveling* to the right. There are beads attached to the string at locations 1 and 2.

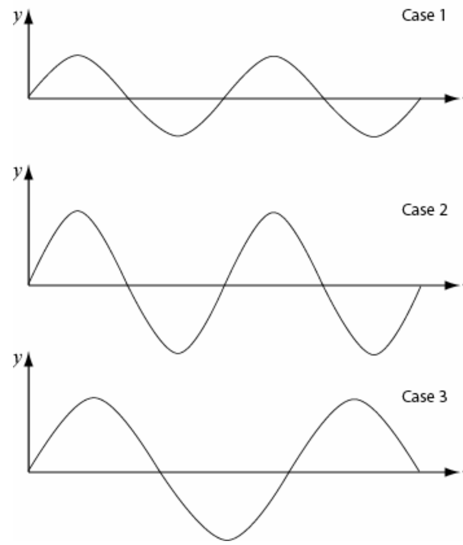
1. Compare the positions of beads 1 and 2 at this instant in time.



2. How, if at all, will beads 1 and 2 move as the wave travels to the right?

3. How would your answer to #2 change if this was a *standing* wave?

C. Three waves on strings are represented below, all traveling with the same velocity *v*. The diagrams are drawn to the same scale.



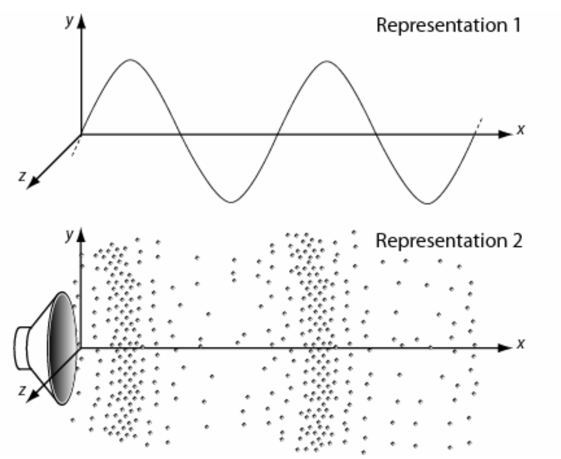
1. Compare the *amplitude* of the wave in Case 1 to that of Case 2. Explain how you can tell from the diagrams.

2. Compare the *wavelength* of the wave in Case 2 to that of Case 3. Explain how you can tell from the diagrams.

3. Compare the *frequency* of the wave in Case 2 to that of Case 3. Explain how you figured this out.

<u>Sound Waves</u>

A. Shown below are two representations of a sound wave. Representation 1 uses a *sine wave* to represent the sound at one instant in time. Representation 2 shows the arrangement of air particles at this instant in time.



1. On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2.

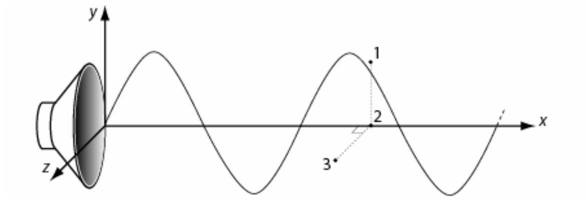
2. How does representation 1 relate to representation 2? What do the *peaks* and *troughs* of the wave in representation 1 indicate?

3. As the speaker moves, a *traveling* sound wave is created. Which direction does the *wave* move? Which direction do the *air particles* move?

4. Is this wave transverse or longitudinal? How can you tell?

B. The figure below shows a sound wave at one instant in time. The wave is *traveling* to the right. Three points in space are labeled 1,2, and 3. Points 1 and 2 lie in the *x-y* plane; point 3 lies in the *x-z* plane (coming out of the page in the *z*-direction). All three points have the same *x*-coordinate, and are separated by a small distance.

1. Compare the sound *pressure* at each of the three points at this instant in time. *(Hint: the pressure is proportional to the density of air particles.)*



2. A short time later, the wave has traveled to the right. How, if at all, will the pressure at the three points change as the wave travels?

3. Suppose the speaker in the diagram above was a *real stereo speaker* and you were standing in front of it. How, if at all, would the sound you hear change if you moved from point 1 to point 2 to point 3? How does your answer here compare to your answer to #1?

<u>Electromagnetic Waves</u>

I. Representations of electromagnetic waves

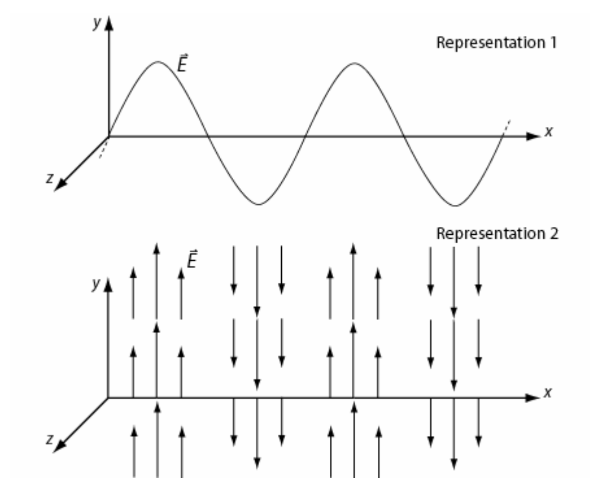
A. Shown below are mathematical and pictorial representations of an electromagnetic plane wave propagating through empty space. Representation 1 uses a *sine wave* to represent the electromagnetic wave at one instant in time; representation 2 uses a *vector field* to represent the wave for several *y*-coordinates.

$$\vec{E}(x,y,z,t) = E_0 \sin(kx+\omega t)\hat{y} \qquad \vec{B}(x,y,z,t) = B_0 \sin(kx+\omega t)\hat{z}$$

1. On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2.

2. How does representation 1 relate to representation 2? What do the *peaks* and *troughs* of the wave in representation 1 indicate?

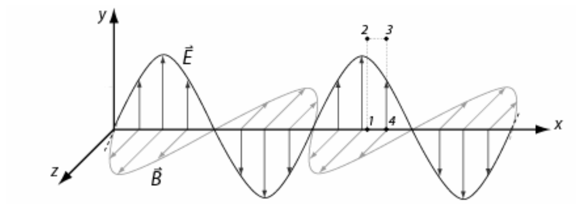
3. Is this wave transverse or longitudinal? How can you tell?



B. The figure below shows an electromagnetic wave at one instant in time. The wave is *traveling* to the right. This figure is a combination of representations 1 and 2 from part A. The electric field, $\vec{E}$, is parallel to the *y*-axis; the magnetic field, $\vec{B}$, is parallel to the *z*-axis.

1. Four points in space (labeled 1,2, 3 and 4) lie in the *x*-*y* plane.

   For the instant shown, rank these points according to the magnitude of the *electric field* at each of the four points.

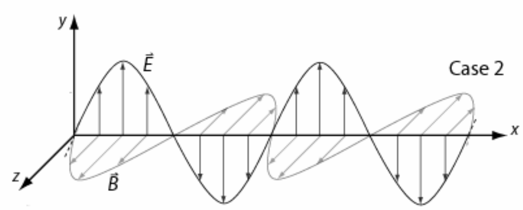


2. For the instant shown, rank points 1-4 according to the magnitude of the *magnetic field* at each of the four points.

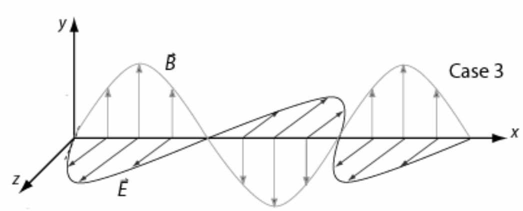
3. How, if at all, will the magnitudes of the electric and magnetic fields change at points 1-4 as the wave travels to the right?

C. Three light waves are represented at right. The diagrams are drawn to the same scale.

1. How is the wave in case 1 different from the wave in case 2? Explain how you can tell from the diagrams.



2. If the wave in case 2 were green light, could the wave in case 3 be *red light* or *blue light*? Explain.

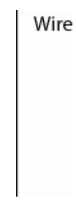


II. Detecting electromagnetic waves

A. Write expressions for the force exerted on a charge, *q*, by (1) an electric field, $\vec{E}$, and (2) a magnetic field, $\vec{B}$.

If an electric field and a magnetic field were both present, would a force be exerted on the charge even if the charge were initially not moving? Explain.

B. Imagine that the electromagnetic wave in section I, part B, is a radio wave. A long, thin conducting wire (see figure at right) is placed in the path of the wave.

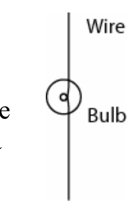


1. Suppose that the wire were oriented parallel to the *y*-axis.

   As the wave propagates past the wire, would the *electric field* due to the radio wave cause the charges in the wire to move? If so, would the charges move in a direction along the length of the wire? Explain.

   As the wave propagates past the wire, would the *magnetic field* due to the wave cause the charges in the wire to move in a direction along the length of the wire? Explain.

2. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)



   If the wire were placed in the path of the radio wave and oriented parallel to the *y*-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

   How, if at all, would your answer change if the wire were oriented:

   - parallel to the *x*-axis? Explain.

   - parallel to the *z*-axis? Explain.

Chapter 6 – No-analogy Tutorial

Name ____________________________ SID____________________________

Introduction
In this activity, you will learn about electromagnetic waves. As you work though the activity, think about how the word "wave" can be applied to this phenomenon.

Electromagnetic waves are often approximated as *plane waves*. In a plane wave, the *wavefronts* propagate in *planes* and the *amplitude* of the wave does not change as the wave propagates forward. Light from the sun and radio waves are examples of electromagnetic waves. You probably know that light from the sun or radio waves from a broadcast antenna gets weaker as you move far away from the source. However, if you only move a small distance compared to your distance from the source, the strength of the wave does not change very much. That is *why* we use the approximation of a plane wave.

The range of wavelengths for *visible light* is about 400-750 nm. If you moved around over a distance of a few wavelengths of visible light, would you notice any difference in the *brightness* of the light?

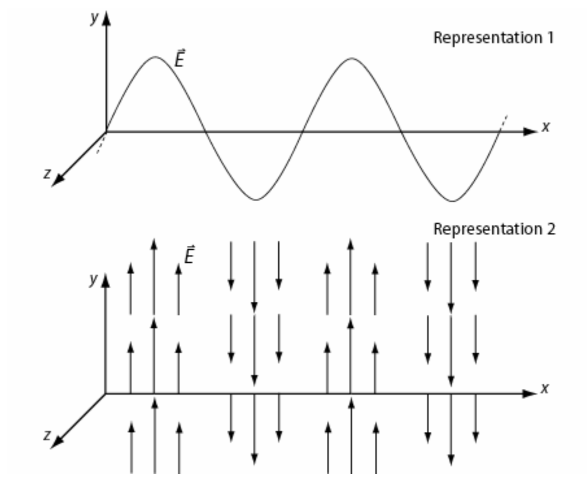
In the space below, draw a picture showing why a *plane wave* is a good way to approximate the light waves traveling from the sun to the earth. (*Hint:* What would the wavefronts look like?)

Electromagnetic Waves

I. Representations of electromagnetic waves

A. Shown below are mathematical and pictorial representations of an electromagnetic plane wave propagating through empty space. Representation 1 uses a *sine wave* to represent the electromagnetic wave at one instant in time; representation 2 uses a *vector field* to represent the wave for several *y*-coordinates.

$$\vec{E}(x,y,z,t) = E_0 \sin(kx + \omega t)\,\hat{y} \qquad \vec{B}(x,y,z,t) = B_0 \sin(kx + \omega t)\,\hat{z}$$



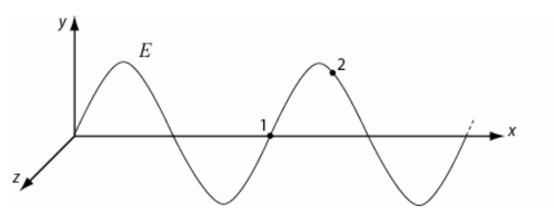
1. On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2.

2. How does representation 1 relate to representation 2? What do the *peaks* and *troughs* of the wave in representation 1 indicate?

3. This electromagnetic wave is *traveling* to the right. Which direction does the wave move? In which direction is the *electric field* changing?

4. Is this wave transverse or longitudinal? How can you tell?

B. The figure below shows an electromagnetic wave at one instant in time. The wave is *traveling* to the right. Two points in space are labeled 1 and



1. Compare the magnitude of the *electric field* at points 1 and 2 at this instant in time.

2. How, if at all, will the electric field at points 1 and 2 change as the wave travels to the right?

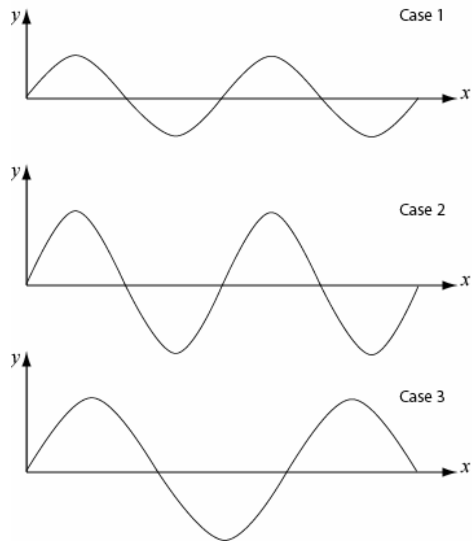
3. How would your answer to #2 change if this was a *standing* wave?

C. Three electromagnetic waves are represented below, all traveling with the same velocity *c*. The diagrams are drawn to the same scale.

1. Compare the *amplitude* of the wave in Case 1 to that of Case 2. Explain how you can tell from the diagrams.

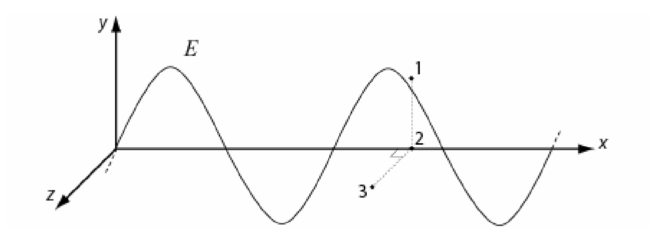
2. Compare the *wavelength* of the wave in Case 2 to that of Case 3. Explain how you can tell from the diagrams.

3. Compare the *frequency* of the wave in Case 2 to that of Case 3. Explain how you figured this out.



D. The figure below an electromagnetic wave at one instant in time. The wave is *traveling* to the right. Three points in space are labeled 1,2, and 3. Points 1 and 2 lie in the *x-y* plane; point 3 lies in the *x-z* plane (coming out of the page in the *z*-direction). All three points have the same *x*-coordinate, and are separated by a small distance.

1. Compare the magnitude of the *electric field* at each of the three points at this instant in time.



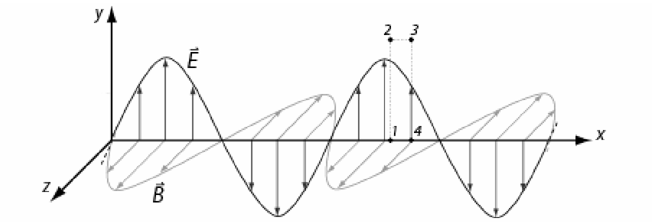
2. How, if at all, will the *electric field* at the three points change as the wave travels to the right?

3. Suppose you put a detector at each of the points. How, if at all, would the electric field you measure change as you moved from point to point?

E. The figure below shows an electromagnetic wave at one instant in time. The wave is *traveling* to the right. This figure is a combination of representations 1 and 2 from part A. The electric field, $\vec{E}$, is parallel to the *y*-axis; the magnetic field, $\vec{B}$, is parallel to the *z*-axis.

1. Four points in space (labeled 1,2, 3 and 4) lie in the *x*-*y* plane.

   For the instant shown, rank these points according to the magnitude of the *electric field* at each of the four points.



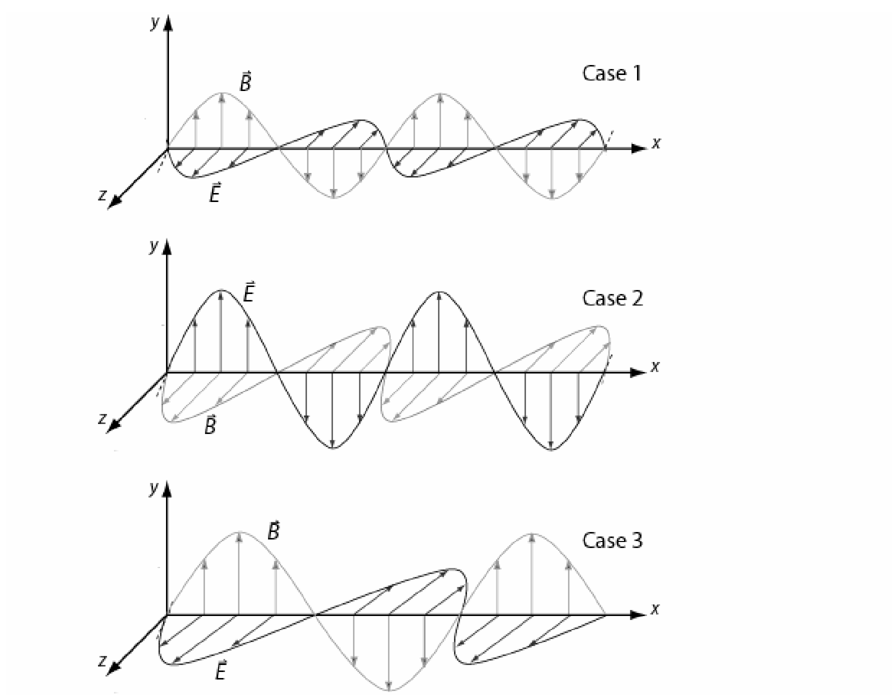
2. For the instant shown, rank points 1-4 according to the magnitude of the *magnetic field* at each of the four points.

3. How, if at all, will the magnitudes of the electric and magnetic fields change at points 1-4 as the wave travels to the right?

F. Three light waves are represented at right. The diagrams are drawn to the same scale.

1. How is the wave in case 1 different from the wave in case 2? Explain how you can tell from the diagrams.

2. If the wave in case 2 were green light, could the wave in case 3 be *red light* or *blue light*? Explain.

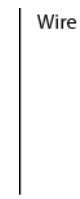


II. Detecting electromagnetic waves

A. Write expressions for the force exerted on a charge, q, by (1) an electric field, $\vec{E}$, and (2) a magnetic field, $\vec{B}$.

If an electric field and a magnetic field were both present, would a force be exerted on the charge even if the charge were initially not moving? Explain.

B. Imagine that the electromagnetic wave in section I, part E, is a radio wave. A long, thin conducting wire (see figure at right) is placed in the path of the wave.

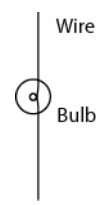


1. Suppose that the wire were oriented parallel to the *y*-axis.

   As the wave propagates past the wire, would the *electric field* due to the radio wave cause the charges in the wire to move? If so, would the charges move in a direction along the length of the wire? Explain.

   As the wave propagates past the wire, would the *magnetic field* due to the wave cause the charges in the wire to move in a direction along the length of the wire? Explain.

2. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)

   If the wire were placed in the path of the radio wave and oriented parallel to the *y*-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

   How, if at all, would your answer change if the wire were oriented:

   - parallel to the *x*-axis? Explain.

   - parallel to the *z*-axis? Explain.

Chapter 7 - Abstract Tutorial

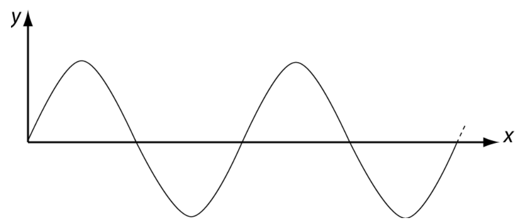
Name ___________________________ Student ID_________________________ Section ________

Introduction
In this activity, you will learn about different types of waves. As you work though the activity, think about how the word "wave" can be applied to these different phenomena.

Waves on a String

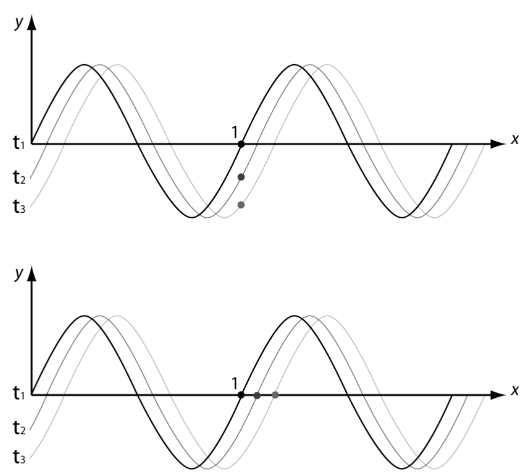
A. The representation below uses a *sine wave* to represent a wave on a string at one instant in time.

What do the peaks and troughs of the wave tell you about the positions of the string segments?



B. A hand wiggles the left end of a string up and down. The figure below shows snapshots of the wave on the string at *three* instants in time (t_1, t_2, t_3) as the wave travels in the +*x*-direction. There is a dot painted on the string at point 1.

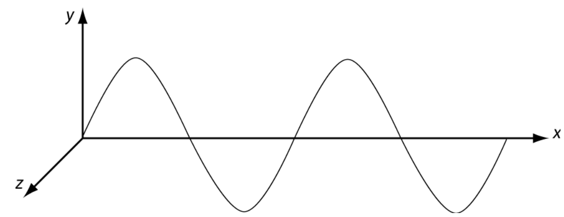
The pictures to the right show two possible ways the dot might move as the wave travels on the string. Which picture shows the correct motion of the dot? If you think the dot does not move, state that explicitly. Explain.



Sound Waves

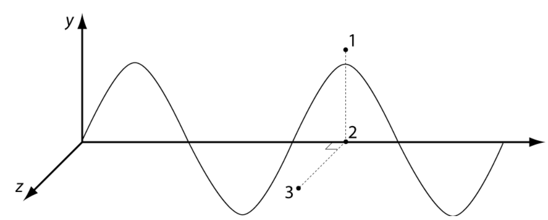
A. The representation below uses a *sine wave* to represent a sound wave at one instant in time.

A sound wave is made up of moving air particles. What do the peaks and troughs of the wave tell you about the density of the air particles?



B. The figure below shows a sound wave at one instant in time. Three points in space are labeled 1,2, and 3. Points 1 and 2 lie in the *x*-*y* plane; point 3 lies in the *x*-*z* plane (coming out of the page in the *z*-direction). All three points have the same *x*-coordinate, and are separated by a small distance.

Compare the sound *pressure* at each of the three points at this instant in time. Explain how your answer makes sense by relating to the figure above. *(*The *pressure* is proportional to the *density* of air particles.*)*

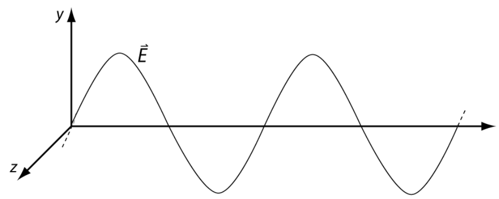


Electromagnetic Waves

I. Representations of electromagnetic waves

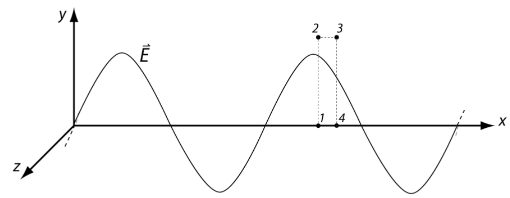
A. The figure below uses a *sine wave* to represent an electromagnetic wave at one instant in time.

1. What does this representation of the wave tell you about the strength of the electric field?



B. The figure below shows an electromagnetic wave at one instant in time. The wave is *traveling* to the right. Four points in space (labeled 1,2, 3 and 4) lie in the *x-y* plane.

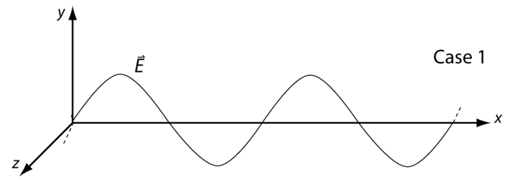
1. For the instant shown, rank these points according to the magnitude of the *electric field* at each of the four points. (*Hint: if this was a* sound wave*, what would the pressure be at the four points?*)



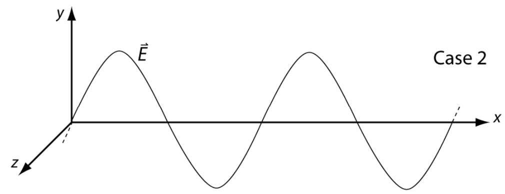
2. How, if at all, will the magnitude of the electric field change at points 1-4 as the wave travels to the right? (*Hint: what happens as a wave travels on a* string*?*)

C. Three electromagnetic waves are represented at right. The diagrams are drawn to the same scale.

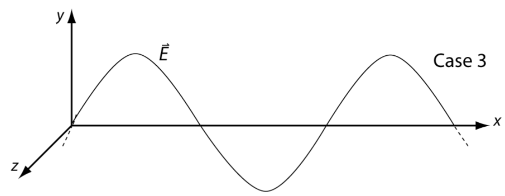
1. Is the *amplitude* of the wave greater in Case 1 or Case 2? Explain how you can tell.



2. Is the *wavelength* of the wave greater in Case 2 or Case 3? Explain how you can tell.

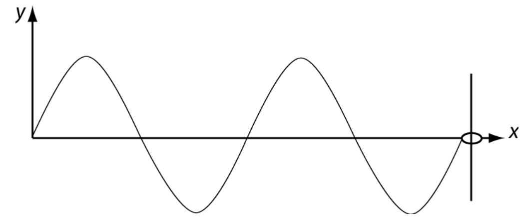


3. Is the *frequency* of the wave greater in Case 2 or Case 3? Explain how you can tell.



II. Detecting electromagnetic waves

A. The figure on the right shows a string with one end attached to a ring. The ring is free to move along the length of the metal rod.

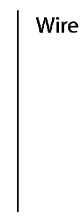


1. Will wiggling the left end of the string up and down cause the ring to move up and down on the rod?

2. Suppose you turned the rod so that it was parallel to the *z*-axis (coming out of the page). If you wiggle the string up and down (along the *y*-axis), will this cause the ring move along the length of the rod?

B. Write an expression for the force exerted on a charge, *q*, by an electric field, ***E***.

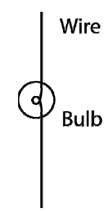
C. Imagine that the electromagnetic wave in section I, part B, is a radio wave. A long, thin conducting wire (see figure at right) is placed in the path of the wave.



Suppose that the wire is oriented parallel to the *y*-axis.

1. As the wave propagates past the wire, would the *electric field* due to the radio wave cause the electrons in the wire to move? If so, would the electrons move in a direction along the length of the wire? Explain.

2. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)



If the wire were placed in the path of the radio wave and oriented parallel to the *y*-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

How, if at all, would your answers to 1 and 2 change if the wire were oriented:

- parallel to the *x*-axis? Explain.

- parallel to the *z*-axis? Explain.

Chapter 7 - Concrete Tutorial

Name ___________________________ Student ID_________________________ Section ________

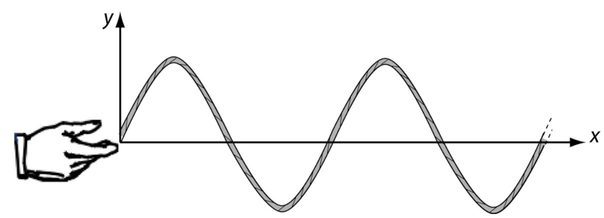
Introduction

In this activity, you will learn about different types of waves. As you work though the activity, think about how the word "wave" can be applied to these different phenomena.

Waves on a String

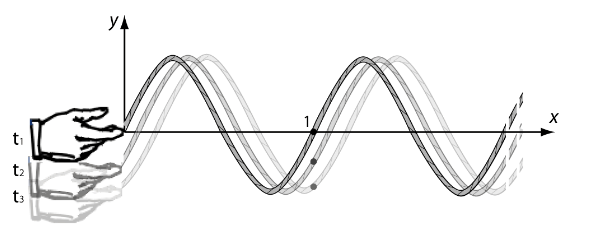
A. The representation below shows a wave on a string at one instant in time.

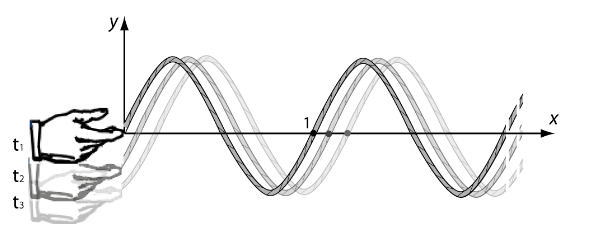
What do the peaks and troughs of the wave tell you about the positions of the string segments?



B. A hand wiggles the left end of a string up and down. The figure below shows snapshots of the wave on the string at *three* instants in time (t_1, t_2, t_3) as the wave travels in the +*x*-direction. There is a dot painted on the string at point 1.

The pictures to the right show two possible ways the dot might move as the wave travels on the string. Which picture shows the correct motion of the dot? If you think the dot does not move, state that explicitly. Explain.

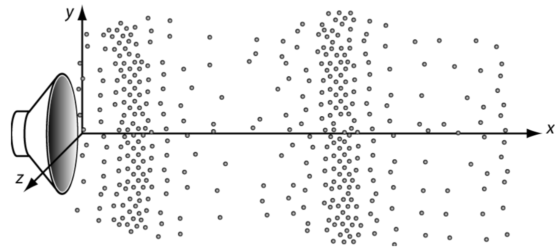




Sound Waves

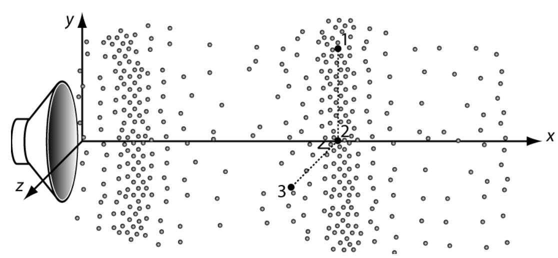
A. The representation below shows the arrangement of air particles at one instant in time.

A sound wave is made up of moving air particles. What does this representation of the wave tell you about the density of the air particles?



B. The figure below shows a sound wave at one instant in time. Three points in space are labeled 1,2, and 3. Points 1 and 2 lie in the *x*-*y* plane; point 3 lies in the *x*-*z* plane (coming out of the page in the *z*-direction). All three points have the same *x*-coordinate, and are separated by a small distance.

Compare the sound *pressure* at each of the three points at this instant in time. (The *pressure* is proportional to the *density* of air particles.)

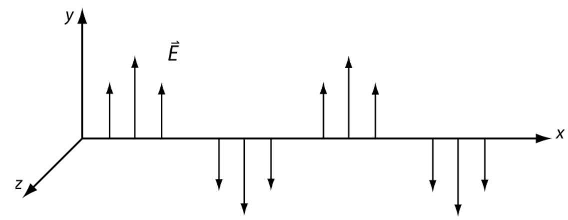


<u>Electromagnetic Waves</u>

I. Representations of electromagnetic waves

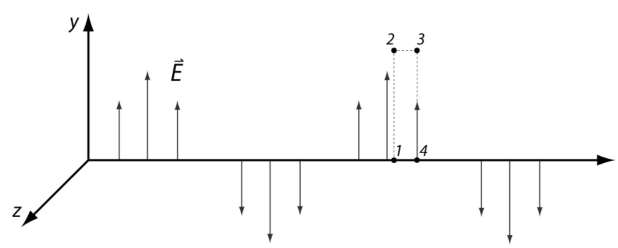
A. The figure below uses *vectors* to represent the electromagnetic wave at one instant in time.

What does this representation of the wave tell you about the strength of the electric field?



B. The figure below shows an electromagnetic wave at one instant in time. The wave is *traveling* to the right. Four points in space (labeled 1,2, 3 and 4) lie in the *x*-*y* plane.

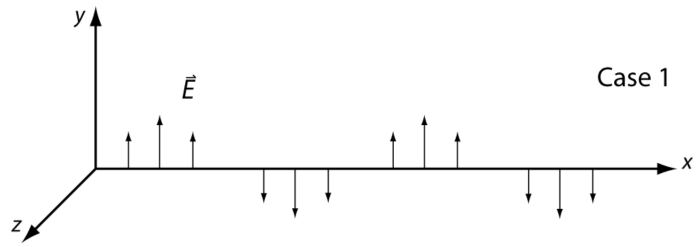
1. For the instant shown, rank these points according to the magnitude of the *electric field* at each of the four points. (*Hint: if this was a* sound wave*, what would the pressure be at the four points?*)



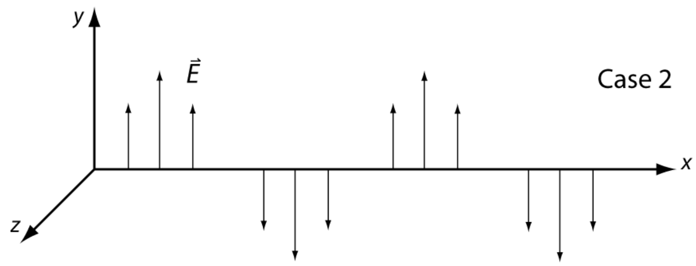
2. How, if at all, will the magnitude of the electric field change at points 1-4 as the wave travels to the right? (*Hint: what happens as a wave travels on a* string*?*)

C. Three electromagnetic waves are represented at right. The diagrams are drawn to the same scale.

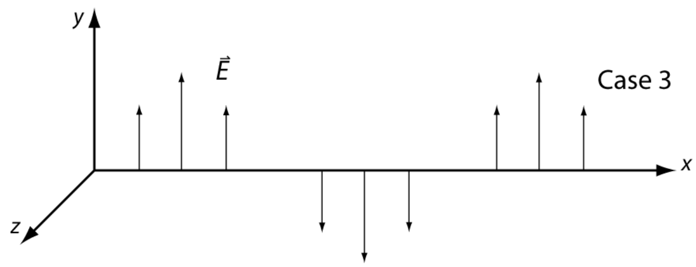
1. Is the *amplitude* of the wave greater in Case 1 or Case 2? Explain how you can tell.



2. Is the *wavelength* of the wave greater in Case 2 or Case 3? Explain how you can tell.

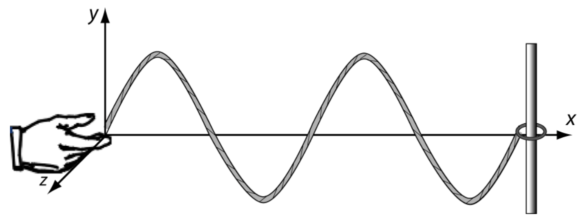


3. Is the *frequency* of the wave greater in Case 2 or Case 3? Explain how you can tell.



II. Detecting electromagnetic waves

A. The figure on the right shows a string with one end attached to a ring. The ring is free to move along the length of the metal rod.

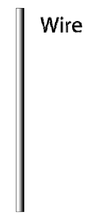


1. Will wiggling the left end of the string up and down cause the ring to move up and down on the rod?

2. Suppose you turned the rod so that it was parallel to the *z*-axis (coming out of the page). If you wiggle the string up and down (along the *y*-axis), will this cause the ring move along the length of the rod?

B. Write an expression for the force exerted on a charge, *q*, by an electric field, ***E***.

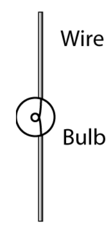
C. Imagine that the electromagnetic wave in section I, part B, is a radio wave. A long, thin conducting wire (see figure at right) is placed in the path of the wave.



Suppose that the wire is oriented parallel to the *y*-axis.

1. As the wave propagates past the wire, would the *electric field* due to the radio wave cause the electrons in the wire to move? If so, would the electrons move in a direction along the length of the wire? Explain.

2. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)



   If the wire were placed in the path of the radio wave and oriented parallel to the *y*-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

   How, if at all, would your answers to 1 and 2 change if the wire were oriented:

   - parallel to the *x*-axis? Explain.

   - parallel to the *z*-axis? Explain.

Chapter 7 - Blend Tutorial

Name ___________________________ Student ID_________________________ Section ________

Introduction

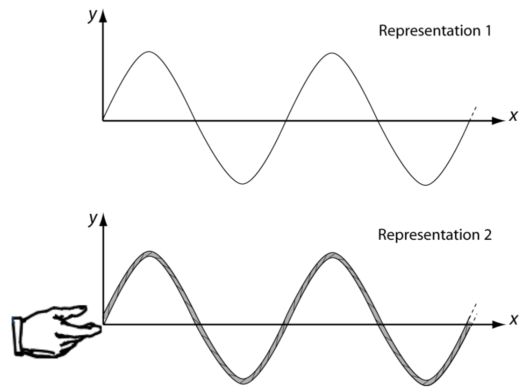
In this activity, you will learn about different types of waves. As you work though the activity, think about how the word "wave" can be applied to these different phenomena.

Waves on a String

A. Shown below are two representations of a wave on a string. Representation 1 uses a *sine wave* to represent the string at one instant in time. Representation 2 shows a picture of the string at this instant in time.

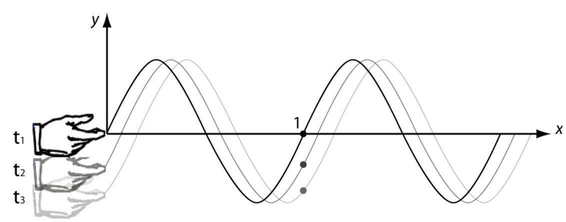
On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2.

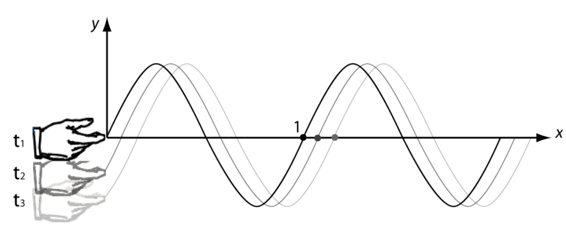
What do the peaks and troughs in *representation 1* tell you about the positions of the string segments?



B. A hand wiggles the left end of a string up and down. The figure below shows snapshots of the wave on the string at *three* instants in time (t_1, t_2, t_3) as the wave travels in the +*x*-direction. There is a dot painted on the string at point 1.

The pictures to the right show two possible ways the dot might move as the wave travels on the string. Which picture shows the correct motion of the dot? If you think the dot does not move, state that explicitly. Explain.



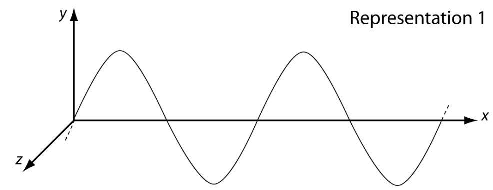


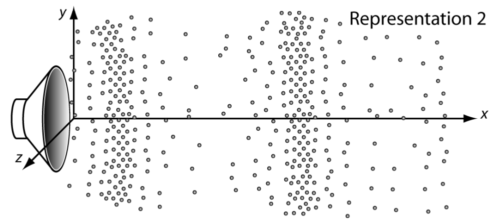
Sound Waves

A. Shown below are two representations of a sound wave. Representation 1 uses a *sine wave* to represent the sound at one instant in time. Representation 2 shows the arrangement of air particles at this instant in time.

On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2.

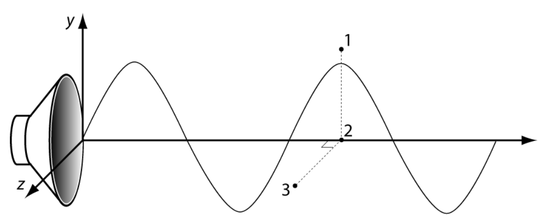
What do the peaks and troughs in *representation 1* tell you about the density of the air particles?





B. The figure below shows a sound wave at one instant in time. Three points in space are labeled 1,2, and 3. Points 1 and 2 lie in the *x*-*y* plane; point 3 lies in the *x*-*z* plane (coming out of the page in the *z*-direction). All three points have the same *x*-coordinate, and are separated by a small distance.

Compare the sound *pressure* at each of the three points at this instant in time. Explain how your answer makes sense by relating to the figures above. (The *pressure* is proportional to the *density* of air particles.)



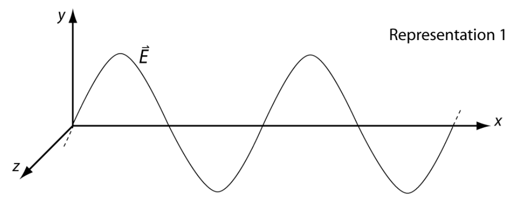
<u>Electromagnetic Waves</u>

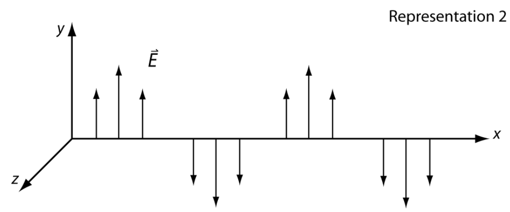
I. Representations of electromagnetic waves

A. Shown below are pictorial representations of an electromagnetic plane wave propagating through empty space. Representation 1 uses a *sine wave* to represent the electromagnetic wave at one instant in time; representation 2 uses a *vector field* to represent the wave.

On the diagram, *draw the wave* in representation 1 on top of the wave in representation 2.

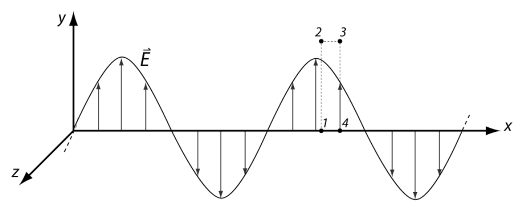
What does *representation 1* tell you about the strength of the electric field?





B. The figure below shows an electromagnetic wave at one instant in time. The wave is *traveling* to the right. Four points in space (labeled 1,2, 3 and 4) lie in the *x-y* plane.

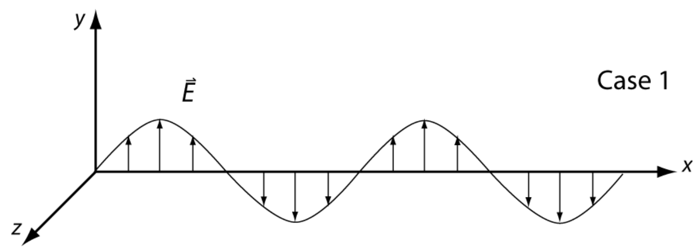
1. For the instant shown, rank these points according to the magnitude of the *electric field* at each of the four points. (*Hint: if this was a* sound wave*, what would the pressure be at the four points?*)



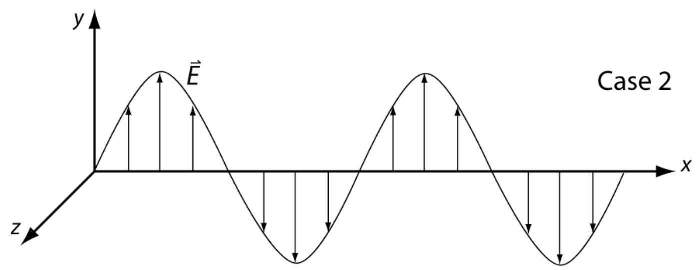
2. How, if at all, will the magnitude of the electric field change at points 1-4 as the wave travels to the right? (*Hint: what happens as a wave travels on a* string*?*)

C. Three electromagnetic waves are represented at right. The diagrams are drawn to the same scale.

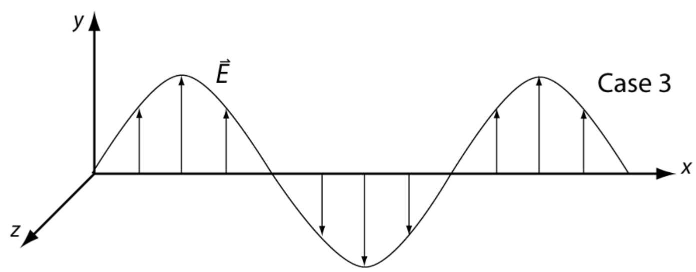
1. Is the *amplitude* of the wave greater in Case 1 or Case 2? Explain how you can tell.



2. Is the *wavelength* of the wave greater in Case 2 or Case 3? Explain how you can tell.

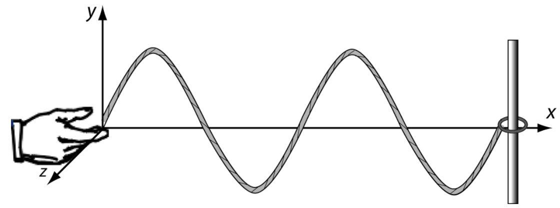


3. Is the *frequency* of the wave greater in Case 2 or Case 3? Explain how you can tell.



II. Detecting electromagnetic waves

A. The figure on the right shows a string with one end attached to a ring. The ring is free to move along the length of the metal rod.

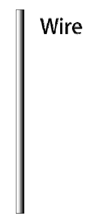


1. Will wiggling the left end of the string up and down cause the ring to move up and down on the rod?

2. Suppose you turned the rod so that it was parallel to the *z*-axis (coming out of the page). If you wiggle the string up and down (along the *y*-axis), will this cause the ring move along the length of the rod?

B. Write an expression for the force exerted on a charge, *q*, by an electric field, ***E***.

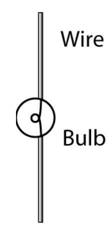
C. Imagine that the electromagnetic wave in section I, part B, is a radio wave. A long, thin conducting wire (see figure at right) is placed in the path of the wave.



Suppose that the wire is oriented parallel to the *y*-axis.

1. As the wave propagates past the wire, would the *electric field* due to the radio wave cause the electrons in the wire to move? If so, would the electrons move in a direction along the length of the wire? Explain.

2. Imagine that the thin conducting wire is cut in half and that each half is connected to a different terminal of a light bulb. (See diagram at right.)



If the wire were placed in the path of the radio wave and oriented parallel to the *y*-axis, would the bulb ever glow? Explain. (*Hint:* Under what conditions can a bulb glow even if it is not part of a closed circuit?)

How, if at all, would your answer change if the wire were oriented:

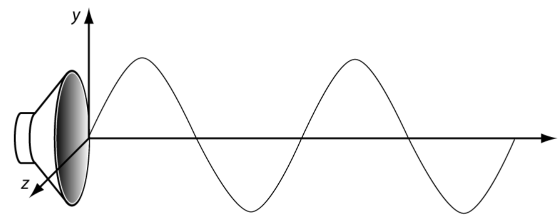
- parallel to the *x*-axis? Explain.

- parallel to the *z*-axis? Explain.

Chapter 7 - Abstract Sound Waves Quiz

Name ______________________________ Student ID __________________ Section ____

The diagram to the right uses a *sinewave* to represent a sound wave at one instant in time.

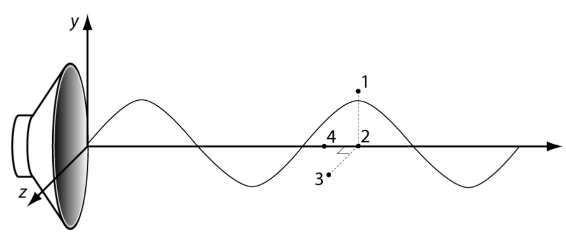


1. Consider the following four analogies for a sound wave:
   a. A crowd in a stadium doing "the wave".
   b. A wave on a string.
   c. A long row of people passing footballs from person to person.
   d. A wave made with a stretched slinky.
   e. Something else.

   Which analogy or analogies (you may use more than one) seem the best for describing a sound wave? Explain your reasoning. Note there is no "correct answer" – it is up to your interpretation.

2. The diagram on the right shows four points (labeled 1-4) in space in front of a speaker. The points are separated by a small distance (less than the size of the speaker.) Points 1, 2, and 4 lie in the x-plane. Point 3 has the same x-coordinate as 1 and 2, but lies out of the page (in the z-direction).

   Which of the following is the best ranking of magnitude of the *pressure* at the four points? **Note the pressure is proportional to the *density* of the air particles.**



   a. 1>2=4>3
   b. 1=2=3>4
   c. 4>1=2=3
   d. 1=2=4>3
   e. 1=2>4>3
   f. 1=2>4=3

3. In the diagram on the right, a dust particle sits directly in front of a speaker. The speaker plays a sound of constant frequency. Which choice below best describes the motion of the dust particle?

   a. Oscillating up and down

   b. Moving to the right away from the speaker

   c. Oscillating left and right

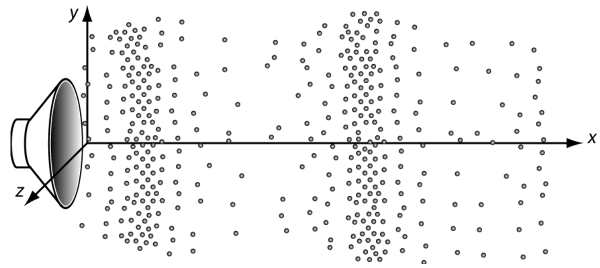
   d. The dust particle will not move

   e. None of the above

Chapter 7 - Concrete Sound Waves Quiz

Name ______________________________ Student ID __________________ Section ____

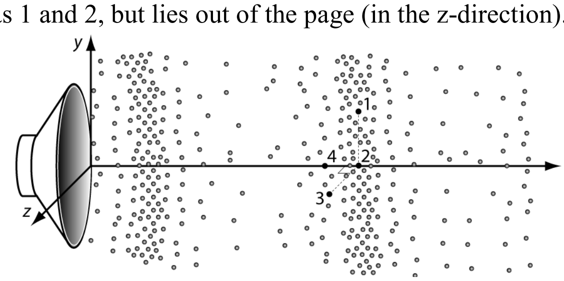
The diagram to the right shows the arrangement of air particles in a sound wave at one instant in time.



1. Consider the following four analogies for a sound wave:
   a. A crowd in a stadium doing “the wave”.
   b. A wave on a string.
   c. A long row of people passing footballs from person to person.
   d. A wave made with a stretched slinky.
   e. Something else.

   Which analogy or analogies (you may use more than one) seem the best for describing a sound wave? Explain your reasoning. Note there is no “correct answer” – it is up to your interpretation.

2. The diagram on the right shows four points (labeled 1-4) in space in front of a speaker. The points are separated by a small distance (less than the size of the speaker.) Points 1, 2, and 4 lie in the x-plane. Point 3 has the same x-coordinate as 1 and 2, but lies out of the page (in the z-direction).



   Which of the following is the best ranking of magnitude of the *pressure* at the four points? **Note the pressure is proportional to the *density* of the air particles.**

   a. 1>2=4>3
   b. 1=2=3>4
   c. 4>1=2=3
   d. 1=2=4>3
   e. 1=2>4>3
   f. 1=2>4=3

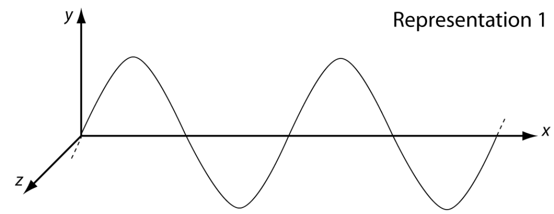
3. In the diagram on the right, a dust particle sits directly in front of a speaker. The speaker plays a sound of constant frequency. Which choice below best describes the motion of the dust particle?

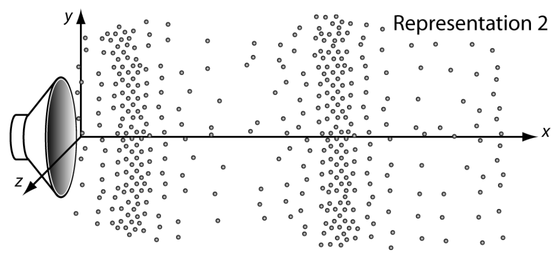
   a. Oscillating up and down
   b. Moving to the right away from the speaker
   c. Oscillating left and right
   d. The dust particle will not move
   e. None of the above

Chapter 7 - Blend Sound Waves Quiz

Name ______________________________ Student ID __________________ Section ____

The diagrams to the right show a sound wave at one instant in time. Representation 1 uses a *sinewave* to represent a sound wave at this instant. Representation 2 shows the arrangement of air particles at this instant. **Note both diagrams are drawn to the same scale.**





1. Consider the following four analogies for a sound wave:
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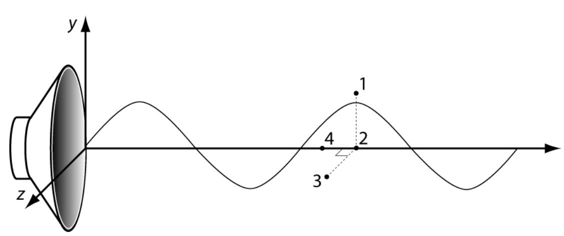
   b. A wave on a string.
   c. A long row of people passing footballs from person to person.
   d. A wave made with a stretched slinky.
   e. Something else.

   Which analogy or analogies (you may use more than one) seem the best for describing a sound wave? Explain your reasoning. Note there is no "correct answer" – it is up to your interpretation.

2. The diagram on the right shows four points (labeled 1-4) in space in front of a speaker. The points are separated by a small distance (less than the size of the speaker.) Points 1, 2, and 4 lie in the x-plane. Point 3 has the same x-coordinate as 1 and 2, but lies out of the page (in the z-direction).

   Which of the following is the best ranking of magnitude of the *pressure* at the four points? **Note the pressure is proportional to the *density* of the air particles.**

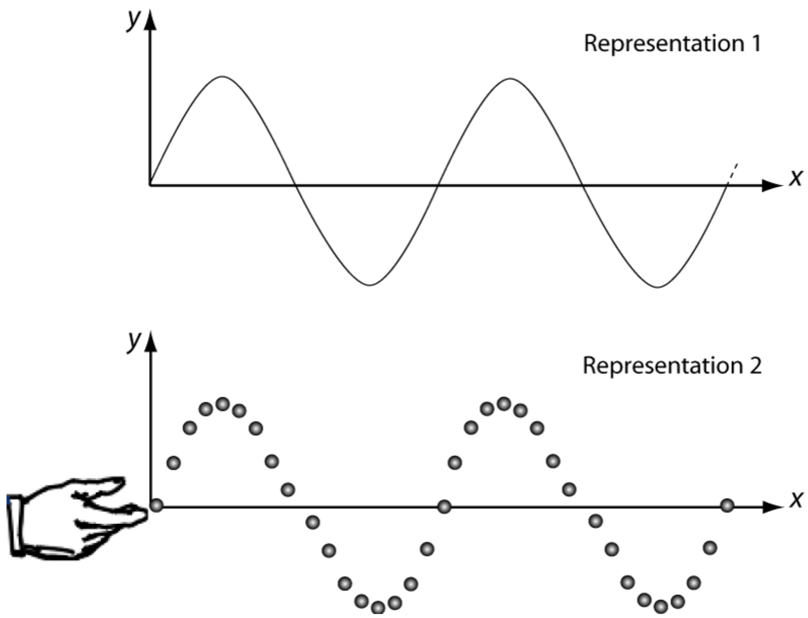


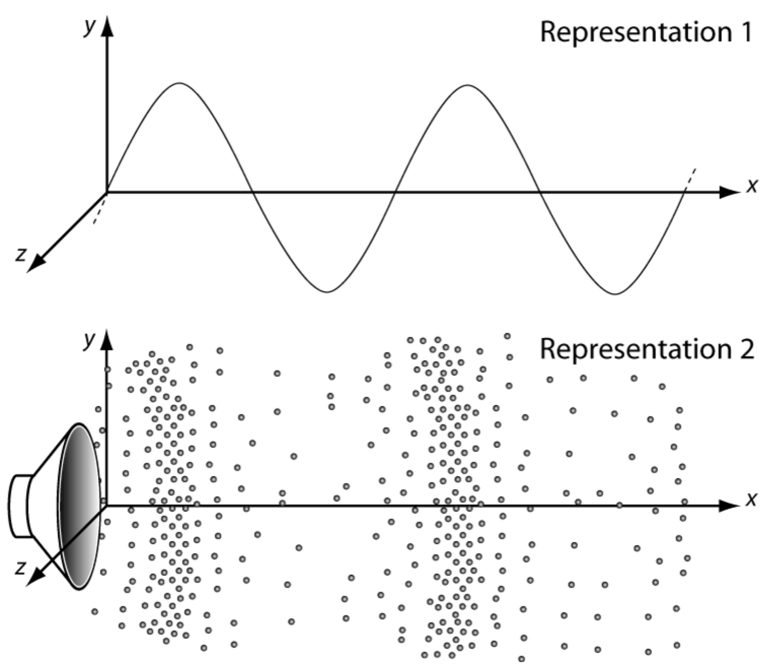
   a. 1>2=4>3
   b. 1=2=3>4
   c. 4>1=2=3
   d. 1=2=4>3
   e. 1=2>4>3
   f. 1=2>4=3

3. In the diagram on the right, a dust particle sits directly in front of a speaker. The speaker plays a sound of constant frequency. Which choice below best describes the motion of the dust particle?

   a. Oscillating up and down
   b. Moving to the right away from the speaker
   c. Oscillating left and right
   d. The dust particle will not move
   e. None of the above

Chapter 8 – Wave on a String (top) and Sound Wave (bottom) Representations from Student Interview





Appendix B. Annotated Bibliography

This section provides supplemental notes for a selection of work.

Ambrose, B.S, Heron, P.R.L, Vokos, S. & McDermott, L.C. (1999) Student understanding of light as an electromagnetic wave: Relating the formalism to physical phenomena, *American Journal of Physics 67(10), 891*

> Investigated student understanding of EM waves, particular interpretation of the canonical EM wave representation (crossed E- and B-fields plotted as sine waves and vectors in x-y-z space). Identified a number of student difficulties with EM wave concepts, particularly related to EM wave diagrams. Claim some of these student difficulties transcend the representation. Developed tutorial to address student difficulties, particular 3D nature of EM plane-waves and oscillations in time.

Atkins, L., Ph.D. dissertation

> Analogy as assertion of category membership. Draws on several lines of research including category, prototype theory, metaphor, and analogy. Promotes analogical competence is goal of instruction (not just tool for teaching).

Bartlett, D. (2004). Analogies between electricity and gravity. *Metrologia 41:S115-S124*

> Example of modern physicist, from University of Colorado at Boulder, explicitly using analogy to generate new theory of gravity.

Blanchette, I. & Dunbar, K. (2000) How Analogies are Generated: The Roles of Structural and Superficial Similarity. *Memory & Cognition 29, 730-735*

> "Reception paradigm" (experimenter provides base and target) vs. "Production paradigm" (experimenter provides target, subject generates base).
>
> 3 experiments:
>
> - "Zero deficit issue" (in the sense of politics, economy) – do subjects in a production paradigm experiment generate analogies based on structural matches rather than surface matches? > Yes, majority of analogies generated did not share surface similarity. I.e., sources were not economics, politics, finance, etc. What problem features to subjects use to generate analogies? Used groups of subjects (based on other work by Dunbar).

- Does group reasoning lead to increased processing capacity which would in turn increase the ability to use structural features? > Surface similarity constrains individuals even less than groups (individuals produced more varied analogies). Also, results of experiment 1 were not an artifact of group work.
- Problem content or production paradigm: Was the real world problem, which people are familiar (somewhat expert), the reason for the ability to use structural (over surface) matches? Used reception paradigm; subjects were first presented with a number of sources. > In this case, majority of retrievals were based on surface (65%) rather than structural (16%). Production paradigm appears to be the reason for structural retrieval.

Brown, D.E. (1992) Using Examples and Analogies to Remediate Misconceptions in Physics: Factors Influencing Conceptual Change, *Journal of Research in Science Teaching 29(1), 17-34*

> Found that examples are more effective when they help students draw on and analogically extend existing valid physical intuitions in constructing a new conceptual model of a target situation. Study included 21 high-school students, interviewed for approximately 45 minutes each. Students were interviewed on Newton III concepts via bridging analogies of springs, springy table, and a microscopic model of springy table. Suggest that to help students in this construct knowledge via analogy:

- The example used must be understandable and believable to the students, no simply to the teacher or textbook author.
- Even when an example is compelling to the student, it may not be seen as analogous to t target problems drawing out a misconception. In that case, analogy relations need to be explicitly developed.
- Qualitative, visualizable models may need to be developed which give mechanistic explanations for phenomena.

Brown, D.E. (1994) Facilitating conceptual change using analogies and explanatory models, *International Journal of Science Education 16(2), 201*

> A large scale study of bridging, shows bridging can be used to teach in large scale.

Brown, D.E. & Clement, J. (1989) Overcoming misconceptions via analogical reasoning: abstract transfer versus explanatory model construction, *Instructional Science 18, 237*

> Key bridging paper. Four student interviews, 2 successful, 2 not.

Catrambone, R. & Holyoak, K.J. (1989) Overcoming Contextual Limitations on Problem Solving Transfer, *Journal of Experimental Psychology: Learning, Memory, and Cognition 15(6), 1147-1156*

Five experiments investigating transfer from multiple analogs to superficially dissimilar target problems. Pre-hint transfer was enhanced, even after a context shift and a week-long delay between reading the source analogs and solving the problem, when the following conditions were met:

i. The target problem was reworded slightly to emphasize a structural feature that it shared with the analogs.
ii. Three rather than two source analogs were provided.
iii. Detailed, schema-oriented questions were used to help subjects focus on the problem-relevant aspects of the stories. Although spontaneous transfer between small numbers of dissimilar analogs is difficult to obtain, it can be achieved by manipulations that foster abstraction of a problem schema from the training examples.

Break down analogy use into two steps: noticing and applying.

i. Notice and apply is goal.
ii. Nice framing of the problem: If subject does not notice, but can apply after given a hint, this leads to the inert knowledge hypothesis. Subjects have the requisite knowledge to solve problem via analogy, but do not know to do so. Subjects also know what to map (transfer) once they know to do a mapping (note that the hint does not convey what to map.)

Only manipulations that permit transfer after a significant delay (days or weeks) are likely to be of value for instruction.

Experiment 1

i. Repeat Gick and Holyoak, but include groups with multiple analogs but not induced to make a comparison. This tests whether schema induction, rather than exposure to multiple examples per se, is a crucial determinant of transfer.
ii. N=77 college students, 4 groups: comparison vs. no-comparison instruction; two analog vs. one analogy plus disanalogous story.
iii. Comparison instruction lead to significantly more pre-hint solutions (47% vs. 16%), but no significant difference pre-hint with no comparison instruction. Post-hint, one analog increased significantly in comparison instruction, and two analogies in no-comparison, but no significant increase for one-analog no-comparison instruction. Comparison instruction of two analogs increases rate of noticing and application. Comparison of one analog (and one disanalogous) results in application, but not initial noticing. Two analogies without instruction results in application,

but not initial noticing. One analogy without comparison instruction does not lead to noticing or application (even with hint).

Experiment 2

i. Testing delay: 30 min vs. 1 week
ii. Subjects write summaries with comparing situations or for each situation individually (no instruction to compare).
iii. Comparison does better (but only after hint) than no-comparison over both time scales.
iv. Fewer solutions, in both comparison and non-comparison, for 1-week delay group.
v. Even a delay of 30 min seems to eliminate spontaneous use of analogy (i.e., pre-hint solution – note experiment 1 had essentially no delay).

Experiment 3

i. Change wording of target problem to better cue solution relevant features of the prior stories
ii. Still low pre-hint solutions (though 20% now in comparison condition).
iii. However, now no effect of delay on post-hint solutions.

Experiment 4

i. Subjects told solution to analogs, and given another analogous problem and told how to solve it by analogy.
ii. Solutions pre-hint significantly increased after both 30 min and 1 week delays.
iii. Overall – When the source and target analog share many salient surface properties, spontaneous transfer is quite likely to occur even in the absence of a hint—even, in fact, when the analogy is misleading. Multiple examples often allow transfer without a hint when the context is relatively constant, whereas a single source example typically would not suffice. However, processing multiple source analogs is not sufficient to ensure transfer under more demanding conditions in which the context is changed.

Chi, M., Feltovich, P., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science, 5(2), 121-152.*

Expert vs. novice categorization of physics problems, focusing on surface features vs. deep structure. Had novices (students) and experts (physics graduate students and professors) engage in a sorting task, arranging physics problems written on cards into piles based on perceived similarity (piles stand for categories). Found that novices and experts differently categorized the problems – the novices focused on surface features (e.g., incline plane problems) while experts focused on "deep structural features" (e.g., conservation of energy problems). In their paper, Chi et al depict the problems used with accompanying pictorial representations. However, the source cited for these problems did not include such pictures with the problems. It is difficult to say whether the problems used by Chi et al had pictures or only words. In either case, Chi et al do not appear to refer specifically to the pictures when describing surface features of these problems – instead, the researchers seem to mean surface features of the situation described in the problem, irrespective of representation.

Clement, J. (1988). Observed Methods for Generating Analogies in Scientific Problem Solving. *Cognitive Science. 12(4):563-586*

Interviewed 10 expert physicists to explore the spontaneous generation of analogy by these experts. The subjects were given a problem involving the stretch of two springs of different diameters. Clement found that a substantial number of analogies were generated spontaneously by these experts and formulated step-wise methods for generating analogy as well as three types of analogical retrieval which complement, but are different from, Gentner's three categorizations of domain comparison.

Clement, J. (1993). Using Bridging Strategies and Anchoring Intuitions to Deal with Students' Preconceptions in Physics. *Jour. Res. in Sci. Teaching. 30(10):1241-1257*

- Three examples of bridging strategies
    i. static normal forces (book on table)
    ii. frictional forces (brush fibers)
    iii. Newton III for moving objects (springs on train cars)
- Large differences in pre-post gains in favor of experimental group (2-3x gains)
- High school students in physics, experimental group N=150, control N=55
- Key points (from abstract)
    i. Lessons have a more complex structure than a simple model of analogy use
    ii. Rational methods using analogy and other plausible reasoning processes that are neither proof based nor directly empirical can play a very important role in science education.
    iii. Much more effort than is usually allocated should be focused on helping students make sense of an analogy.

iv. Researchers and developers of curriculum should be focusing at least as much attention on students' useful prior knowledge as they are on students' alternative conceptions.

- Anchoring conceptions – preconceptions that are largely in agreement with accepted scientific theory. Note that preconceptions are not necessarily misconceptions.
- By using the term misconceptions, there might be a negative connotation towards students' self-constructed ideas and thought processes – therefore, use the phrase alternative conceptions. (Note that this does not mean that all conceptions are equally useful in all contexts.)
- Book on table – students presented with hand on spring and book on table generally do not see the two situations as analogous (pilot interviews conducted by D. Brown). [Seems to be that spring has agency, it can act, while the table is inert or "dead".] Want to give students the intuition that the table can push up (like the spring).
- Paradox of prior knowledge. Anchoring intuitions may be a way around this – student knowledge is small pieces, instead of richly connected packages of experts. Different pieces can be activated in different contexts.
- Explanatory models – instead of specific analogous cases. "..an image of a mechanism that is assumed to be present in many cases." Not just a set of common features abstracted from observed phenomena (e.g., cannot observe atoms). They are "imagined constructions."

diSessa, A.A. (1988) Knowledge in Pieces. In G. Forman & P.B. Pufall (Eds.) *Constructivism in the Computer Age.* Lawerence Erlbaum Associates. Hillsdale, New Jersey.

Introduces the "pieces" view of knowledge and learning, aligned with a resource model. Presents the ideas of phenomenological primitives (p-prims), fine-grained analysis of student reasoning, and suggests student reasoning is fractured and not as structured, robust, coherent, and/or stable across time and contexts as some theories prescribe.

Dunbar, K. (1999) How Scientists Build Models: InVivo Science as a Window on the Scientific Mind. In *Model-based reasoning in scientific discovery.* Magnani, L., Nercessian, N., and Thagard, P. ed. Plenum Press.

Observation of biologists using analogies in situ. Found that scientists regularly use analogies in their normal work. Defined near vs. far analogy. Near analogy involves base and target from the same or closely related domain (e.g., a newly discovered bacterium is like an already understood bacterium; could describe the use of example problems in physics, e.g. this

inclined plane problem is like another inclined plane problem). Far analogy involves base and target from substantially different domains (e.g., a cell is like a factory; in physics, the atom is like the solar system).

Elby, A. (2000) What students' learning of representation tells us about constructivism, *Journal of Mathematical Behavior 19, 481*

Introduces the cognitive mechanism of *What-You-See-Is-What-You-Get* (WYSIWYG) and the *compelling visual attribute*. WYSIWYG elicits literal interpretations of diagrams and other representations along the lines of "up-is-up", or more generally "x-is-x". For instance, students may interpret a graph of velocity vs. time that is shaped like a hill as a real hill, rather than the more abstract meaning of this graph. Elby distinguishes fine-grained constructivism from misconceptions constructivism and points out that a fine-grained approach makes predictions about student problem solving in physics that a misconceptions approach cannot. Demonstrates the utility of the fine-grained approach through several experiments involving student interpretations of diagrams in physics problems.

Fauconnier, G. and Turner, M. (2003) *The Way We Think: Conceptual Blending and the Mind's Hidden Complexities*, Basic Books

This is THE book on blending theory. The authors describe conceptual blending, a *multi-domain* model of cognition. At a basic level, blending involves several *mental spaces*, including two input spaces which blend, via selective projection, to a third blend space. Input spaces can have *organizing frames* which dictate the structure of each mental space. A *generic* space partially directs the blending process. *Vital relations*, such as time, space, or representation, are *compressed* in a blend. For instance, *the atom is like a tiny solar system* analogy may be recast as a blend where the atom and the solar system comprise two input spaces, whereby the solar system frame is compressed over space to the microscopic dimension of the atom, retaining the overall structure. The generic space here would include a central object and orbiting objects (generic versions of the sun/nucleus and planets/electrons). Blend spaces can become input spaces to new blends, allowing for layering and building up of more complex blends.

Fauconnier, G. (2001) Conceptual blending and analogy. In D. Gentner, K.J. Holyoak, and B.N. Kokinov (eds.), *The Analogical Mind: Perspectives from Cognitive Science*. MIT Press, Cambridge, MA.

This chapter of *The Analogical Mind* is on blending and analogy. Fauconnier claims that analogy is a subset of blending, and points out limitations of traditional theories of analogy including the notion of two domain vs. multi-domain. Traditional mapping theories of analogy are two domain (base –

target) models, whereas blending theory is a multi-domain model (e.g., blends can come from any number of input spaces.)

Gentner, D. & Gentner, D.R. (1983). Flowing waters or teeming crowds: Mental models of electricity. In Gentner, D. and Stevens, A., (eds.), *Mental Models*. Lawrence Erlbaum Press.

Describes structure mapping theory (see Gentner (1983) and experiments meant to validate this model. Found differences in student performance on questions about batteries or resistors for students who generated electric circuit analogies from water or moving objects, respectively. However, they found that teaching these two analogies was not nearly as productive. The key points are that analogies can be generative of ideas in a target domain (not merely surface terminology), but, importantly, analogies were not effective (generative) when taught to students (i.e., instructor generated not effective in this case).

Gentner, D (1983) Structure-mapping: A theoretical framework for analogy. *Cognitive Science, 7:155-170.*

This is the key structure mapping paper, laying out the theory in detail. Analogy is framed as a mapping (i.e., isomorphism) between two cognitive structure, a base domain to a target domain. For instance, the base domain of the solar system maps to the target domain of the atom. Gentner defines objects and predicates (descriptors) in each structure. Predicates can be attributes (e.g., YELLOW(sun)) or relations (e.g., REVOLVES AROUND(sun , planet)), or *higher-order relations* which are relations between relations.

Gentner, D. (1999) Analogy. In R.A. Wilson & F.C. Keils (eds.) *The MIT Encyclopedia of the Cognitive Sciences*. Cambridge, MA: MIT Press.

An overview of analogy research, obviously from Gentner's point of view. Structure mapping and mapping (or abstract transfer) theories are central here. This piece is a nice review of the state of analogy research as of the late 1990's.

Gentner, D., Bowdle, B.F., Wolff, P., & Boronat, C. (2001) Metaphor is like analogy. In D. Gentner, K.J. Holyoak, and B.N. Kokinov (eds.), *The Analogical Mind: Perspectives from Cognitive Science*. MIT Press, Cambridge, MA.

Consider the metaphor *Love is a journey* or *The mind is a computer* under the structure mapping framework. Research suggests that novel metaphors can be modeled by an extension of structural mappings between domains, but not for conventional metaphors. (I think this means metaphors that share relational structure are analogies (e.g., "My job is a jail", but metaphors (e.g., "tires are

like shoes") are not analogies because they are based on surface similarity. *This seems questionable to me – tires are like shoes could share both. Certainly, tires are not simply another example of shoes.*) Some literary metaphors may require processes such as metonymy or "phenomenological matching" in addition to alignment and mapping.

Gick, M.L. & Holyoak, K.J. (1980) Analogical problem solving. *Cognitive Psychology 12, 306-355*

The original Gick and Holyoak study of the tumor convergence problem and fortress analogy. Found low rates of spontaneous use of the analogy, but substantially increased rates after a hint was given to subjects. Replicated and expanded upon in Gick and Holyoak (1983).

Gick, M.L. & Holyoak, K.J. (1983) Schema induction and analogical transfer. *Cognitive Psychology 15(1):1-38*

Follow up study of the fortress problem with groups of N~20. 3 key experiments are described in this paper.

Experiment 1: Giving subjects a single story analogy did not promote transfer (the general invading a fortress) (32% producing convergence solution before hint, 48% *more* after hint – note this does not mean they explicitly used the analogy).

Experiment 2: Story plus diagrams. No better with diagrams (however, diagrams alone did very poorly, only 1/15 subjects, and he said he did not use the diagrams). Similar %'s after hint (but diagrams only significant lower).

Experiment 3: Two stories: the general and the commander (attacking an island) – i.e. similar analogs, 39%. The general and firefighters (oil wells) – i.e. dissimilar analogs, 52%. One analogy plus control, 21%. Similar %'s after hint.

Key finding that subjects significantly more likely to transfer when given two source analogs (both before and after hint) than when given only one source analog. Hypothesis is suggested that abstract schema induction occurs more readily from two source analogs. *[I note that with the hint, people use the schema – so perhaps it is not the schema that is better formed, but the ability to use it. This is kind of a weird result – leads to idea of salience – the more salient schema is induced by comparison of 2 analogs? Suggests the need to tease apart abstraction of schema and salience of schema.]*

Givry, D. & Roth, W.M. (2006) Toward a New Conception of Conceptions: Interplay of Talk, Gestures, and Structures in the Setting. *J. of Research in Science Teaching*, 43(10):1086-1109

This paper goes beyond analysis purely of language in analysis of students' conceptions and conceptual change to draw on talk, gestures, and semiotic resources in the setting to "propose a redefinition of the nature of conception." They videotaped 14 student interviews (1-hr each), 48 hours of classroom videotape, 420 questionnaires, and 160 pages of worksheets. Examine students conceptions of pressure (with piston and gauges) and find an analysis incorporating talk, gesture, and other semiotic resources (such as diagrams) not only better describes "concepts", but includes features of concepts that an analysis of talk alone does not. See also the work of Goldin-Meadow on gesture analysis.

Glynn, S.M. (1991). Explaining science concepts : a Teaching-With-Analogies mode. In S.M. Glynn, R.H. Yeany and B.K. Britton (Eds.), *The Psychology of Learning Science* (pp. 219-239). Hillsdale, NJ, Erlbaum.

This chapter describes a step-by-step process of teaching with analogy.

Goldstone, R.L. and Sakamoto, Y. (2003) The transfer of abstract principles governing complex adaptive systems, *Cog. Psych. 46, 414-446*

Key finding that concrete reps facilitate learning within domain, but abstract reps facilitate transfer (but can hinder also). This effect found more for low achieving students compared to high achieving students. *[I note here that this may mean high-achieving students have already created their own blends or abstract schemata.]* They conducted four experiments using computer simulations of complex systems (e.g., dropping balls into an area with several local minima). Experiments 1-3 showed better transfer of abstract principles across situations that were relatively dissimilar, and that this effect was due to participants who performed relatively poorly on the initial simulation. Experiment 4, subjects showed better understanding of a simulation when depicted with concrete rather than abstract graphical elements. However, for poor performers, the idealized (abstract) version transferred better to a new simulation governed by the same abstraction. Results interpreted in terms of competition between concrete and abstract. 'Individuals prone to concrete construals tend to overlook abstractions when concrete properties or superficial similarities are salient."

Hesse, M. (1966) *Models and Analogies in Science.* University of Notre Dame Press

This is an often cited book on analogy use in the sciences. Suggest analogy as a key process of scientific reasoning and scientific advance. Hesse argues for analogy as a heuristic for scientific practice, communication and understanding by practitioners.

Hewitt, P.G. (1987) *Conceptual Physics*, p513, Addison-Wesley: Menlo Park, CA.

Contains a water circuit analogy for electric circuit, with diagram showing both systems. The two systems are drawn to cue similarities and cross-space mappings between the two domains by means of spatial proximity of the features to map. E.g., the valve is in the same place in the water circuit as the switch in the electric circuit.

Holyoak, K.J. & Koh, K. (1987) Surface and structural similarity in analogical transfer. *Memory and Cognition, 15(4) 332-340*

Showed that spontaneous transfer can be obtained even after a delay of several days between presentation of the source and the target, if the source and target have at least one salient surface similarity. They conducted two experiments. Experiment 1: demonstrated spontaneous analogical transfer after a delay of several days. Experiment 2: both structural and salient surface features influence spontaneous selection of an analog. Structural features have a greater impact than do surface features on a problem solver's ability to use an analogy once its relevance has been pointed out. Suggest four steps to using analogy

i. Constructing mental representations of the source and target
ii. Selecting the source as a potentially relevant analog to the target (this is perhaps the least understood step)
iii. Mapping the components of the source and target
iv. Extending the mapping to generate a solution to the target (Holyoak, 1984)

[I note this assumes problem solving as goal. Also inherently linear – this approach may be oversimplification.]

Interesting point made on p. 2 "Gick and Holyoak (1983) interpreted these and other more detailed results to indicate that induction of an explicit schema facilitates transfer. Once a person has induced a schema from initial examples, novel problems that can be categorized as instances of the schema can be solved without necessarily directly accessing representations of the initial analogues. It follows that although experiments illustrating the role of schemata demonstrate spontaneous inter-domain transfer, they do not provide clear evidence *of analogical* transfer, in the sense of direct transfer from a representation of a particular prior situation to a novel problem. A major goal of the present study was to identify conditions under which spontaneous analogical transfer in fact occurs." Shared features serve as retrieval cues in a "content-addressable memory system". *[I note, we should be careful about positing mental representations as overly coherent, or unitary. This framing seems to suggest that there are preexisting chunks of significant enough size to contain individual features and represent whole situations. It may be, rather, that mental representation or patterns of activation are fleeting, much less stable.]*

Introtroduce the idea of s*tructure-preserving difference* – difference in surface similarity that does not change structural similarity. (Contrast with *structure-violating* difference.)

Experiment 1: source is fixing a broken light bulb filament with a laser, but the laser has to be divided and recombined to avoid breaking the glass (structural analog to x-rays divided and recombined to destroy tumor). Delay of several days. They get >81% spontaneous transfer in analogy group, 10% in the control group.

Experiment 2: 4 versions of light bulb story to examine surface vs. structural similarity

i. Story 1: same as experiment 1, laser, fragile glass (88% spontaneous solution)

ii. Story 2: laser, insufficient intensity (40%)

iii. Story 3: Ultrasound used to break filament apart, fragile glass (56%)

iv. Story 4: Ultrasound, insufficient intensity (13%)

v. Note significant increase after hint.

Conclusion: both surface and structural similarity affects retrieval.

Holyoak K.J. & Thagard, P. (1997) The Analogical Mind. *Am Psychol. Jan;52(1):35-44.*

This is one of the key papers on multi-constraint theory. Multi-constraint theory introduces three *constraints* on analogical mapping (a la Gentner): structure, similarity, and purpose. The most interesting of these is *purpose* which claims that the purpose, or goal, of an analogy can guide which analogy is chosen and even which mappings are chosen. Notably, different mappings can occur between the same two domains depending on the purpose constraint.

Hrepic, Z., Zollman, D. & Robello, S. (2005) Eliciting and representing hybrid mental models, *Proceedings of the NARST 2005 Annual Meeting.*

Introduces hybrid-mental models, which seem to be akin to blends. Focus on student understanding of sound waves based on different analogies and combinations of these analogies. Student models of sound may be meta-stable and different models (i.e., analogies such as people passing footballs) may appear depending on context of probing student thinking. I used some of these models to create the sound waves quiz question on analogies to sound.

Iding, M.K. (1997). How analogies foster learning from science texts. *Instructional Science. 25:233-253*

Survey of analogy use in textbooks. Find that textbooks use many analogies in a number of ways. Delineates a categorization for types of analogies used in textbooks.

Keane, M. (1987) On Retrieving Analogues When Solving Problems, *Quarterly Jour of Experimental Psychology 39A, 29-41*

Two experiments – similarity vs. analogy in tumor problem.

Experiment 1 – people readily use story about a surgeon to solve tumor problem (88%) (structural and surface similarity matches); people do not readily use fortress analogy to solve tumor problem (12%) (structural matches only), but do use when given hint

Experiment 2 – people more likely (~50%) to use a story about shooting lasers through the atmosphere to solve tumor analogy; however, no difference between whether lasers are called "rays" (exact semantic match for x-rays) or "beams" (??) (conceptual match but not exact semantic)

Pedone, R., Hummel, J. E., & Holyoak, K. J. (2001). The use of diagrams in analogical problem solving. *Memory & Cognition, 29, 214--221.*

Role of representations in analogy and Dunker's radiation problem. Found that certain diagrams could enhance spontaneous transfer, but some did not. Animations significantly increased transfer. *[I note an incomplete theoretical understanding of why these results on analogy and representation.]*

Roth, W.M. and Bowen, G.M. (1999) Complexities of graphical representations during lectures: A phenomenological approach, *Learning and Instruction 9, 235*

This paper introduces the semiotic triangle; sign, referent, interpretant *[I note that analogical scaffolding replaces interpretant with schema, drawing on analogy work on schema induction schema abstraction. See Gick and Holyoak (1983)]* 39 lectures of a course on ecology were recorded and analyzed, 36 seminars of students problem solving, and 14 scientists videotaped interpreting graphs Key findings:

v. The normally existing mutually-constitutive relationship between phenomena and their graphical representation is not sufficiently elaborated.

vi. Important relationships proper to ecology are not maintained when mundane examples are chosen ad hoc.

What makes it so difficult to understand graphs in lectures? Graphing as a social, semiotic practice. Graphs are useful to scientists because they (a) constitute the best tools to represent covariation between continuous measures and (b) are useful to summarize large amounts of data in economical ways.

They suggest that there is a belief that there is an isomorphism of the type world <-> mathematics (nature has a mathematical structure, see also romance of mathematics in Lakoff and Nunez). To quote Roth and Bowen, "…any such isomorphism is established by means of agreed upon practices (Latour, 1993). First, there are multiple transformations across ontological gaps that separate any two representations of some phenomenon. Then, descriptions of these practices are removed so that they no longer appear in discovery accounts. Finally, because the transformation practices are invisible, the isomorphism between the experienced world and mathematics appears to have existed all along."

Introduce the reflexive relationship between graphs and ecological phenomena. Data and graphs are "mutually constitutive: scientists understand familiar situations because of the graphs and understand graphs because of their experience with familiar situations." Use *Peircean semiotics*, interpretation is played out in the relations between a sign (text), a thing referred to by the sign (referent), and a gloss on the sign in its relation to the interpreter (interpretant). The primary relation to be explained and elaborated is that between a sign vehicle (S) and a referent (R). A second interpretant can mediate the first S-I relationship. The S-I association however realized is rendered possible only in a community with shared experiences (Ricoeur, 1991). Semiosis is the process of producing ever new S-I relationships. R-I is phenomenological experience, S-R is hermeneutic experience. A final good quote is, "Using this triangle as a frame allows us to identify critical issues arising from understanding graphs when they are presented during and as part of lecture."

Sandifer, S. (2004) Spontaneous Student Generated Analogies. *Proceedings of the 2003 Physics Education Research Conference, vol. 720, p. 93-96*

Studied student generated analogies. Found several factors that can promote spontaneous generations of analogies – interestingly, one of the key factors is group work and sharing of student ideas between the students.

Sloutsky, V. M., Kaminsky, J.A. and Heckler, A.F. (2005) The Advantage of Simple Symbols for Learning and Transfer, *Psychonomic Bulletin and Review, 12 (3), 508-513*

Studied abstract vs. concrete reps, transfer, and learning mathematics. Focused on learning modulo 3 arithmetic using different representations: abstract (simple shapes like circles or diamonds), and concrete (pictures of insects). Found that concrete representations lead to significantly less transfer to modulo 3 arithmetic problems compared to abstract. Intended to question the notion that concrete representations are more interesting or exciting to students and lead to better learning. This paper demonstrated that for learning abstract mathematics, irrelevant concrete is actually harmful to learning the

abstractions. (I.e., pretty pictures are not only not useful but can actually be harmful).

Spiro, R.J., Feltovich, P.J., Coulson, R.L., & Anderson, D.K. (1989) Multiple analogies for complex concepts: antidotes for analogy-induced misconception in advanced knowledge acquisition. In Vosniadou, S. and Ortony, A. (eds.), *Similarity and analogical reasoning.* Cambridge University Press, Cambridge.

Propose using multiple analogies for teaching in order to overcome problems or shortcomings of individual analogies. This paper outlines a teaching lesson in biology using analogies for the working of muscle fibers. They do not present any empirical data in this paper.

Sutton, C.R. (1978). Metaphorically Speaking: The role of metaphor in teaching and learning science. *Leicester University School of Education Occasional Papers*

- Metaphor, analogy, simile, narrative defined and discussed.

Turner, M. (2006) Compression and representation. *Language and Literature, 15(1) 17-27*

- Example of seeing rain and mental schema of rain, blended with a picture of rain falling – how the picture is blended with the phenomenon.
- Question of whether the picture blends with the phenomenon, or the mental representation of the picture blends with the mental representation of the phenomenon (safer to stick with the latter interpretation – but the former may be in line with a view that considers the blending system as including the sign, referent, and individual simultaneously. Situated cog view?)

Van Heuvelen, A. & Zou, X. (2001) Multiple representations of work-energy processes. *Am. J. Phys. 69(2):184-194*

Teaching thermodynamics concepts with layered representations of increasing abstraction.

References

1 Seuss, D. (1949) *Bartholomew and the Oobleck.* Random House Books for Young Readers.

2 Podolefsky, N.S., Zhong, S., & McNamera, A.D. (2004) The anisotropic and rheological structure of the oceanic upper mantle from a simple model of plate shear." *Geophysical Journal International* 158, 1

3 Maxwell, J.C. (1885). Abstract of Paper 'On Faraday's Lines of Force', in W. D. Niven (ed.) *The Scientific Papers of James Clerk Maxwell (*p367-369) Cambridge, MA: Cambridge University Press, 1890; New York, NY: Dover, 1952.

4 Rutherford, E. (1911) The Scattering of α and β Particles by Matter and the Structure of the Atom, *Philos. Mag.*, 6, 21

5 Hesse, M.B. (1966). *Models and analogies in science*. Notre Dame, IN: University of Notre Dame Press.

6 Nersessian, N. (2002) Maxwell and "the Method of Physical Analogy": Model-based Reasoning, Generic Abstraction, and Conceptual Change. In D.B. Malament (ed.) *Reading Natural Philosophy: Essays in the History and Philosophy of Science and Mathematics* (pp. 129-166). Chicago, IL: Open Court.

7 Sutton, C. (1978) Metaphorically Speaking: The Role of Metaphor in Teaching and Learning Science. *An Occasional Paper – School of Education Series*. University of Leicester School of Education.

8 Brookes, D.T. & Etkina, E. (2007) Using conceptual metaphor and functional grammar to explore how language used in physics affects learning. *Physical Review ST Phys. Educ. Res.* 3, 010105

9 Curtis, R.V. & Reigeluth, C.M. (1984) The use of analogies in written text. *Instructional Science* 13:99-117

10 Iding, M.K. (1997) How analogies foster learning from science texts. *Instructional Science*. 25:233-253

11 Halliday, D., Resnick, R., & Walker, J. (2004) *Fundamentals of physics (7th Edition)*. John Wiley & Sons, Inc.

12 Maxwell, J.C. (1881) *Elementary Treatise on Electricity.* Oxford, England: Clarendon Press.

13 Hammer, D., Elby, A., Scherr, R.E., & Redish, E.F. (2005) Resources, framing, and transfer, in J. Mestre (ed.) *Transfer of Learning from a Modern Multidisciplinary Perspective* (pp. 89-120). Greenwich, CT: Information Age Publishing

14 Gentner, D., Bowdle, B.F., Wolff, P. & Boronat, C. (2001) Metaphor is like Analogy. In D. Gentner, K.J. Holyoak, & B.N. Kokinov (eds.) *The analogical*

mind: Perspectives from cognitive science. (pp. 199-253) Cambridge, MA: MIT Press

15 Lakoff, G (1993). *The Contemporary Theory of Metaphor*. In A. Ortony (ed.), *Metaphor and Thought, Second Edition.* New York: Cambridge University Press.

16 Lakoff, G. & Johnson, M. (1980) *Metaphors we Live By*. Chicago, IL: University of Chicago Press

17 Lakoff, G. & Nunez, R. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*. New York: Basic Books

18 Gentner, D. (1998). Analogy. In W. Bechtel & G. Graham (eds.), *A companion to cognitive science* (pp. 107-113). Oxford: Blackwell.

19 Gentner, D. (1999) Analogy. In R.A. Wilson & F.C. Keil (eds.) *The MIT encyclopedia of the cognitive sciences* (pp. 17-20). Cambridge, MA: MIT Press.

20 Rumelhart, D.E. & McClelland, J.L. (1986) *Parallel Distributed Processing: Explorations in the microstructure of cognition*. Cambridge, MA: MIT Press

21 Gentner, D (1983) Structure-mapping: A theoretical framework for analogy. *Cognitive Science*, 7:155-170.

22 Holyoak K.J. & Thagard, P. (1997) The Analogical Mind. *Am. Psychol.* Jan, 52(1):35-44.

23 Holyoak, K.J. (2005) Analogy. In K.J. Holyoak & R.G. Morrison (eds.) *The Cambridge Handbook of Thinking and Reasoning*, Cambridge University Press.

24 Spellman, B.A. & Holyoak, K.J. (1992) If Saddam is Hitler then who is George Bush? Analogical mapping between systems of social roles. *Journal of Personality and Social Psychology*, 62, 913-933.

25 Brown, D.E., & Clement, J. (1989) Overcoming misconceptions via analogical reasoning: Factors influencing understanding in a teaching experiment. *Instructional Science*, 18:237-261.

26 Chalmers, D.J., French, R.M., & Hofstadter, D.R. (1992) High-level Perception, Representation, and Analogy: A Critique of Artificial Intelligence Methodology, *Journal of Experimental and Theoretical Artificial Intelligence* 4, 185

27 Brown, D.E. (1992) Using Examples and Analogies to Remediate Misconceptions in Physics: Factors Influencing Conceptual Change, Journal of Research in Science Teaching 29(1), 17-34

28 Brown, D.E. (1994) Facilitating conceptual change using analogies and explanatory models, International Journal of Science Education 16(2), 201

29 Fauconnier, G. & Turner, M. (2003) *The Way We Think: Conceptual Blending and the Mind's Hidden Complexities*, Basic Books

30 Hobbes, T. (1651) *The Leviathan*.

31 Locke, J. (1689) *An Essay Concerning Human Understanding*.

32 Petrie, H.G. & Oshlag, R.S. (1993). Metaphor in Education. In A. Ortony (ed.), *Metaphor and Thought, Second Edition.* New York: Cambridge University Press.

33 Brophy, S. P., & Schwartz, D. L. (1998) Interactive analogies. In D. Edelson & E. Domeshek (eds.), *Proceedings from the 1996 International Conference on the Learning Sciences* (pp. 351-356). Evanston, IL: Association for the Advancement of Computing in Education.

34 Paatz, R., Ryder, J., Schwedes, H., & Scott, P. (2004) A case study analyzing the process of analogy-based learning in a teaching unit about simple electric circuits. *International Journal of Science Education*. 26(9):1065-1081

35 Reiner, M., Slotta, J. D., Chi, M. T. H., & Resnick, L. B. (2000) Naïve physics reasoning: A commitment to substance-based conceptions. *Cognition and Instruction*, 18, 1-34.

36 Simanek, D.E. *The Dangers of Analogies*. http://www.lhup.edu/~dsimanek/scenario/analogy.htm

37 Gentner, D. & Gentner, D.R. (1983). Flowing waters or teeming crowds: Mental models of electricity. In D. Gentner & A. Stevens (eds.), *Mental Models*. Lawrence Erlbaum Press.

38 Clement, J. (1988). Observed Methods for Generating Analogies in Scientific Problem Solving. *Cognitive Science.* 12(4):563-586

39 Sandifer, S. (2004) Spontaneous Student Generated Analogies. *Proceedings of the 2003 Physics Education Research Conference*, vol. 720, p. 93-96

40 Glynn, S.M. (1991). Explaining science concepts : a Teaching-With-Analogies mode. In S.M. Glynn, R.H. Yeany and B.K. Britton (eds.), *The Psychology of Learning Science* (pp. 219-239). Hillsdale, NJ: Erlbaum.

41 Kurtz, K.J., Mao, C., Gentner, D. (2001) Learning by Analogical Bootstrapping. *The Journal of the Learning Sciences*, 10(4):417-446

42 Catrambone, R. & Holyoak, K.J. (1989) Overcoming Contextual Limitations on Problem Solving Transfer, Journal of Experimental Psychology: Learning, Memory, and Cognition 15(6), 1147-1156

43 Dunker, K. (1945) On problem solving. *Psychological monographs*, 58 (Whole No. 270).

44 Gick, M.L. & Holyoak, K.J. (1980) Analogical problem solving. *Cognitive Psychology*, 12, 306-355.

45 Gick, M.L. & Holyoak, K.J. (1983) Schema Induction and Analogical Transfer. *Cognitive Psychology*, 15, 1-38.

46 Pedone, R., Hummel, J. E., & Holyoak, K. J. (2001). The use of diagrams in analogical problem solving. *Memory & Cognition, 29, 214--221.*

47 Keane, M. (1987) On Retrieving Analogues When Solving Problems, *Quarterly Jour of Experimental Psychology 39A, 29-41*

48 Chi, M., Feltovich, P., & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science*, 5(2), 121-152.

49 Redish, E.F. (2003) *Teaching Physics with the Physics Suite*. John Wiley & Sons, Inc.

50 Holyoak, K.J. & Koh, K. (1987) Surface and structural similarity in analogical transfer. Memory and Cognition, 15(4) 332-340

51 Blanchette, I. & Dunbar, K. (2000) How Analogies are Generated: The Roles of Structural and Superficial Similarity. *Memory & Cognition 29, 730-735*

52 Clement, J. (1993). Using Bridging Analogies and Anchoring Intuitions to Deal with Students' Preconceptions in Physics, *Journal of Research in Science Teaching*, 30(10), 1241-1257

53 Meltzer, D.E. (2005) Relationship between students' problem-solving performance and representational format. *Am. J. Phys.* 73**(**5), 463-478

54 Kohl, P.B. & Finkelstein, N.D. (2005) Student representational competence and self-assessment when solving physics problems, *Phys. Rev. ST Phys. Educ. Res.* 1, 010104

55 Kohl, P.B. & Finkelstein, N.D. (2006) Effects of representation on students solving physics problems: A fine-grained characterization, *Phys. Rev. ST Phys. Educ. Res.* 2, 010106

56 Kohl, P.B. & Finkelstein, N.D. Patterns of Multiple Representation Use by Experts and Novices During Physics Problem Solving. *In review at Phys. Rev. ST Phys. Educ. Res.*

57 diSessa, A.A. (2004) Metarepresentation: Native Competence and Targets for Instruction. *Cognition and Instructions*, 22(3):293-331.

58 diSessa, A.A. & Sherin, B.L. (2000) Meta-representation: an introduction, *Journal of Mathematical Behavior* 19, 385

59 Ochs, E., Gonzales, P., Jacoby, S. (1986) "When I come down I'm in the domain state": grammar and graphic representation in the interpretive activity of physicists. In E. Ochs, E. Schegloff, & S. Thompson (eds.) *Interaction and grammar*. Cambridge: Cambridge University Press.

60 diSessa, A. A., Hammer, D., Sherin, B., & Kolpakowski, T. (1991). Inventing graphing: Metarepresentational expertise in children. *Journal of Mathematical Behavior* 10: 117-160.

61 Schwartz, D.L. (1993). The Construction and Analogical Transfer of Symbolic Visualizations. *Journal of Research in Science Teaching*. 30(10):1309-1325

62 Podolefsky, N.S. & Finkelstein, N.D. (2006) Use of Analogy in Learning Physics: The Role of Representations, *Phys. Rev. ST Phys. Educ. Res.* 2, 020101

63 Ambrose, B.S., Heron, P.R.L., Vokos, S. & McDermott, L.C. (1999). Student understanding of light as an electromagnetic wave: Relating the formalism to physical phenomena, *Am. J. Phys.* 67(10), 891

64 Mazur, E. (1997) *Peer Instruction: A User's Manual* New Jersey: Prentice Hall

65 H-ITT: see http://www.h-itt.com

66 McDermott, L.C. & Schaffer, P.S. (2001) *Tutorials in Introductory Physics*, New Jersey: Prentice Hall

67 Hestenes, D., Wells, M. & Swackhammer, G. (1992) Force Concept Inventory, *The Physics Teacher* 30, 141

68 Bao, L. & Redish, E.F. (2000) What can you learn from a (good) multiple-choice exam? Contributed paper, *GIREP Conference Physics Teacher Education beyond 2000*, Barcelona Spain, August 2000.

69 http://www.theonion.com

70 Hrepic, Z., Zollman, D., & Robello, S. (2005) Eliciting and representing hybrid mental models, *Proceedings of the NARST 2005 Annual Meeting*.

71 diSessa, A.A. (1998) Knowledge in pieces, in G. Foreman & P. Pufall (eds.) *Constructivism in the computer age*.

72 J. Lemke, (1998) Multiplying Meaning: Visual and Verbal Semiotics in Scientific Text, in J.R. Martin and R. Veels (eds.), *Reading Science: Critical and Functional Perspectives on Discourses of Science*. London: Routledge

73 Roth, W.M. & Bowen, G.M. (1999) Complexities of graphical representations during lectures: A phenomenological approach, *Learning and Instruction* 9, 235

74 Kozhevnikov, M., Hegarty, M., & Mayer, R.E. (2002) Revising the Visualizer-Verbalizer Dimension: Evidence for Two Types of Visualizers, *Cognition and Instruction* 20(1), 47

75 Elby, A. (2000) What students' learning of representation tells us about constructivism, *Journal of Mathematical Behavior* 19, 481

76 diSessa, A.A. & Sherin, B.L. (1998) What changes in conceptual change? *International Journal of Science Education* 20(10), 1155

77 Wittmann, M.C. (2002) The object coordination class applied to wavepulses: Analyzing student reasoning in wave physics, *International Journal of Science Education* 24(1), 97

78 Podolefsky, N.S. & Finkelstein, N.D. Reframing analogy: framing as a mechanism of analogy use, *Proceedings of the 2006 Physics Education Research Conference*, AIP Press

79 Feynman, R.P. (1965) *The Character of Physical Law*. Cambridge, MA: MIT Press.

80 Podolefsky, N.S. & Finkelstein, N.D. (2006) Use of Analogy in Learning Physics: The Role of Representations, *Phys. Rev. ST Phys. Educ. Res.* 2, 020101

81 Spiro, R.J., Feltovich, P.J., Coulson, R.L,. & Anderson, D.K. (1989) Multiple analogies for complex concepts: antidotes for analogy induced misconception in advanced knowledge acquisition. In S. Vosniadou, & A. Ortony (eds.) *Similarity and analogical reasoning*. Cambridge: Cambridge University press

82 Turner, M. & Fauconnier, G. (1995) Conceptual Integration and Formal Expression. Metaphor and Symbol, 10(3):183-204 (1995)

83 Fauconnier, G. (2001) Conceptual blending and analogy. In D. Gentner, K.J. Holyoak, & Kokinov (eds.) *The Analogical Mind.* Cambridge, Mass.:MIT Press

84 Namy, LL. & Gentner, D. (2002) Making a Silk Purse Out of Two Sow's Ears: Children's' Use of Comparison in Category Learning. *Jour. of Experimental Psych: General*, 131(1), 5-15

85 Goldstone, R.L. & Sakamoto, Y. (2003) The transfer of abstract principles governing complex adaptive systems. *Cog. Psych.*, 46, 414-466

86 Sloutsky, V.M., Kaminsky, J.A., & Heckler, A.F. (2005) The Advantage of Simple Symbols for Learning and Transfer, *Psychon. Bull. and Rev.*, 12 (3), 508-513

87 Brookes, D.T. & Etkina, E. (2006) Do our words really matter? Case Studies From Quantum Mechanics. *Proceedings of the 2005 Physics Education Research Conference*, AIP Press

88 Schwartz, D.L. (1995) Reasoning about the referent of a picture versus reasoning about the picture as the referent: an effect on visual realism. *Memory and Cognition*, 23(3), 709-22

89 Tornkvist, S., Pettersson, K.A. & Transtromer, G. (1993) Confusion by representation: On students' comprehension of the electric field concept. *Am. J. Phys.* 61 (4)

90 Rumelhart, D.E. (1980) Schemata: The Building Blocks of Cognition. In R.J. Spiro, B.C. Bruce, & W.F. Brewer (eds.), *Theoretical Issues in Reading Comprehension* (pp. 38-58). Hillsdale, NJ: Erlbaum.

91 Turner, M. (2006) Compression and representation. *Language and literature*, 15(1):17-27

92 Wittmann, M.C. (2006) Using resource graphs to represent conceptual change. *Phys. Rev. ST Phys. Educ. Res.* 2, 020106

93 Hayakawa, S.I. (1972) Language in Thought and Action (3rd Edition), Harcourt Brace Jovanovich, Inc.

94 Strike K.A. & Posner G.J. (1985) A conceptual change view of learning and understanding. In L.H.T. West & A.L. Pines (eds.) *Cognitive Structure and Conceptual Change* (pp. 211-231). New York: Academic Press.

95 McCloskey, M. (1983) Intuitive Physics. *Scientific American*, 249, 122.

96 Attributed, found at http://wikiquote.org

97 Allie, S. University of Capetown, SA, personal communication.

98 Lemke, J. (1998) Multiplying Meaning: Visual and Verbal Semiotics in Scientific Text, in J.R. Martin & R. Veels (eds.), *Reading Science: Critical and Functional Perspectives on Discourses of Science*. London: Routledge

99 Dunbar, K. (1999) How Scientists Build Models: InVivo Science as a Window on the Scientific Mind. In L. Magnani, N. Nercessian, & P. Thagard (eds.) *Model-based reasoning in scientific discovery*. Plenum Press.

100 Givry, D. & Roth, W.M. (2006) Toward a New Conception of Conceptions: Interplay of Talk, Gestures, and Structures in the Setting. *J. of Research in Science Teaching*, 43(10):1086-1109

101 Podolefsky, N.S. & Finkelstein, N.D. (2007) *Analogical Scaffolding and the Learning of Abstract Ideas in Physics: An example from electromagnetic waves*, Phys. Rev. ST Phys. Educ. Res. 3, 010109

102 Van Heuvelen, A. & Zou, X. (2001) Multiple representations of work-energy processes. *Am. J. Phys.* 69(2):184-194.

103 Touger, J. (2005) *Introductory Physics: Building Understanding*, John Wiley & Sons

104 www.lon-capa.org

105 Attributed, found at http://thinkexist.com

106 Lave, J. & Wenger, E. (1991) Legitimate Peripheral Participation. Cambridge, MA: Cambridge University Press.

107 Wenger, E. (1999) Communities of Practice. Cambridge, MA: Cambridge University Press.

108 Hutchins, E. (1995) How a cockpit remembers its speeds. *Cognitive Science*, 19(3): 265-288

109 Vosniadou, S. & Ortony, A. (eds.) (1989) *Similarity and Analogical Reasoning*. Cambridge University Press: Cambridge.

110 Vosniadou, S. (1989) Analogical reasoning as a mechanism in knowledge acquisition: a developmental perspective. In S. Vosniadou and A. Ortony (eds.) *Similarity and analogical reasoning* (pp. 413-437), Cambridge: Cambridge University Press