
Historical Methods: A Journal of Quantitative and Interdisciplinary History

ISSN: 0161-5440 (Print) 1940-1906 (Online) Journal homepage: https://www.tandfonline.com/loi/vhim20

# Increasing returns to scale in the towns of early Tudor England

Rudolf Cesaretti, José Lobo, Luis M. A. Bettencourt & Michael E. Smith

HISTORICAL METHODS: A JOURNAL OF QUANTITATIVE AND INTERDISCIPLINARY HISTORY
https://doi.org/10.1080/01615440.2020.1722775



# Increasing returns to scale in the towns of early Tudor England

Rudolf Cesaretti[a], José Lobo[b], Luis M. A. Bettencourt[c,d], and Michael E. Smith[a]

[a]School of Human Evolution and Social Change, Arizona State University; [b]School of Sustainability, Arizona State University; [c]Mansueto Institute for Urban Innovation, University of Chicago; [d]Santa Fe Institute

**ABSTRACT**

Urban agglomeration economies make cities central to theories of modern economic growth. There is historical evidence for the presence of Smithian growth and agglomeration effects in English towns c.1450-1670, but seminal assessments deny the presence of agglomeration effects and productivity gains to Early Modern English towns. This study evaluates the presence of increasing returns to scale (IRS) in aggregate urban economic outputs—the empirical signature of feedbacks between Smithian growth and agglomeration effects—among the towns of 16th century England. To do so, we test a model from settlement scaling theory against the 1524/5 Lay Subsidy returns. Analysis of these data indicates that Tudor towns exhibited IRS—a finding that is robust to alternative interpretations of the data. IRS holds even for the smallest towns in our sample, suggesting the absence of town size thresholds for the emergence of agglomeration effects. Spatial patterning of scaling residuals further suggests regional demand-side interactions with Smithian-agglomeration feedbacks. These findings suggest the presence of agglomeration effects and Smithian growth in pre-industrial English towns. This begs us to reconsider the economic performance of Early Modern English towns, and suggests that the qualitative economic dynamics of contemporary cities may be applicable to premodern settlements in general.

**KEYWORDS**

Settlement scaling; Early Modern Europe; 1524/5 Lay Subsidy; agglomeration economies; Smithian growth; urban economics

## Introduction

Cities are central to theories of modern economic growth because of the invention, innovation, and specialization generated by urban agglomeration economies (Lucas 1988; Romer 1990; Weil 2008). Agglomeration economies are cost savings and non-market-mediated benefits produced by a dense network of individuals, firms, and institutions concentrated in space (Fujita, Krugman, and Venables 1999; Glaeser and Gottlieb 2009; Storper and Venables 2004). These include logistical cost savings via spatial proximity, as well as economies of scale in access to human capital, knowledge, institutions, infrastructure, and other public goods (Duranton and Puga 2004; Glaeser and Gottlieb 2009; Storper 2010, 2013). Since larger cities command greater spatial economies of scale, a crucial outcome of agglomeration effects is *increasing returns to scale* (IRS) in aggregate urban economic output (Bettencourt 2013; Fujita, Krugman, and Venables 1999; Lobo et al. 2013; Weil 2008). IRS in aggregate urban economic output (hereafter simply IRS) is the result and empirical signature of feedbacks between agglomeration effects and Smithian growth[1] (Batty 2013; Bettencourt, Samaniego, and Youn 2014; Fujita, Krugman, and Venables 1999; Fujita and Thisse 2002; Krugman 1991; Lobo et al. 2013; Ottaviano and Thisse 2004).

The economic performance of towns in Early Modern England is a matter of debate because of its implications for the drivers of Smithian growth leading to the Industrial Revolution. While many of the qualitative outcomes of agglomeration economies are historically documented in Early Modern English towns, the paucity of adequate data has so far inhibited the direct statistical identification of either agglomeration effects or their economic outcomes. Seminal theorists have argued that preindustrial English towns exhibited insignificant productivity gains due to inherent limitations in technology, institutions, and the organization of production (Mokyr 1990, 1995; Ogilvie 2004; Wrigley 1978, 1990). Other work has suggested that modern agglomeration economies were nonexistent in Britain before the 18th century (Hohenberg and Lees 1995; Jedwab and Vollrath 2015; Mokyr 1995). As a result, preindustrial urbanization in Britain has primarily been seen as a

**CONTACT** Rudolf Cesaretti rcesaret@asu.edu School of Human Evolution and Social Change, 900 S. Cady Mall, Tempe, AZ 85281, USA.

Supplemental materials for this article are available at https://doi.org/10.1080/01615440.2020.1722775.

consequence of productivity growth in other sectors rather than a cause of productivity growth in its own right (Broadberry et al. 2015; Mokyr 1999, 2002, Wrigley 1985, 2004). Yet the statistical identification of agglomeration effects like IRS in Early Modern English towns would indicate that they were also a mechanism of Smithian growth. Given the major episode of urbanization in England c.1450-1670, this could mean that premodern towns were a driver of the British economic takeoff.

In this paper, we evaluate the presence of IRS in aggregate urban economic outputs among the towns[2] of early 16th century England. Due to the hazards of estimating production functions for 16th century England, we instead use the analytical framework of settlement scaling theory to evaluate the presence of IRS. Settlement scaling theory posits that IRS emerge from the dense spatial agglomeration of networked socioeconomic interactions rather than specific economic mechanisms. This means that IRS can be measured as a function of settlement population size and aggregate economic output (Bettencourt 2013; Lobo et al. 2013). This is made possible through analysis of the 1524/5 Lay Subsidy tax returns, which uniquely meet the necessary data requirements. The 1524/5 Subsidies provide directly corresponding population and economic output proxy data for a large sample of towns—assessed in the same way, at the same time, at the level of individual settlements, across the urban system. After demonstrating the robust existence of IRS in early Tudor towns, we then analyze its relationship with town size and regional demand, before finally discussing their implications for English economic history.

## The economic performance of English towns, c.1450-1670

English towns c.1450-1670 have long been wrapped-up in pessimistic debates about urban economic stagnation. Leading scholars have hypothesized that urban productivity growth was limited before the Industrial Revolution, after which technological progress enabled the emergence of efficiency-driven industries based on in capital-intensive factory production (Crafts and Harley 1992; Mokyr 1995, 1999, Wrigley 2006, 2010). From this perspective, major technological breakthroughs and firm-level economies of scale were necessary prerequisites for agglomeration effects. The small-scale organization of urban production in towns hindered Smithian growth, such that any gains from specialization were small and attributable to increased working hours (Mokyr 1995; Wrigley 1978, 1990). Productivity gains from innovation and specialization were therefore negligible before machine technology, resulting in negligible added output per capita in specialized urban industries before the 18th century (Crafts and Harley 1992; Mokyr 1995). Many also argue that backward institutions like guilds and corporate towns raised transaction costs and resisted innovation via monopolistic controls over production and labor, shifting investment to rural proto-industry (Mokyr 1997; Ogilvie 2004, 2007).

Yet other scholars have cast serious doubt on the validity of these theoretical assertions. First, the small-scale organization of urban production dominated English towns during the 18th century urban takeoff. Small-scale specialized artisans were the basis of secondary production from the 16th to the mid-19th century introduction of the factory system (Berg 1993, 1994b; Crouzet 1985; Hudson 1986, 1989). The vast majority of innovation c.1450-1670 and c.1670-1840 were "micro-inventions"[3] driven by small-scale urban artisans in guilds, among whom competition fostered inventiveness, patent activity, intense copying, imitation, and the stealing of innovations (Berg 1993, 1994a, 1999; Britnell 2000a; Clifford 1995; Mokyr 1990). Moreover, urban guilds provided the essential system of vocational training until the 1830 s (Ben-Amos 1994; Brooks 1994; Epstein 1998; Epstein and Prak 2008; Lane 1996; Prak 2018). Rather than institutional transaction costs, the higher cost of town labor was due to higher-skill, full-time specialized production—as opposed to low skill, off-season rural labor used in proto-industry (Britnell 2000a; Clay 1984a).

Unfortunately, the fragmentary quantitative data on town economies is neither sufficiently consistent through time nor comparable across cases to falsify hypotheses about English towns' economic performance. Without good economic data, historians have pointed to various socioeconomic trends as evidence of towns' stagnating economic performance. English towns of this period are best known for their loss of trade and mercantile autonomy to London (Britnell 2000a; Nightingale 1996), lower unskilled wages compared to London (Allen 2009; Wrigley 1985), and for the decline of their textile export industries to rural proto-industry (Britnell, 2000; Clay 1984a, 1984b). Only London has been hailed as an "engine of economic growth" from the 15th century forward due to its size, astronomical growth, and high-wage economy (Allen 2009; Fisher 1990; Wrigley 1967). Most assessments of town economies in England have ascribed the onset of productivity growth and agglomeration economies to the "new" towns of the 18th century (Balderston 2009; Crafts and Harley 1992; Hohenberg and Lees 1995; Mokyr 1995, 1999; Trinder 2000; Wrigley 1985, 1990).

As a result, theories of Smithian productivity growth in Early Modern England have not been extended to towns. Before the 18th century takeoff, this has mostly been attributed to agriculture and leading rural industries (Allen 2009; Broadberry et al. 2015; Overton 1996; Wrigley 1985, 2010). This has shifted scholarly focus onto urban macroeconomic contributions to demand (Broadberry et al. 2015; Wrigley 1985, 2006). Urbanization ratios[4] thus serve as proxies for non-urban productivity (Malanima 2009; see Persson 2010; Wrigley 1985). From this perspective, towns are primarily characterized as *loci* of consumption and distribution, and thus passive drivers of demand—reduced to stimulating productivity increases in agriculture and rural industry (Mokyr 1999, 2002, Wrigley 1985, 2004). These assumptions are embedded in leading national accounting models of preindustrial growth (e.g., Broadberry et al. 2015).

## *Agglomeration effects and Smithian growth*

Smithian growth is intimately connected to the origins of agglomeration effects. At the macro-level, specialization is driven by the growth of markets and demand (Kelly 1997; Persson 2010). But at the micro-level, specialization is driven by "learning by doing"–the generation of specialized knowledge that enables production to be focused on increasingly specific outputs. This involves imitating and learning from others, refining processes, streamlining logistics, and innovating production practices (Arrow 1962; Solow 1997). Agglomeration effects are a catalyst of learning by doing. Whereas Smithian growth involves learning by doing within a formal occupation, agglomeration effects enhance the flow of information and skills among them. Agglomeration effects produce a feedback that diffuse specialized knowledge throughout the local network of individuals, firms, and institutions. Urban agglomeration economies are thus a non-market-mediated catalyst of Smithian growth, and the result of this Smithian-Agglomeration positive feedback is IRS (Bettencourt 2013; Bettencourt, Samaniego, and Youn 2014; Fujita, Krugman, and Venables 1999; Lobo et al. 2013).

A growing number of studies have argued that Early Modern cities exhibited agglomeration effects (e.g., Davids and De Munck 2014; O'Brien 2001; van Zanden 2009). Wahl (2016) has argued that urban economic growth in Medieval Europe was driven by IRS from Smithian-Agglomeration feedbacks. Specifically, agglomeration economies accelerated the specialization of the division of labor and urbanization, fostered by the growth of macro-level forces like markets, trade, and regional demand. The resulting urban productivity growth and agglomeration effects circularly reinforced the growth of regional demand, trade, and markets. Over time, Smithian-Agglomeration feedbacks developed geographical specializations and enduring core-periphery patterns (Wahl 2016).

There is substantial historical evidence to suggest similar Smithian-Agglomeration feedbacks among English towns c.1450-1670. Town economies became increasingly specialized in high-skill industries, often connected to the burgeoning regional specializations of rural industry (Britnell 1993, 2000a, Clay 1984a, 1984b; A. Dyer, 1995; Goose 1982, 1986; Keibek 2016; Patten 1978; Slack 2000). This involved small-scale urban specialists who drove commercial innovations at the apex of regional production chains (Britnell 2000a; Clay 1984a, 1984b; Duncan 1976; Thirsk 1973). For example, the urban textile industry created specialized commercial innovations like the 'New Draperies' to capture foreign demand (Coleman 1969; A. Dyer, 1995; Munro 1997), as well as novel clothing styles to spur local consumption (Clay 1984a; Harte 1991; Spufford 1984, 2000). Likewise, town mercantile trade the transport industries grew specialized regional networks c.1500-1700, and their innovations (technology, infrastructure, logistics, institutions, new occupations) engendered a revolution in the efficiency of overland trade and transport (Chartres 1977; Clay 1984a; Gerhold 1993; Slack 2000). These advances were made possible by human capital development via the proliferation of novel urban institutions such as the apprentice system, schools, civic libraries, booksellers, and tutors (Brooks 1994; Clay 1984b; Glennie and Whyte 2000; Humphries 2003; Slack 2000).

There is also evidence that town specialization resulted in Smithian growth. Allen (2009) has argued that the New Draperies underwent productivity growth before the 18th century, and that town specialization "led directly to greater efficiency and higher wages." Goldstone (1984) has argued that the English price revolution c.1500-1650 was an outcome of an exponential increase in monetary velocity stemming from Smithian growth and IRS in towns. At the level of national accounting, Broadberry and colleagues (2015) have found evidence of considerable productivity growth in both the 'trade and transport' and 'specialty consumer industries' sectors before 1700. By correcting for by-employment biases in probate inventories, Keibek (2016) has demonstrated that town occupational specialization was very high, and that the English secondary sectoral shift actually occurred in the 16th and 17th centuries. This suggests that national accounting estimates of Smithian growth c.1500-1700 are too low, and that the parallel growth of towns was an important locus of these processes.

The demand forces postulated by Wahl (2016) are well documented. Inland and coastal trade underwent a massive proliferation c.1500-1700, resulting in gradual domestic market integration (Chartres 1977; Clay 1984a; Gerhold 1993). The expansion of overseas trade connected England to global markets (Allen 2009; Cuenca Esteban 2004). Demand for domestic and imported consumer goods grew considerably c.1500-1700 (Clay 1984b), and probate inventories suggest that consumption of luxury manufactures was growing steadily by 1600 (Overton et al. 2004). Productivity growth in agriculture and rural industry further stimulated rural demand for urban manufactures, and there is evidence for a growth in rural working days per capita to increase consumption (Broadberry et al. 2015; Clay 1984a, 1984b; Spufford 1984; Wrigley 1985).

## Settlement scaling theory

Settlement scaling theory provides a formal framework that can both quantitatively identify IRS and explain the causes of its emergence in pre-industrial urban economies. This begins with the recognition of quantitative empirical regularities in the average properties of human settlements. Whereas settlements' physical infrastructure and settled area exhibit economies of scale with respect to population, various socioeconomic outputs (e.g., GDP, patents) exhibit IRS with respect to population (Bettencourt 2013). Researchers have long been aware of such scaling relationships (e.g., Batty 2008; Glaeser and Gottlieb 2009; Glaeser and Sacerdote 1999; Nordbeck 1971; Samaniego and Moses 2009), which exhibit specific scale-invariant exponents (or 'elasticities'). Using the power law form $Y(N) = Y_0\, N^{\,b}$, where $N$ is settlement population and $Y$ is some settlement property, infrastructure and settled area have characteristic scaling exponents $b$ of about 5/6, and socioeconomic outputs have exponents of about 7/6 (Bettencourt 2013; Bettencourt et al. 2007; Bettencourt et al. 2010; Lobo et al. 2019). The scale-invariance of these properties means that are evident throughout the urban hierarchy, from small villages to the largest metropolis. In addition to modern cities (Bettencourt 2013; Bettencourt and Lobo 2016), these scaling regularities have been observed in diverse premodern contexts including Medieval Europe (Cesaretti et al. 2016), Classical Antiquity (Hanson and Ortman 2017; Hanson, Ortman, and Lobo 2017), and the Prehispanic Americas (Ortman et al. 2016; Ortman et al. 2014, 2015; Ortman and Coffey 2017).

The models of settlement scaling theory derive and predict these elasticities as a function of generic socioeconomic network interactions embedded in physical space. These 'socioeconomic' benefits are not limited to market-mediated economic transactions of traditional economic models, but include other non-market-mediated benefits like new information and knowledge, skills and innovations, social connections, and political favors that are crucial to the production of agglomeration effects (Fujita, Krugman, and Venables 1999; Glaeser and Gottlieb 2009; Storper and Venables 2004). The mechanisms of settlement scaling theory models are thus sufficiently general to be applied to premodern settlements regardless of technology, level of economic development, or the character of institutions.

Settlement scaling theory models also employ extensive (aggregate) metrics of urban variables rather than intensive (per capita) ones. This is necessary because intensive metrics conflate scale-driven effects—primarily driven by the non-market-mediated mechanisms of agglomeration economies and learning by doing—with the market-driven mechanisms of productivity growth central to traditional economic models (i.e., technology, firm-level economies of scale, etc.). The framework proposes that any extensive property of the $i$th settlement, $Y_i$, is a power law function of its population, $N_i$, such that

$$Y_i = Y_0 N_i^{\beta} e^{\xi}, \tag{1}$$

where $Y_0$ is a baseline prefactor, $\beta$ is a dimensionless scaling exponent, and $\xi$ represents the deviation of $Y_i$ from the expected system-wide, scale-free relationship. When $Y_i$ is some socioeconomic output (e.g., GDP), the prefactor $Y_0$ captures average intensive economic performance, produced by macroeconomic characteristics of the urban system. Deviation from this trend is governed by $e^{\xi}$ —representing the settlement-specific variability ('error') in intensive economic performance not produced by scale-driven effects (i.e., market mechanisms).

The mechanics of the settlement scaling model begin by positing that all human settlements can be characterized as social networks embedded in physical space. As such, settlements are taken to be containers of frequent social network interaction, where the aggregate number of interactions within settlements is assumed to be much greater than the total number of external interactions. Within this spatially embedded social network, people organize their lives around a set of economic, social, and energetic (or 'technological') constraints to balance the socioeconomic costs and benefits of urban life. This can be seen as a short-term spatial equilibrium produced by constraints derived as settlement-wide averages.

To model this short-term spatial equilibrium, the average costs of mobility through urban space, $c$, must equal the per capita net benefits of socioeconomic interactions, $y$. The costs of spatial movement can be approximated by typical length scale of the city, $L = \sqrt{A}$, such that

$$c = \varepsilon A^{1/2}, \tag{2}$$

where $A$ is the area of the city, and ε is the cost of movement per unit length. Assuming that all social interactions have socioeconomic benefits, $y$ can be written as is a function of the density of social interactions per unit time, which can be approximated by population density, such that

$$y = G\frac{N}{A}, \tag{3}$$

where $N/A$ is the average population density and $G$ represents the average gross socioeconomic benefit of social interactions. The parameter $G$ is itself a function of average per capita gross socioeconomic output, $g$, the average individual interaction strength with others, $a_0$, and the average distance traveled per person across settlement area, $l$, so that

$$y = G\frac{N}{A} = ga_0 l\frac{N}{A} \tag{4}$$

By equating these costs (Eq. 2) and benefits (Eq. 4), we can derive the optimal spatial extent of the city as a function of its population size, $N$, such that

$$A(N) = \left(\frac{G}{\varepsilon}\right)^{2/3} N^{2/3} = \alpha N^{2/3} \tag{5}$$

The settled area *A(N)* therefore increases with more productive interactions, and the population density decreases with more productive interactions. This means that productivity growth and/or decreasing transportation costs enable cities to increasingly sprawl out across the landscape—a pattern observed across cities worldwide (Angel et al., 2012).

Because infrastructure modifies the costs of mobility, the 'space' created by these infrastructure networks is dimensionally distinct from the physical area of a settlement. We refer to this as "infrastructural area," $A_I$, which is the primary network utilized by people to conduct their socioeconomic interactions given its cost efficiency. Assuming that there is a quantity of infrastructure per capita, $d$, that is proportional to circumscribing area:

$$d \sim \left(\frac{A(N)}{N}\right)^{1/2}, \tag{6}$$

Assuming the network space is proportional to population, we can derive the scaling relation:

$$\begin{aligned} A_I \approx Nd = N\left(\frac{A(N)}{N}\right)^{1/2} &= A(N)^{1/2}N^{1/2} \\ = \left(\alpha N^{2/3}\right)^{1/2}N^{1/2} &= \alpha^{1/2}N^{5/6}, \end{aligned} \tag{7}$$

Assuming that the socioeconomic outputs, $Y$, of settlements are proportional to social network interactions, it follows that gross socio-economic outputs are simply a function of per capita net benefits of socioeconomic interactions, $y$, multiplied by population, $N$, such that

$$Y(N) = yN \tag{8}$$

Assuming that social networks in any settlement will be as well-mixed as possible given the above spatial cost-benefit constraints, we can write

$$Y(N) = \frac{GN(N-1)}{A} \approx \frac{GN^2}{A} \tag{9}$$

Substituting the value of $A_I$ from Eq. (7) for $A$ in Eq. (9) we can derive expected scaling outputs:

$$Y(N) = \frac{GN^2}{\alpha^{1/2}N^{5/6}} = Y_0 N^{7/6} \tag{10}$$

Settlement scaling theory makes the explicit prediction that the scaling coefficients for socioeconomic outputs should be $\beta_{output} = 7/6 \cong 1.17$. A scaling coefficient greater than 1 thus indicates IRS in economic output with respect to population size (Bettencourt 2013). Per equation (10), IRS (superlinear scaling effects) emerge from the ability of larger population agglomerations to sustain larger and denser socioeconomic networks.

## The 1524/5 Lay Subsidy dataset

The Lay Subsidies of 1524 and 1525 were taxes levied by the Crown on the private wealth of England,[5] and include both taxpayer counts and tax receipts for a large sample of towns.[6] Levied at the municipal level, each over a one-year period, the 1524/5 subsidies have a fine spatial and temporal resolution given their geographical scope. Those liable to taxation included all men over age 15 and female heads of household whose assessed 'moveable goods' (including coin, rents on property, and credit) or annual wages were worth £1 or more (Schofield 2004). The tax rolls count all but the very poor, most servants, the clergy, children, and attached women (A. Dyer 2000a; Goose and Hinde 2007). Annual wages and 'moveable goods' were assessed as separate categories, each placed in value brackets (£1–2, £2–10, £10–20, £20–40, etc.), with a flat payment for each bracket. Individuals assessed in more than one category would only be taxed under the category yielding the highest payment (Schofield 2004).

The 1524/5 Lay Subsidies are unique in their ability to evaluate the presence of IRS in Early Modern towns. Reliable taxation data proportional to economic output were rarely assessed for all social strata, in the same way, at the same time, at the level of individual settlements, across an urban system. Even more rare are data with directly corresponding population and tax estimates. Population is usually only estimable from records where people were not assessed and taxed in proportion to their wealth/income (e.g., censuses, poll or hearth taxes). In cases where taxpayers were reliably assessed in proportion to wealth/income, the number of taxpayers is either not recorded or only county-level returns survive. This means that population and economic output must be estimated from different taxes more than 50 or 100 years apart. For some regions, population and socioeconomic output must be estimated for each individual settlement from one or more idiosyncratic local taxes, at different dates, assessing different things for different wealth strata, and often only the largest towns have data to assemble in either manner (e.g., Italy and the Holy Roman Empire). Not only can the 1524/5 Subsidies overcome these issues, they were comparatively comprehensive and accurate, and their well-studied biases make it possible to subject them to contextually valid sensitivity analysis (*see* Sections Accuracy and Possible Systematic Biases from Exemption).

The core data used in this paper comprise a sample of the largest 93 towns with both taxpayer and tax data in either 1524 or 1525.[7] These data are available to the reader in the online supplementary materials. The largest surviving return from either year was used in all cases. Tax receipts are rounded to the nearest Pound, and a lower threshold of 150 taxpayers was used to bound the sample (see below). Numerous small towns and a few larger towns (Southwark, Newcastle, Chester, Durham, Kendal and Rye) were either not included in the subsidies or lack surviving taxpayer counts. Thus, the data analyzed in this paper is a sample, the comprehensiveness of which increases going up the urban hierarchy.

### *Interpretation*

The 1524/5 Lay Subsidies have been widely used by historians as demographic and economic metrics for England's provincial towns. As a demographic metric, taxpayer counts are thought to be proportional to town populations, and the 1524/5 taxpayer figures are the most frequently used data for estimating contemporary town populations (e.g., A. Dyer, 1995, Dyer 2000c; C. Dyer, 1995; Goose and Hinde 2007; Rigby 2010). As an economic measure, historians have traditionally treated the 1524/5 tax receipts as proportional to household 'wealth,' used to evaluate the distribution of private wealth across space (e.g., Darby et al. 1979; Schofield 1965; Sheail 1968), among towns (e.g., Bridbury 1962; Britnell 2000a; A. Dyer, 1995; Sheail 1968), and within towns (e.g., Cornwall 1988; Hoskins 1976).

However, the 1524/5 returns do not measure 'wealth' because land and other real estate constituted the vast majority of capital before the 18th century (Piketty 2014; Piketty and Zucman 2014). The 1524/5 Subsidies taxed all wages, moveable capital, coin, credit and rents—including all incomes earned from land and other real estate. This makes them an ideal proxy for economic output in premodern economies constrained by capital immobility, high liquidity preference, low effective demand, and low velocity of circulation (Keynes 1964[1936]; Malanima 2009b). As such, the 1524/5 returns provide a proxy metric more along the lines of income-based GDP.[8]

### *Validity*

Controversy about the validity of the 1524/5 Lay Subsidies primarily surrounds their direct comparability with the 1377 Poll Tax and the 1334 Lay Subsidy (Bridbury 1962, 1992, A. Dyer, 1995, Dyer 2000c, Rigby 1979, 2010). We sidestep this debate by restricting their use to the immediate 1524-5 period, shifting focus onto their internal coherence.

In order to cross-check the validity of the 1524/5 Lay Subsidy data, we first subjected the univariate distributions of taxes and taxpayers to analysis. A detailed discussion of this analysis is available in section S1.1 of the online supplementary materials. Log-normal distributions indicate that the data's underlying generative processes are unconstrained (i.e., strongly interacting) and multiplicative, which is expected for distributions of urban population and economic output (Batty 2006; Gibrat 1931; Limpert, Stahel, and Abbt 2001; Montroll and Shlesinger 1982). The sample of tax returns ($n = 93$) is approximately log-normal, and a two-sided Kolmogorov-Smirnov (KS) test for normality of the logged tax returns ($D = 0.08$; $p > 0.73$) failed to reject the Gaussian null hypothesis. While the sample of taxpayer counts ($n = 93$) is *not* clearly log-normal, it does exhibit a strong Zipfian fat right tail ($b = -0.35$). Zipfian distributions have frequently been shown to be a pronounced feature of the right tails of log-normal distributions for modern urban variables with threshold size cutoffs like our sample threshold of 150 taxpayers (Adamic and Huberman 2002; Batty 2006; Gabaix 1999; Pumain 2000). As expected, an expanded sample of towns from the 1524/5 Lay Subsidy with more than 50 taxpayers ($n = 134$) is approximately lognormal, such that a two-tailed KS normality test of the logged taxpayer counts ($D = 0.06$; $p > 0.65$) fails to reject the Gaussian null hypothesis. This finding of both log-normality and Zipfian behavior for the sample data indicates that they exhibit statistical properties typical of modern urban data. This suggests that the 1524/5 Lay Subsidy data are proportional to socioeconomic output and population.

### Accuracy

The accuracy of the 1524/5 Lay Subsidy returns has also been called into question for several reasons, which we review here to place controls on the data for scaling analysis. To do so, the cases most susceptible to these errors are isolated and subdivided from the total into corresponding error-type subsets. All possible combinations of these error-type subsets can then be analyzed to evaluate the sensitivity of our results to each potential source of error. A more detailed summary and analysis these issues is available in Section S1.2 of the online supplementary materials.

First, it has been suspected that municipalities across England were not assessed consistently (e.g., A. Dyer 2000c; Goose and Hinde 2006). However, Roger Schofield's analysis of the subsidy yields of 1524-72 relative to their corresponding probate inventories indicates that the 1524/5 subsidies were the most accurate taxes of the era (R. Schofield 2004, pp. 98–102, 168–77, 201–18). Considering the distributional analysis above, we find no reason to believe that the suspected errors invalidate the data for scaling analysis. Inconsistent assessments should result in essentially random errors and lower correlations.

Second, 16 towns have been explicitly identified as having problematic taxpayer numbers and/or tax receipts due to underassessment or deficient returns (A. Dyer 2000a, 2000c; Goose and Hinde 2006; Sheail 1968). We therefore group these 16 "problematic" cases together as a subset for use in sensitivity analysis.

Third, the 1524/5 tax returns are vulnerable to municipalities that included both towns and their rural hinterlands (see Goose and Hinde 2006). The 25 towns with under 200 taxpayers are therefore grouped together with the five cases named above. Together these form the "small town/rural boundary" error-type subset for sensitivity analysis. This issue also sets constraints on the smallest valid town size that should be included in our dataset, which is why the sample analyzed here ($n = 93$) was limited to towns with more than 150 taxpayers.

### Possible systematic biases from exemption

The number of people not counted by the taxes is also a potential source of error. This is the proportion of town populations that were not counted in the 1524/5 taxpayer rolls, either due to exemption from poverty or because their demographic category was not liable to taxation. To estimate the total population, historians always multiply the taxpayer counts by constant 'modifiers' derived from other available demographic data (see e.g., A. Dyer 2000c; Goose and Hinde 2007; Rigby 2010). The use of constant multipliers assumes that the proportion of uncounted persons does not covary with town population. If this assumption is valid, such that taxpayer counts are roughly proportional to population (with some degree of unstructured variability), then only the actual taxpayer counts are necessary to conduct scaling analysis.

However, some scholars have suggested that larger towns may have had greater proportions of exempted taxpayers due to higher poverty rates (Cornwall 1962b; Rigby 2010). If so, then the undercounting of potential taxpayers in larger towns would cause the systematic underestimation of their population. If this error were was large, then the use of raw taxpayer counts, or estimates of population with a constant multiplier, could systematically bias regression results

**Table 1.** Scaling analysis of error-type subsets in 1524/5 Lay Subsidy. Log-linear OLS regression of log tax on log taxpayer count.

| Subset | $n$ | $R^2$ | $\beta$ | $\beta$ 95% C.I | $\alpha$ | $\alpha$ 95% C.I |
|---|---|---|---|---|---|---|
| All Cases | 93 | 0.676 | 1.270 | [1.10, 1.44] | 0.051 | [0.02, 0.14] |
| Without Municipal/Rural Error Subset | 68 | 0.677 | 1.255 | [1.07, 1.44] | 0.056 | [0.02, 0.17] |
| Without Problem Error Subset | 77 | 0.719 | 1.229 | [1.05, 1.41] | 0.065 | [0.02, 0.18] |
| Excluding Both Error Subsets | 55 | 0.739 | 1.284 | [1.09, 1.48] | 0.047 | [0.01, 0.15] |

by increasing the value of estimated scaling ($\beta$) coefficients. This feature of the Lay Subsidy town data has not previously been addressed by historians in any systematic way. In order to control for the possibility of systematic error, we undertake a sensitivity analysis of the data in Section Sensitivity analysis, below.

Another potential source of systematic bias is the exemption of the clergy and ecclesiastical institutions. The distribution of clerical wealth across English towns in the early 16th century remains unstudied (Jurkowski 2016), but it is possible that larger towns had greater untaxed income/moveables on average than small towns (Cornwall 1988). However, correcting such a bias for larger towns would only increase estimated scaling exponents in favor of IRS. (*See the online supplementary materials for details*).

## Scaling analysis

To evaluate the 1524/5 Lay Subsidies data, we first compare the predictions of scaling theory to estimated regression parameters. If the provincial towns of Tudor England exhibited IRS, then the estimated scaling exponent $\beta$ will be $> 1$. If this estimated scaling exponent conforms to the predictions of settlement scaling theory, then $\beta$ should be roughly 7/6. Scaling parameters were estimated through OLS of the natural log-transformed variables:

$$\ln(tax_i) = \alpha + \beta \ln(taxpayers_i) + \xi \tag{11}$$

where $i$ indexes an individual town and $\xi$ denotes stochastic variation.[9] Eq. (11) was estimated using OLS with the Huber/White correction for heteroscedasticity in R using the "sandwich" package (R Core Team 2016; Zeileis 2004).

### *Results by Error-Type subset*

Table 1 presents the log-linear OLS estimated scaling coefficients for all error-type subsets. Four ($n = 4$) different analyses were conducted using combinations of the error subsets. All four analyses exhibit IRS in aggregate urban economic performance in accordance with the predictions of settlement scaling theory. The estimated $\beta$ coefficients are all strongly superlinear, and their 95% C.I.s exclude 1 in all cases (Table 1). All estimated $\beta$ coefficients exceed the theoretically predicted value of 7/6 ($\approx$1.167). While high, these estimates are well within the realm of possibility, as the scaling of socioeconomic rates in modern cities has been observed as high as 1.47 (Lobo et al. 2019). Moreover, the estimated $\beta$ and $\alpha$ coefficients of all error-type subset combinations are clustered closely together, indicating that these subsets had a relatively small effect on the results. Some of the error subset cases are outliers, but many others are located near the OLS line (Figure 1). Taken together, this result indicates that Tudor towns exhibited strong IRS that is robust to known sources of error in the data.

## Sensitivity analysis

As noted above, it is possible that that larger towns had greater proportions of poverty-based taxpayer exemptions. This could systematically bias regression results by increasing the value of estimated scaling coefficients. To test the robustness of our scaling results to this possibility, we employed several methods to model "worst case scenario" (WCS) population datasets. These WCS population models were specifically designed to "stack the cards against" IRS by systematically making the largest towns larger and the smallest towns smaller.

The WCSs were produced using three different methods. The first method, dubbed "WCS1," is based on ranges of modifier values estimated by scholars for converting taxpayers into population counts. These modifier ranges are then extended well beyond the original ranges estimated by scholars, and linearly extrapolated onto the range of 1524/5 town taxpayers—such that the largest town has the highest modifier, and the smallest town has the lowest modifier. This linear extrapolation greatly exaggerates the degree of systematic error.

The second method, WCS2, uses available historical data on taxpayer exemption rates and average household size in Tudor towns. This conforms to the assumptions of historians who interpret Lay Subsidy taxpayers as households (e.g., A. Dyer 2000c; Patten 1978). As with the modifier ranges, a large range of taxpayer exemption rates is directly and linearly

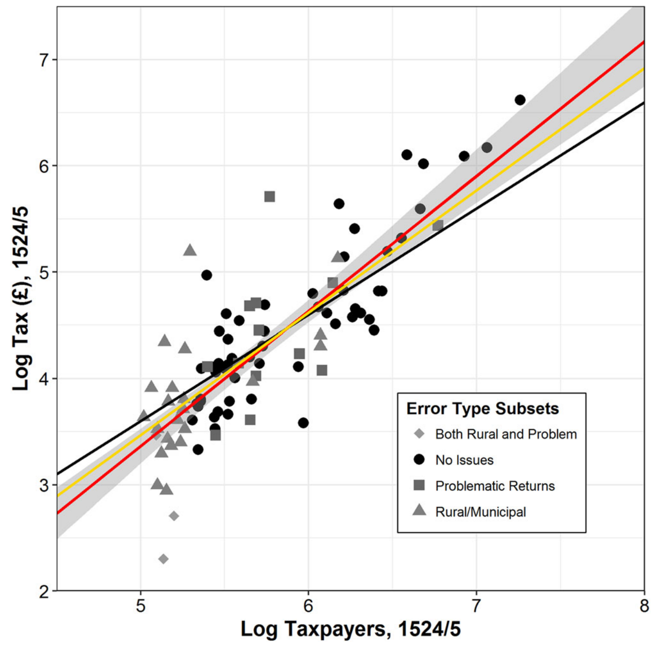


**Figure 1.** Estimation of 1524/5 Tax ~ Taxpayer scaling relation for all towns (n = 93). the figure legend indicates the error subset status of individual cases. The black line represents proportionate (linear) scaling; the yellow line is the theoretical prediction where $\beta = 7/6$; the gray shaded region is the 95% confidence interval for the estimated OLS parameters; the red line is the OLS estimate using the log-transformed data.

extrapolated onto taxpayer counts. Taxpayers are then converted into population numbers using period demographic statistics on the mean household size for differently sized towns.

WCS3 is very similar to WCS2, except that it conforms to the assumptions of historical demographers who interpret taxpayers as adult males (e.g., Goose and Hinde 2006, 2007). As such, the same range of taxpayer exemption rates is directly and linearly extrapolated onto taxpayer counts, but taxpayers are converted into population numbers using period demographic statistics on the age structure and sex ratios of differently sized towns.

The WCS datasets deliberately make exaggerated assumptions compared to the accepted methods of English historical demography. WCS town populations were to intentionally destroy the superlinearity of the estimated scaling coefficients by making the population of towns directly covary with the number of taxpayers. The magnitude of this covariation was modeled to the greatest extent possible from the existing evidence, and for many towns the WCS populations extend considerably beyond realm of possibility. (See Sections S2 through S5 of the online supplementary materials for details).

### *Results*

Table 2 summarizes the results of the sensitivity analysis, reporting the average correlation coefficients and parameter values for the WCSs (datasets D through L in Table 2). Included for comparison are the original taxpayer dataset (A) and two realistic datasets based on Dyer's (2000c) population estimates (B and C). Each of these 12 datasets was also evaluated for each of the 4 error-type subsets from Section Scaling analysis, resulting in 48 regressions.

Estimated scaling coefficients were superlinear ($\beta > 1$) in all 36 WCS cases *despite our deliberate attempt to eliminate their superlinearity*. This indicates that the finding of IRS in early Tudor towns is robust. The estimated WCS scaling coefficients range from 1.06 to 1.17, with a mean of 1.12. The distribution of estimated coefficients is unimodal and two-tailed. The WCS $\beta$ estimates overlap with the settlement scaling theory prediction of 7/6 ($\approx$1.167), as well as most IRS scaling coefficients observed for the socioeconomic rates of modern industrialized cities.[10] Given that these estimates come from 'worst case scenarios,' this suggests that the IRS of Tudor towns is very robust to a wide range of extreme demographic assumptions about the 1524/5 Lay Subsidies.

## IRS and town size

Threshold population sizes are commonly used to classify settlements as 'urban' for use in proxy metrics of the scale and organization of the urban economy. A key issue here is qualitatively distinguishing rural

**Table 2.** Regression results summarized by dataset. Mean values for correlation coefficients, parameter estimates, and confidence intervals used to summarize the results of each dataset for all error-type subsets (n = 4). Each of the 12 datasets in this table (labeled A-L) summarizes 4 regressions. Full regression results available in Table A9 of the online appendix.

| | Dataset | Method | Modifier Range | Mean $R^2$ | Mean $\beta$ | Mean $\beta$ 95% C.I.* | Mean $\alpha$ |
|---|---|---|---|---|---|---|---|
| A | Taxpayers | Raw Data | – | 0.71 | 1.26 | [1.08, 1.44] | 0.055 |
| B | Dyer (2000c)** | Detailed | 6 | 0.67 | 1.23 | [1.02, 1.44] | 0.007 |
| C | Dyer (2000c) Ext† | Estimates | 6 | 0.67 | 1.19 | [1.00, 1.39] | 0.009 |
| D | WCS1 | | [4, 6] | 0.71 | 1.10 | [0.94, 1.24] | 0.029 |
| E | WCS1 | Extended Modifier | [5, 7] | 0.71 | 1.12 | [0.97, 1.28] | 0.019 |
| F | WCS1 | Ranges | [5.5, 7.5] | 0.71 | 1.13 | [0.97, 1.29] | 0.016 |
| G | WCS1 | | [6, 8] | 0.71 | 1.14 | [0.98, 1.30] | 0.014 |
| H | WCS1 | | [7, 9] | 0.71 | 1.15 | [0.99, 1.32] | 0.010 |
| I | WCS2 | Exemption % | [5.94, 7.66] | 0.71 | 1.13 | [0.98, 1.29] | 0.014 |
| J | WCS2 Honor‡ | and Households | [5.5, 9.05] | 0.71 | 1.09 | [0.93, 1.23] | 0.027 |
| K | WCS3 | Exemption % | [4.52, 6.55] | 0.72 | 1.12 | [0.97, 1.28] | 0.016 |
| L | WCS3 Honor‡ | and Adult Males | [4.22, 7.42] | 0.72 | 1.08 | [0.93, 1.22] | 0.029 |

*Calculated by separately taking the means of the upper and lower 95% C.I. limits estimated for each dataset.

**The population estimates of A. Dyer (2000c) use a constant modifier, but are 'corrected' case-by-case using contextual information from other historical sources ($n = 63$). This is the only dataset analyzed that does not use the full 1524/5 Lay Subsidy town dataset ($n = 93$).

†This dataset ($n = 93$) extends Dyer's (2000c) original dataset ($n = 63$) to the remaining 30 small towns in our dataset using his modifier of 6x to convert taxpayers into populations.

‡These WCS dataset variants use the same methods, with the exception that towns with proxy data for their 1524/5 exemption rates are used instead of the linearly extrapolated estimates. These exemption data include wide outlier cases, which has the effect of further extending the modifier range and exaggerating the WCS assumptions. The inclusion of this proxy data in place of the linear model is referred to as 'honoring the data.'

villages from urban economies characterized by agglomeration effects like IRS. Most prior work has used arbitrary threshold population levels to classify urban centers, based loosely on assumptions about nonagricultural employment and occupational structure (e.g., Bairoch 1991; De Vries 1984; Wrigley 1985). Estimated thresholds often range from 2000-5000. More recently, Ploeckl (2017) has argued that urban economies have sufficiently complex divisions of labor when town growth is independent of local agricultural conditions, which should serve as a method to identify thresholds. In all cases, the implication is that small towns (below the threshold) are part of the rural economy—important as marketing centers but lacking urban economic performance.

To identify town population thresholds at which IRS emerge, scaling analysis was performed for different town size ranges. As in Section Sensitivity analysis, this only intended as a robustness test to determine whether IRS disappears for smaller subsets of towns. Our sample is much less complete for small towns, so subsets of small towns greatly magnify the sampling error. We are therefore careful not to overinterpret these regression results, especially those with lower $R^2$.

Table 3A shows log-linear OLS results for Early Tudor towns divided by size into three equal groups ($n = 31$, to satisfy the central limit theorem). Large, middling, and small towns all exhibit IRS ($\beta > 1$) with $\beta$ estimates comparable to those found in modern cities (Lobo et al. 2019). Middling and small towns exhibit much lower $R^2$, but the models exhibit good visual fit. This suggests that even small towns with fewer than 1,000 inhabitants exhibited IRS.

To double-check this result, Table 3B shows regression results for a stepwise removal of larger towns to detect a threshold size at which IRS disappears. Removing larger towns does not destroy IRS, and subsets of progressively smaller towns exhibit minor $\beta$ estimate deviations from the estimates in Section Scaling analysis (well within the margin of error for range-restricted subsamples). Reasonable $R^2$, visual fit, and plausible $\beta$ estimates are maintained until the subset falls well below the sample size needed to satisfy the central limit theorem ($n = 18$, $\beta = 2.6$). This demonstrates that IRS is evident throughout the settlement hierarchy, and not an artifact of the large towns in our sample.

This suggests that that there was no town size threshold for IRS in early Tudor towns. At minimum, any such threshold was below 1,000 for the early Tudor urban system. The small towns of our sample were highly dependent on local agricultural conditions, challenging Ploeckl's (2017) theory that this obstructs economic performance by constraining the complexity of town divisions of labor. Towns with fewer than 2,000 people also generally had higher fractions of primary sector employment than larger towns (A. Dyer 2000b; Glennie and Whyte 2000; Keibek 2016), challenging the idea that occupational structure set a qualitative threshold on town economic performance.

This result conforms to the predictions of settlement scaling theory, which suggests that agglomeration effects like IRS are scale-free properties of urban systems. Rather than the type of economic activity controlling a qualitative threshold, productivity

**Table 3.** Scaling analysis of town size ranges in 1524/5 lay subsidy. Log-Linear OLS regression of log tax on log taxpayer count.

| Analysis | Rank Range (Pop) | N | Taxpayer Range | Population Range (6x Taxpayers) | $\beta$ | $\alpha$ | $R^2$ | Avg. Tax per Taxpayer |
|---|---|---|---|---|---|---|---|---|
| (A) Towns Split into Thirds | 1–31 | 31 | 1423–391 | 8538–2346 | 1.54 | 0.0090 | 0.71 | 0.288 |
| | 32–62 | 31 | 380–231 | 2280–1386 | 1.13 | 0.1200 | 0.37 | 0.268 |
| | 63–93 | 31 | 230–151 | 1380–906 | 1.16 | 0.0900 | 0.31 | 0.246 |
| (B) Stepwise Removal of Larger Towns | 1–93 | 93 | 1423–151 | 8538–906 | 1.27 | 0.0500 | 0.67 | 0.267 |
| | 6–93 | 88 | 784–151 | 4704–906 | 1.16 | 0.0900 | 0.55 | 0.258 |
| | 11–93 | 83 | 611–151 | 3666–906 | 1.11 | 0.1200 | 0.48 | 0.253 |
| | 16–93 | 78 | 531–151 | 3186–906 | 1.19 | 0.0810 | 0.46 | 0.257 |
| | 21–93 | 73 | 480–151 | 2880–906 | 1.15 | 0.1000 | 0.40 | 0.251 |
| | 26–93 | 68 | 433–151 | 2598–906 | 1.23 | 0.0600 | 0.41 | 0.252 |
| | 31–93 | 63 | 382–151 | 2292–906 | 1.23 | 0.0600 | 0.41 | 0.256 |
| | 36–93 | 58 | 307–151 | 1842–906 | 1.22 | 0.0700 | 0.41 | 0.245 |
| | 41–93 | 53 | 289–151 | 1734–906 | 1.22 | 0.0700 | 0.41 | 0.243 |
| | 46–93 | 48 | 267–151 | 1602–906 | 1.22 | 0.0700 | 0.42 | 0.246 |
| | 51–93 | 43 | 250–151 | 1500–906 | 1.23 | 0.0600 | 0.46 | 0.246 |
| | 56–93 | 38 | 237–151 | 1422–906 | 1.22 | 0.0700 | 0.45 | 0.242 |
| | 61–93 | 33 | 231–151 | 1386–906 | 1.25 | 0.0600 | 0.35 | 0.242 |
| | 66–93 | 28 | 221–151 | 1326–906 | 1.18 | 0.0800 | 0.31 | 0.233 |
| | 71–93 | 23 | 207–151 | 1242–906 | 1.05 | 0.1700 | 0.35 | 0.238 |
| | 76–93 | 18 | 199–151 | 1194–906 | 2.66 | 4.0e-05 | 0.10 | 0.226 |

appears to continuously covary with the scale of town divisions of labor. This can be seen in Table 3, where the average tax per taxpayer suggests a gradual increase in productivity moving up the urban hierarchy. As elaborated in Section The economic performance of English towns, c.1450-1670, the growth of English towns c.1500-1700 tended to involve the growth and specialization of their secondary and tertiary sectors. Our analysis suggests that Smithian-Agglomeration feedbacks were an underlying driver of urban growth and the sectoral shift of town economies, yielding gradual productivity gains at the urban level as smaller towns developed larger and more complex divisions of labor.

## Scaling residuals and regional demand

As noted above, leading scholars have argued that growing demand for urban goods and services did not lead to meaningful productivity growth in preindustrial English towns (e.g., Mokyr 1999, 2002, Wrigley 1985, 2004). In contrast, Wahl (2016) argues that regional demand played a key role in Medieval urban economic growth by stimulating Smithian-Agglomeration feedbacks. If Smithian-Agglomeration feedbacks resulted in productivity growth, then the economic performance of early Tudor towns should covary with regional demand.

To test this possibility, scaling residuals from Section Results by error-type subset were first tested for spatial autocorrelation. The regression residuals of scaling analysis are an aggregate measure of economic output, standardized to factor-out the influence of scale on productivity (i.e., city size, land area). As such, the scaling residuals represent the economic success or failure of towns relative to other towns, enabling direct comparison (Bettencourt et al. 2010). The scaling residuals for early Tudor towns ($n = 93$) displays clear spatial autocorrelation. Global Moran's I produced a significant standard deviate of 2.1 ($p = 0.017$), indicating considerably greater (positive) autocorrelation than would be expected at random. This indicates that town economic performance was spatially organized and that town economic performance may have been connected to regional demand.

To explore this further, we analyze the relationship between the scaling residuals and the available county-level proxy data on regional demand. To do so, scaling residuals were aggregated to the county-level, and the following equation was estimated using OLS:

$$Resid = \alpha + \beta_1 Tax + \beta_2 Markets + \beta_3 \%TownTax + \xi \tag{12}$$

where $Tax$ is the aggregate county 1524/5 Lay Subsidy return (Sheail 1968), $Markets$ is the number of markets per km$^2$ in the county in 1588 (A. Dyer 2000b), $\%TownTax$ is the percent of total county tax (the 1524/5 Lay Subsidy return) that comes from the towns in our sample, and $\xi$ is the error term. $Tax$ serves as a proxy for the aggregate regional demand, $Markets$ is a proxy for the spatial density of rural demand, and $\%TownTax$ is a proxy for per capita county demand for urban goods and services (akin to the urbanization ratio, see Wrigley 1985).

As seen in Table 4, there is a significant positive relationship between the scaling residuals and county-level demand proxies. The demand proxies do reasonably well at predicting the county sum and maximum scaling residuals, suggesting that town economic

Table 4. Influence of demand proxies on town scaling residuals at the county-level. Multiple linear OLS regression of county demand proxies on town scaling residuals aggregated to the county-level.

| | Model 1 Sum of Town Residuals | Model 2 Max of Town Residuals |
|---|---|---|
| Intercept ($\alpha$) | −2.009*** | **−0.7816*** |
| | (0.4758) | **(0.2085)** |
| Aggregate County Tax 1524/5 (*Tax*; $\beta_1$) | 0.00023* | 0.00025*** |
| | (0.00011) | (0.000047) |
| Number of Markets in County per km$^2$ in 1588 (*Markets*; $\beta_2$) | 207.8* | 24.92 |
| | (79.9) | (35.01) |
| Percent of County Tax 1524/5 from Towns (*%TownTax*; $\beta_3$) | 3.237* | 2.427*** |
| | (1.51) | (0.6614) |
| Observations ($n$) | 31 | 31 |
| Adjusted $R^2$ | 0.40 | 0.64 |

Standard errors reported in parentheses.
*$p < 0.05$;
**$p < 0.01$;
***$p < 0.001$.

performance was indeed connected to regional demand. Aggregate and per capita county demand are better predictors than the spatial density of rural demand. Based on the spatial distribution of residuals, the lower $R^2$ for Model 1 is likely due to boundary effects and the coarser spatial resolution of counties. Indeed, the sub-county distribution of 1524/5 tax revenue could be quite variable and agglomerate across borders (Sheail 1968, 1972). The higher $R^2$ for Model 2 suggests that the highest performing towns were better connected to a greater proportion of county-level demand.

The covariance of town economic performance with regional demand proxies suggests that Smithian-Agglomeration feedbacks resulted in productivity growth. The English countryside experienced substantial Smithian growth c.1500-1700 that stimulated demand for urban goods and services (Allen 2009; Broadberry et al. 2015; Wrigley 2004). Our results here imply that this growth in demand was capable of stimulating productivity growth in towns as well. Rather than only being a constraint on town economies, this suggests that town integration with agricultural hinterlands could drive growth (cf. Ploeckl 2017).

## Discussion

Historians have widely argued that the economic performance of English towns was meager before the 18th century. At the time of the 1524/5 Lay Subsidy, the specialization of town economies was constrained by low effective demand (Britnell 1993, 2000a; Rigby 2010). Depressions in foreign demand starved urban textile industries (Britnell 2000a; A. Dyer, 1995). Town elites begged the crown for tax relief and assistance with urban poverty and underemployment (Cornwall 1988; Lilley 2000; Palliser 1992). Using urbanization ratios, qualitative records, population trends, and scattered economic data, many have suggested that the early 16$^{th}$ century was a low point of town economic performance (e.g., Clark and Slack 1976; Goose 1986; Palliser 1992). Even the most optimistic assessments hypothesize stagnation (A. Dyer, 1995, Dyer 2000c; Rigby 2010), and Allen's (2009) real wage series for Oxford, York, and London show stagnation prior to c.1550. For all these reasons, the period has famously been referred to as an 'acute urban crisis' (Phythian-Adams 1978).

In spite of this, scaling analysis of Tudor towns from the 1524/5 Lay Subsidy exhibit strong evidence of IRS. Not only are these IRS robust to alternative interpretations of the lay subsidies, they also withstand 'worst case scenario' assumptions designed to destroy the superlinear scaling relationship. The IRS of Tudor towns, without including London, is quantitatively very similar to the IRS of modern agglomeration economies in the UK (Bettencourt 2013; Bettencourt and Lobo 2016). We suspect that the A. Dyer (2000c) population estimates (Table 2, B and C) produce the most probable scaling results for Tudor towns 1524/5 ($1.17 \leq \beta \leq 1.26$) given their basic conformity with the prevailing assumptions of historical demographers. These estimates conform to the quantitative predictions of settlement scaling theory (Bettencourt 2013).

IRS is the result and empirical signature of feedbacks between agglomeration effects and Smithian growth (Bettencourt, Samaniego, and Youn 2014; Fujita, Krugman, and Venables 1999; Lobo et al. 2013). Therefore, the English urban system c.1524/5—including settlements of fewer than 1,000 inhabitants—exhibited the same qualitative dynamics as modern urban economies. The fact that the estimated scaling parameter of early Tudor towns *without London* matches that of modern agglomeration economies begs us to reconsider the productivity growth of towns in Early Modern England.

## *Small towns and urban thresholds*

No town size threshold for the emergence of IRS was detected in our sample of Early Tudor towns, as step-wise removal of larger towns failed to eliminate IRS. Small town agglomeration effects and economic performance were therefore not qualitatively inhibited by their occupational structures or economic dependence on the local primary sector. Rather, increasing large towns, with increasingly specialized divisions of labor, exhibited progressively higher productivities. This result conforms to the predictions of settlement scaling theory, which holds that agglomeration effects like IRS are *scale-free* properties common to villages and big cities alike. Larger cities have (on average) greater productivity precisely because IRS is a generic mechanism of urban growth.

From this perspective, there is no reason to exclude small towns from urban macroeconomic metrics. It is well established that the growth of numerous small towns was critical to Early Modern English market integration and rural economic development (A. Dyer 2000b; Glennie and Whyte 2000; Overton 1996). Many such small towns had high productivity in the c.1524/5 subsidy (proxied by tax per taxpayer), and small towns in 16th and 17th century England experienced growth in both occupational and regional specialization (Britnell 2000a; Clay 1984a; A. Dyer 2000b; Keibek 2016). Our analysis suggests that these towns also functioned as smaller agglomeration economies, such that their growth could have involved productivity gains.

A town size threshold of 5,000 (Wrigley 1985) or 10,000 (Broadberry et al. 2015) would disregard most towns in early Tudor England. The same is true two centuries later, in part because in-migration was frequently restricted to high-skill specialists (Clark and Souden, 1987; Clay 1984a; A. Dyer 2000b; Galley 1998; Souden 1984). While any one small town has little macroeconomic significance by itself, there were over 700 market towns in England c.1600-1700 with fewer than 3,500 people (A. Dyer 2000b). On aggregate, productivity gains from the growth of these small towns could have been significant.

## *Hypothesis of Smithian growth in towns c.1525-1670*

England underwent urbanization and overall demographic growth c.1525-1670. This involved the growth of the urban population, the urbanization ratio, and the urban hierarchy (Broadberry et al. 2015; De Vries 1984; Langton 2000; Patten 1978; Wrigley 1985). According to Wrigley (1985), London's population increased by around 8.5x c.1525-1670, and the population living in towns of 5,000 people or more increased by around 3x. As noted above, including towns with under 5,000 people would greatly increase this figure. Despite unduly pessimistic assessments of intensive performance, there is a general consensus that the demographic growth of towns c.1525-1670 involved the *extensive* economic growth (Britnell 2000a, 2000b; Broadberry et al. 2015; Clay 1984a; Corfield 1976; A. Dyer, 1995; Glennie and Whyte 2000; Goose 1984, 1986; Palliser 1992; Slack 2000; Wrigley 1990).

This leads us to formulate a speculative hypothesis about urban intensive growth in Early Modern England. Since Tudor towns exhibited IRS even during the 'acute urban crisis' of the early 16th century, we might therefore infer Smithian-Agglomeration feedbacks in English towns over the entire Early Modern period. It follows that town extensive growth c.1525-1670 resulted in productivity growth at the aggregate level of town economies. The evidence presented in Section Agglomeration effects and Smithian growth for Smithian-Agglomeration feedbacks in English towns before 1670 supports this contention. Likewise, the correlation of town economic performance with regional demand proxies found in Section Scaling residuals and regional demand suggests that the demand forces postulated by Wahl (2016) stimulated Smithian-Agglomeration feedbacks—even before the Early Modern proliferation of trade, market integration (Britnell 2000a, 2000b; Chartres 1977; Clay 1984a, 1984b; Rigby 2010), or the regional agglomeration of specialized industries (Keibek 2016). This would mean that the demand for urban goods and services stimulated by English rural productivity growth c.1500-1700 also stimulated the Smithian growth of towns.

The implications of this hypothesis for London are striking. If the urban system of Early Modern England exhibited IRS, then the magnitude of London's agglomeration effects should have been very high. Indeed, this is exactly what we find suggested in the literature (Allen 2009; Barron 2000; Boulton 2000; Clay 1984a; Fisher 1990; O'Brien 2001). This implies that the astronomical growth of Early Modern London was not only an "engine of growth" due to of the massive demand it stimulated (Wrigley 1985, 1987, 2004), but that it was also a major driver of endogenous intra-urban Smithian growth.

This hypothesis in no way contradicts the 18th century takeoff of Smithian growth in the British urban system. As demonstrated by Kelly (1997), Smithian growth dynamics exhibit generic 'threshold' behavior as a function of market growth. The rate of Smithian growth is slower below a threshold level of market expansion and integration, the crossing of which enables a higher degree of specialization and the rapid acceleration of Smithian growth (Kelly 1997). Given the demand constraint over Smithian growth (Chaney and Ossa 2013; Kelly 1997; Persson 2010; Stigler 1951), this suggests that the rapid $18^{th}$ century British Smithian takeoff can be explained by domestic market integration (Bogart 2005, 2009; Buchinsky and Polak 1993; Chartres 1995; Eagly and Smith 1976; Gerhold 1996; Wrigley 2014) and Britain's rise to supremacy over global markets (Allen 2009; Cuenca Esteban 2004; Wallerstein 1989). Rather than the sudden qualitative emergence of Smithian growth and agglomeration effects in $18^{th}$ century towns, this suggests that gradual urban Smithian growth c.1525-1670 *necessarily preceded* its $18^{th}$ century acceleration. We hope that scholars will explore the potential magnitude and macroeconomic implications of urban productivity growth before the $18^{th}$ century.

## Notes

1. productivity growth generated by gains from the specialization of the division of labor
2. Here, we loosely define "town" as nucleated population centers in England (excluding London). This broad definition is justified in sections 7 and 9.1 via the results of our empirical analysis.
3. Small modifications or combinations of existing techniques and technologies progressing via numerous of small changes (Berg 1994a; Mokyr 1990, 2009, von Tunzelmann, 1981, von Tunzelmann 1993)
4. the proportion of the population in urban areas, sometimes referred to as the "urbanization rate."
5. For an overview of the sources, see (Jurkowski, Smith, and Crook 1998, pp. 137–9); for the administration of the tax, see (Schofield 2004, pp. 85–139).
6. Excluding Wales, Cheshire, Westmoreland, Cumberland, Northumberland, the Bishopric of Durham, the towns of Ludlow and the Cinque ports, and all overseas possessions (see A. Dyer 2000a; Schofield 2004, pp. 93–101).
7. These data have been compiled by several different authors, in ways that contain small differences. Our data give preference to Sheail (1968), Rigby (2010), and A. Dyer (2000a).
8. The Subsidies are not quite proportional to income-based GDP of the lay population because of the exempted untaxed incomes of the very poor, and the taxed savings of the very rich accumulated from prior years. However, these errors are opposite and their magnitudes both covary with town size (see Cornwall 1988), thereby mitigating the error on aggregate.
9. Eq. (11) is the log-transformed version of Eq (1), such that $\beta$ (scaling coefficient) is the scaling exponent $b$ in Eq (1). The intercept $\alpha$ in Eq. (11) is the prefactor $Y_0$ in Eq. (1).
10. Including scaling coefficients for GDP, income, and wages for cities in Europe, the United States, and Japan, which range from 1.08-1.20 (see Bettencourt 2013: SM; Bettencourt and Lobo 2016; Lobo et al. 2019).

## Funding

This work was partially supported by the Arizona State University – Santa Fe Institute Center for Biosocial Complex Systems, which funded Rudolf Cesaretti's data collection, analysis, and preparation of the manuscript through a Research Assistantship. This work was partially supported by the James S. McDonnell Foundation [grant number 220020438]. The authors have no competing interests.

## ORCID

Rudolf Cesaretti http://orcid.org/0000-0002-7003-1211
José Lobo http://orcid.org/0000-0002-0814-7168
Luis M. A. Bettencourt http://orcid.org/0000-0001-6176-5160
Michael E. Smith http://orcid.org/0000-0002-0677-8206

## References

Adamic, L. A., and B. A. Huberman. 2002. Zipf's law and the internet. *Glottometrics* 3:143–50.

Allen, R. C. 2009. *The British industrial revolution in global perspective*. Cambridge: Cambridge University Press.

Angel, S. 2012. *Planet of cities*, 360. Cambridge, MA: Lincoln Institute of Land Policy.

Arrow, K. 1962. The economic implications of learning-by-doing. *Review of Economic Studies* 29 (3):155–73. doi: 10.2307/2295952.

Bairoch, P. 1991. *Cities and economic development: From the dawn of history to the present*. Chicago, IL: University of Chicago Press.

Balderston, T. 2009. The economics of abundance: Coal and cotton in Lancashire and the world. *The Economic History Review* 63 (3):569–90. doi: 10.1111/j.1468-0289.2009.00453.x.

Barron, C. 2000. London 1300-1540. In *The Cambridge Urban History of Britain, Volume I: 600-1540*, ed. D. M. Palliser, 395–440. Cambridge: Cambridge University Press.

Batty, M. 2006. Hierarchies in Cities and City-Systems. In *Hierarchy in Natural and Social Sciences*, ed. D. Pumain, 143–68. Dordrecht: Springer.

Batty, M. 2008. The size, scale, and shape of cities. *Science* (New York, N.Y.) 319 (5864):769–71. doi: 10.1126/science.1151419.

Batty, M. 2013. A theory of city size. *Science* 340 (6139): 1418–9. doi: 10.1126/science.1239870.

Ben-Amos, I. K. 1994. *Adolescence and youth in early modern England*. New Haven: Yale University Press. doi: 10.1086/ahr/100.3.896.

Berg, M. 1993. Factories, Workshops and Industrial Organisation. In *The Economic History of Britain Since 1700, Volume 1: 1700-1860 (Second)*, ed. R. Floud and D. McCloskey, 123–50. Cambridge: Cambridge University Press.

Berg, M. 1994a. *The age of manufactures, 1700-1820: Industry, innovation and work in Britain (Second)*. London: Routledge.

Berg, M. 1994b. *The age of manufactures* 1700-1820. New York, NY: Barnes & Noble.

Berg, M. 1999. New Commodities, Luxuries and Their Consumers in Eighteenth-century England. In *Consumers and Luxury: Consumer Culture in Europe, 1650–1850*, ed. M. Berg and H. Clifford, 63–85. Manchester: Manchester University Press.

Bettencourt, L. M. A. 2013. The origins of scaling in cities. *Science* 340 (6139):1438–41. doi: 10.1126/science.1235823.

Bettencourt, L. M. A., and J. Lobo. 2016. Urban scaling in Europe. *Journal of the Royal Society Interface* 13 (116): 20160005. doi: 10.1098/rsif.2016.0005.

Bettencourt, L. M. A., J. Lobo, D. Helbing, C. Kühnert, and G. B. West. 2007. Growth, innovation, scaling, and the pace of life in cities. *Proceedings of the National Academy of Sciences of Sciences* 104 (17):7301–6. doi: 10.1073/pnas.0610172104.

Bettencourt, L. M. A., J. Lobo, D. Strumsky, and G. B. West. 2010. Urban scaling and its deviations: revealing the structure of wealth, innovation and crime across cities. *PloS One* 5 (11): e13541. doi: 10.1371/journal.pone.0013541.

Bettencourt, L. M. A., H. Samaniego, and H. Youn. 2014. Professional diversity and the productivity of cities. *Scientific Reports* 4:3–8. doi: 10.1038/srep05393.

Bogart, D. 2005. Turnpike trusts and the transportation revolution in 18th century England. *Explorations in Economic History* 42 (4):479–508. doi: 10.1016/j.eeh.2005.02.001.

Bogart, D. 2009. Turnpike trusts and property income: New evidence on the effects of transport improvements and legislation in eighteenth-century England. *The Economic History Review* 62 (1):128–52. doi: 10.1111/j.1468-0289.2008.00434.x.

Boulton, J. 2000. London, 1540-1700. In *The Cambridge Urban History of Britain, Vol II: 1540-1840*, ed. P. Clark, 315–46. Cambridge: Cambridge University Press.

Bridbury, A. R. 1962. *Economic growth: England in the later Middle Ages*. London: George Allen & Unwin.

Bridbury, A. R. 1992. *The English economy from Bede to the Reformation*. Woodbridge: Boydell Press.

Britnell, R. H. 1993. *The Commercialization of English Society, 1000-1500*. Cambridge: Cambridge University Press.

Britnell, R. H. 2000a. The Economy of British Towns 1300–1540. In *The Cambridge Urban History of Britain, Volume I: 600-1540*, 313–34. Cambridge: Cambridge University Press.

Britnell, R. H. 2000b. Urban demand in the English economy, 1300-1600. In *Trade, Urban Hinterlands and Market Integration c.1300-1600*, ed. J. A. Galloway, 1–21. London: Institute of Historical Research.

Broadberry, S., B. M. S. Campbell, A. Klein, M. Overton, and B. van Leeuwen. 2015. *British Economic Growth*, 1270–870. Cambridge: Cambridge University Press.

Brooks, C. 1994. Apprenticeship, Social Mobility and the Middling Sort, 1550–1800. In *The Middling Sort of People: Culture, Society and Politics in England, 1550–1800*, ed. J. Barry and C. Brooks, 52–83. London: Palgrave Macmillan.

Buchinsky, M., and B. Polak. 1993. The emergence of a National Capital Market in England, 1710-1880. *The Journal of Economic History* 53 (1):1–24. doi: 10.1017/S0022050700012365.

Cesaretti, R., J. Lobo, L. M. A. Bettencourt, S. G. Ortman, and M. E. Smith. 2016. Population-area relationship for medieval European cities. *PloS One* 11 (10):e0162678–27. doi: 10.1371/journal.pone.0162678.

Chaney, T., and R. Ossa. 2013. Market size, division of labor, and firm productivity. *Journal of International Economics* 90 (1):177–80. doi: 10.1016/j.jinteco.2012.11.003.

Chartres, J. A. 1977. *Internal Trade in England, 1500-1700*. London: Macmillan.

Chartres, J. A. 1995. Market integration and agricultural output in Seventeenth-, Eighteenth-, and Early Nineteenth-Century England. *The Agricultural History Review* 43 (2):117–38.

Clark, P., and Souden, D. (Eds.). 1987. *Migration and society in early modern England*. London: Hutchinson.

Clark, P., and P. Slack. 1976. *English Towns in Transition, 1500-1700*. Oxford: Oxford University Press. doi: 10.1086/ahr/82.5.1243.

Clay, C. G. A. 1984a. *Economic Expansion and Social Change: England 1500-1700: Volume I, People, Land and Towns*. Cambridge: Cambridge University Press.

Clay, C. G. A. 1984b. *Economic Expansion and Social Change: England 1500-1700: Volume II, Industry, Trade and Government*. Cambridge: Cambridge University Press.

Clifford, H. 1995. The Myth of the Maker: Manufacturing Networks in the London Goldsmiths' Trade 1750–1790. In *Silver & Jewellery: Production and Consumption since 1750*, ed. K. Quickenden and N. A. Quickenden, 5–12. Birmingham: ARTicle Press.

Coleman, D. C. 1969. An innovation and its diffusion: The "New draperies. *The Economic History Review* 22 (3): 417–29. doi: 10.2307/2594119.

Corfield, P. J. 1976. Urban development in England and Wales in the sixteenth and seventeenth centuries. In *Trade, Government and Economy in Pre-Industrial England*, ed. D. C. Coleman and A. H. John, 214–47. London: Weidenfeld & Nicolson.

Cornwall, J. 1962. The people of Rutland in 1522. *Transactions of the Leicestershire Archaeological and Historical Society* 37:7–28.

Cornwall, J. 1988. *Wealth and society in early sixteenth century England*. London: Routledge.

Crafts, N. F. R., and C. K. Harley. 1992. Output growth and the British industrial revolution: A restatement of the Crafts-Harley view. *The Economic History Review* 45 (4): 703–30. doi: 10.2307/2597415.

Crouzet, F. 1985. *The first industrialists: The problem of origins*. Cambridge: Cambridge University Press.
Cuenca Esteban, J. 2004. Comparative patterns of colonial trade: Britain and its rivals. In *Exceptionalism and Industrialisation: Britain and its European Rivals, 1688-1815*, ed. L. P. de la Escosura. Cambridge: Cambridge University Press.
Darby, H. C., R. E. Glasscock, J. Sheail, and G. R. Versey. 1979. The changing geographical distribution of wealth in England: 1086-1334-1525. *Journal of Historical Geography* 5 (3):247–62. (79)90071-9 doi: 10.1016/0305-7488(79)90071-9.
Davids, K., and B. De Munck. 2014. Innovation and Creativity in Late Medieval and Early Modern European Cities: An Introduction. In *Innovation and Creativity in Late Medieval and Early Modern European Cities*, ed. K. Davids and B. De Munck, 1–34. Burlington, VT: Ashgate.
De Vries, J. 1984. *European Urbanization, 1500-1800*. New York: Routledge.
Duncan, G. D. 1976. *Monopolies under Elizabeth I*. Cambridge: University of Cambridge.
Duranton, G., and D. Puga. 2004. Micro-Foundations of Urban Agglomeration Economies. In *Handbook of Regional and Urban Economics, Vol.4: Cities and Geography*, ed. V. Henderson and J. F. Thisse, 2063–117. Amsterdam: Elsevier.
Dyer, A. 1995. *Decline and Growth in English Towns 1400-1640*. Vol. 13. Cambridge: Cambridge University Press.
Dyer, A. 2000a. Appendix: Ranking Lists of English Medieval Towns. In *The Cambridge Urban History of Britain, Volume I: 600-1540*, ed. D. M. Palliser, 741–70. Cambridge: Cambridge University Press.
Dyer, A. 2000b. Small Market Towns, 1540-1700. In *The Cambridge Urban History of Britain, Vol II: 1540-1840*, ed. P. Clark, 425–50. Cambridge: Cambridge University Press.
Dyer, A. 2000c. Urban Decline" in England, 1377-1525. In *Towns in Decline, AD 100-1600*, ed. T. R. Slater, 266–88. Aldershot: Ashgate.
Dyer, C. 1995. How urbanized was medieval England?. In *Peasants and townsmen in medieval Europe: studia in honorem Adriaan Verhulst*, ed. J.-M. Duvosquel and E. Thoen, 169–83. Ghent: Snoeck-Ducaju & Zoon.
Eagly, R., and V. K. Smith. 1976. Domestic and International Integration of the London Money Market, 1731-1789. *The Journal of Economic History* 36 (1): 198–212. doi: 10.1017/S0022050700094638.
Epstein, S. R. 1998. Craft Guilds, Apprenticeship, and Technological Change in Pre-Industrial Europe. *The Journal of Economic History* 58 (3):684–713. doi: 10.1017/S0022050700021124.
Epstein, S. R., and M. Prak. 2008. Introduction: Guilds, Innovation, and the European Economy, 1400–1800. In *Guilds, Innovation, and the European Economy, 1400–1800*, ed. S. R. Epstein and M. Prak, 1–24. Cambridge: Cambridge University Press.
Fisher, F. J. 1990. *London and the English Economy, 1500-1700*. ed. P. J. Corfield and N. B. Harte. London: Hambledon Press.
Fujita, M., and J.-F. Thisse. 2002. *Economics of Agglomeration: Cities. Industrial Location, and Regional Growth*. Cambridge: Cambridge University Press. http://doi.org/10.1007/s13398-014-0173-7.2.
Fujita, M., P. R. Krugman, and A. Venables. 1999. *The spatial economy: Cities, regions, and international trade*. Cambridge, Mass.: MIT press.
Gabaix, X. 1999. Zipf's Law for Cities: An Explanation. *The Quarterly Journal of Economics* 114 (3):739–67. doi: 10.1162/003355399556133.
Galley, C. 1998. *The Demography of Early Modern Towns: York in the Sixteenth and Seventeenth Centuries*. Liverpool: Liverpool University Press.
Gerhold, D. 1993. Packhorses and wheeled vehicles in England, 1550-1800. *The Journal of Transport History* 14 (1):1–26. doi: 10.1177/002252669301400102.
Gerhold, D. 1996. Productivity change in road transport before and after turnpiking, 1690-1840. *The Economic History Review* 49 (3):491–515. doi: 10.1111/j.1468-0289.1996.tb00579.x.
Gibrat, R. 1931. *Les inégalités économiques: applications: aux inégalitês des richesses, à la concentration des entreprises, aux populations des villes, aux statistiques des familles, etc: d'une loi nouvelle: la loi de l'effet proportionnel*. Librairie du Recueil Sirey.
Glaeser, E. L., and B. I. Sacerdote. 1999. Why is there more crime in cities?. *Journal of Political Economy* 107 (S6): S225–S258. doi: 10.1086/250109.
Glaeser, E. L., and J. D. Gottlieb. 2009. The wealth of cities: Agglomeration economies and spatial equilibrium in the United States. *Journal of Economic Literature* 47 (4): 983–1028. doi: 10.1257/jel.47.4.983.
Glennie, P., and I. Whyte. 2000. Towns in an Agrarian economy, 1540-1700. In *The Cambridge Urban History of Britain, Vol II: 1540-1840*, ed. P. Clark, 167–94. Cambridge: Cambridge University Press.
Goldstone, J. A. 1984. Urbanization and Inflation: Lessons from the English Price Revolution of the Sixteenth and Seventeenth Centuries. *American Journal of Sociology* 89 (5):1122–60. doi: 10.1086/227986.
Goose, N. 1982. English pre-industrial urban economies. *Urban History Yearbook* 9:24–30. doi: 10.1017/S096392680000609X.
Goose, N. 1984. *Economic and social aspects of provincial towns: A comparative study of Cambridge, Colchester and Reading, c. 1500–1700*. Cambridge: University of Cambridge.
Goose, N. 1986. In Search of the Urban Variable: Towns and the English Economy, 1500-1650. *The Economic History Review* 39 (2):165–85. doi: 10.2307/2596148.
Goose, N., and A. Hinde. 2006. Estimating local population sizes at fixed points in time: Part I - General principles. *Local Population Studies* 77:66–74.
Goose, N., and A. Hinde. 2007. Estimating local population sizes at fixed points in time: Part II - Specific sources. *Local Population Studies* 78:74–88.
Hanson, J. W., and S. G. Ortman. 2017. A systematic method for estimating the populations of Greek and Roman settlements. *Journal of Roman Archaeology* 30: 301–24. doi: 10.1017/S1047759400074134.
Hanson, J. W., S. G. Ortman, and J. Lobo. 2017. Urbanism and the division of labour in the Roman Empire. *Journal of the Royal Society Interface* 14 (136):20170367–24. doi: 10.1098/rsif.2017.0367.

Harte, N. B. 1991. The economics of clothing in the late seventeenth century. *Textile History* 22 (2):277–96. doi: 10.1179/004049691793712594.

Hohenberg, P. M., and L. H. Lees. 1995. *The Making of Urban Europe 1000-1994* (2nd Editio). Cambridge, Mass.: Harvard University Press. doi: 10.1086/ahr/91.3.652-a.

Hoskins, W. G. 1976. *The Age of Plunder: King Henry's England 1500-1547*. London: Longman.

Hudson, P. 1986. *The Genesis of Industrial Capital: A Study of the West Riding Wool Textile Industry, c.1750-1850*. Cambridge: Cambridge University Press. doi: 10.1086/ahr/94.3.762.

Hudson, P. 1989. The regional perspective. In *Regions and Industries: A perspective on the industrial revolution in Britain*, ed. P. Hudson, 5–40. Cambridge: Cambridge University Press.

Humphries, J. 2003. English apprenticeship: A neglected factor in the industrial revolution. In *The Economic Future in Historical Perspective*, ed. P. A. David and M. Thomas, 73–102. Oxford: Oxford University Press.

Jedwab, R., and D. Vollrath. 2015. Urbanization without growth in historical perspective. *Explorations in Economic History* 58:1–21. doi: 10.1016/j.eeh.2015.09.002.

Jurkowski, M. 2016. The History of Clerical Taxation in England and Wales, 1173–1663: The Findings of the E 179 Project. *The Journal of Ecclesiastical History* 67 (1): 53–81. doi: 10.1017/S0022046915001608.

Jurkowski, M., C. L. Smith, and D. Crook. 1998. *Lay taxes in England and Wales 1188–1688*. Richmond: Public Record Office.

Keibek, S. 2016. *The male occupational structure of England and Wales, 1600-1850*. Cambridge: University of Cambridge.

Kelly, M. 1997. The dynamics of Smithian growth. *The Quarterly Journal of Economics* 112 (3):939–64. doi: 10.1162/003355397555398.

Keynes, J. M. 1964. *The General Theory of Employment, Interest and Money*. New York: Harcourt.

Krugman, P. 1991. Increasing Returns and Economic Geography. *Journal of Political Economy* 99 (3):483–99. doi: 10.1086/261763.

Lane, J. 1996. *Apprenticeship in England, 1600-1914*. London: UCL Press.

Langton, J. 2000. Urban Growth and Economic Change: From the Late Seventeenth Century to 1841. In *The Cambridge Urban History of Britain, Vol II: 1540-1840*, ed. P. Clark, 453–90. Cambridge: Cabridge University Press.

Lilley, K. D. 2000. Decline or Decay? Urban landscapes in late-medieval England. In *Towns in decline, AD 100-1600*, ed. T. R. Slater, 235–65. Aldershot: Ashgate.

Limpert, E., W. A. Stahel, and M. Abbt. 2001. Log-normal Distributions across the Sciences: Keys and Clues. *BioScience* 51 (5):341–52.

Lobo, J., L. M. A. Bettencourt, D. Strumsky, and G. B. West. 2013. Urban Scaling and the Production Function for Cities. *PloS One* 8 (3):e58407. doi: 10.1371/journal.pone.0058407.

Lobo, J., L. M. A. Bettencourt, M. E. Smith, and S. G. Ortman. 2019. Settlement scaling theory: Bridging the study of ancient and contemporary urban systems. *Urban Studies* 6:1–17. doi: 10.1177/0042098019873796.

Lucas, R. E. J. 1988. On the Mechanics of Economic Development. *Journal of Monetary Economics* 22 (1): 3–42. doi: 10.1016/0304-3932(88)90168-7.

Malanima, P. 2009. *The Pre-Modern European Economy: One Thousand Years (10th-19th Centuries)*. Leiden: Brill.

Mokyr, J. 1990. *The Lever of Riches: Technological Creativity and Economic Progress*. Oxford: Oxford University Press. doi: 10.1086/ahr/96.4.1164.

Mokyr, J. 1995. Urbanization, Technological Progress, and Economic History. In *Urban Agglomeration and Economic Growth*, ed. H. Giersch, 3–38. Heidelberg: Springer.

Mokyr, J. 1997. *The Political Economy of Technological Change: Resistance and Innovation in Economic History*.

Mokyr, J. 1999. Editor's Introduction: The New Economic History and the Industrial Revolution. In J. Mokyr (Ed.), *The British industrial revolution: An Economic Perspective*, 2nd ed., 1–128. Boulder, CO: Westview Press. Retrieved from http://books.google.com/books?id=vyLhAnukIMkC&pgis=1.

Mokyr, J. 2002. *The gifts of Athena: Historical origins of the knowledge economy*. Princeton: Princeton University Press. doi: 10.1086/ahr/109.1.136.

Mokyr, J. 2009. *The Enlightened Economy: Britain and the Industrial Revolution 1700–1850*. London: Penguin.

Montroll, E. W., and M. F. Shlesinger. 1982. On 1/f noise and other distributions with long tails. *Proceedings of the National Academy of Sciences* 79 (10):3380–3. doi: 10.1073/pnas.79.10.3380.

Munro, J. H. 1997. The Origin of the English "New Draperies": The Resurrection of an Old Flemish Industry, 1270–. 1570. In *The New Draperies in the Low Countries and England, 1300–1800*, ed. N. B. Harte, 35–127. Oxford: Oxford University Press.

Nightingale, P. 1996. The Growth of London in the Medieval Economy. In *Progress and Problems in Medieval England*, ed. R. H. Britnell and J. Hatcher, 89–106. Cambridge: Cambridge University Press.

Nordbeck, S. 1971. Urban Allometric Growth. *Geografiska Annaler, 53B* 53 (1):54–67. doi: 10.1080/04353684.1971.11879355.

O'Brien, P. K. 2001. Reflections and meditations on Antwerp, Amsterdam and London in their golden ages. In *Urban Achievement in Early Modern Europe: Golden Ages in Antwerp, Amsterdam and London*, ed. P. K. O'Brien, 3–38. Cambridge: Cambridge University Press.

Ogilvie, S. 2004. Guilds, Efficiency and Social Capital: Evidence from German Proto-Industry. *The Economic History Review* 57 (2):286–333. doi: 10.1111/j.1468-0289.2004.00279.x.

Ogilvie, S. 2007. Whatever is, is right"? Economic Institutions in Pre-industrial Europe. *The Economic History Review* 60 (4):649–84. doi: 10.1111/j.1468-0289.2007.00408.x.

Ortman, S. G., A. H. F. Cabaniss, J. O. Sturm, and L. M. A. Bettencourt. 2014. The pre-history of urban scaling. *PLoS One* 9 (2):e87902. doi: 10.1371/journal.pone.0087902.

Ortman, S. G., A. H. F. Cabaniss, J. O. Sturm, and L. M. A. Bettencourt. 2015. Settlement scaling and increasing returns in an ancient society. *Science Advances* 1 (1): e1400066–8. doi: 10.1126/sciadv.1400066.

Ortman, S. G., and G. D. Coffey. 2017. Settlement Scaling in Middle-Range Societies. *American Antiquity* 82 (4): 662–82. doi: 10.1017/aaq.2017.42.

Ortman, S. G., K. E. Davis, J. Lobo, M. E. Smith, L. M.A. Bettencourt, and A. Trumbo. 2016. Settlement scaling and economic change in the Central Andes. *Journal of Archaeological Science* 73:94–106. doi: 10.1016/j.jas.2016.07.012.

Ottaviano, G., and J.-F. Thisse. 2004. Agglomeration and Economic Geography. In *Handbook of Regional and Urban Economics*, Vol. 4, 2563–608. Amsterdam, Netherlands: Elsevier.

Overton, M. 1996. *The Agricultural Revolution: The Transformation of the Agrarian Economy: 1500–1850*. Cambridge: Cambridge University Press.

Overton, M., J. Whittle, D. Dean, and A. Hann. 2004. *Production and consumption in English households 1600–1750*. London: Routledge.

Palliser, D. M. 1992. *The Age of Elizabeth*. London: Longman.

Patten, J. 1978. *English Towns 1500-1700*. Folkestone: Dawson.

Persson, K. G. 2010. *An Economic History of Europe: Knowledge, institutions and growth, 600 to the present*. Cambridge: Cambridge University Press.

Phythian-Adams, C. 1978. Urban decay in late medieval England. In *Towns in Societies: Essays in Economic History and Historical Sociology*, ed. P. Abrams and E. A. Wrigley, 159–85. Cambridge: Cambridge University Press.

Piketty, T. 2014. *Capital in the 21st Century*. Cambridge, Mass.: Belknap.

Piketty, T., and G. Zucman. 2014. Capital is back: Wealth-income ratios in rich countries 1700–2010. *The Quarterly Journal of Economics* 129 (3):1255–310. doi: 10.1093/qje/qju018.

Ploeckl, F. 2017. Towns (and villages): definitions and implications in a historical setting. *Cliometrica* 11 (2): 269–87. doi: 10.1007/s11698-016-0145-6.

Prak, M. 2018. *Citizens without nations: Urban citizenship in Europe and the world, c. 1000–1789*. Cambridge: Cambridge University Press.

Pumain, D. 2000. Settlement Systems in the Evolution. *Geografiska Annaler, Series B: Human Geography* 82 (2): 73–87. doi: 10.1111/j.0435-3684.2000.00075.x.

R Core Team2016. *R: A language and environment for statistical computing*. Vienna, Austria: R Foundation for Statistical Computing. Retrieved from http://www.r-project.org/.

Rigby, S. H. 1979. Urban decline in the later middle ages: Some problems in interpreting the statistical data. *Urban History* 6:46–59. doi: 10.1017/S0963926800003904.

Rigby, S. H. 2010. Urban population in late medieval England: The evidence of the lay subsidies. *The Economic History Review* 63 (2):393–417. doi: 10.1111/j.1468-0289.2009.00489.x.

Romer, P. M. 1990. Endogenous Technological Change. *Journal of Political Economy* 98 (5, Part 2):S71–S102. 1. doi: 10.1086/261725.

Samaniego, H., and M. E. Moses. 2009. Cities as organisms: Allometric scaling of urban road networks. *Journal of Transport and Land Use* 1:2139. doi: 10.5198/jtlu.v1i1.29.

Schofield, R. 1965. The geographical distribution of wealth in England, 1334–1649. *The Economic History Review* 18 (3):483–510. doi: 10.2307/2592561.

Schofield, R. 2004. *Taxation Under the Early Tudors 1485-1547. Taxation Under the Early Tudors 1485-1547*. Oxford: Blackwell. http://doi.org/10.1002/9780470758298.

Sheail, J. 1968. *The regional distribution of wealth in England as indicated in the 1524/5 lay subsidy returns*. London, UK: University of London.

Sheail, J. 1972. The Distribution of Taxable Population and Wealth in England during the Early Sixteenth Century. *Transactions of the Institute of British Geographers* 55: 111–26. doi: 10.2307/621725.

Slack, P. 2000. Great and Good Towns, 1540-1700. In *The Cambridge Urban History of Britain, Vol II: 1540-1840*, ed. P. Clark, 347–76. Cambridge: Cambridge University Press.

Solow, R. M. 1997. *Learning from "Learning by Doing": lessons for economic growth*. Stanford: Stanford University Press.

Souden, D. 1984. Migrants and the population structure of later seventeenth-century provincial cities and market towns. In *The Transformation of English Provincial Towns 1600-1800*, ed. P. Clark, 133–68. London: Hutchinson.

Spufford, M. 1984. *The Great Reclothing of Rural England: Petty Chapman and Their Wares in the Seventeenth Century*. London: The Hambledon Press.

Spufford, M. 2000. The cost of apparel in seventeenth-century England, and the accuracy of Gregory King. *The Economic History Review* 53 (4):677–705. doi: 10.1111/1468-0289.00174.

Stigler, G. J. 1951. The Division of Labor is Limited by the Extent of the Market. *Journal of Political Economy* 59 (3): 185–93. doi: 10.1086/257075.

Storper, M. 2010. Why Does a City Grow? Specialisation, Human Capital or Institutions?. *Urban Studies* 47 (10): 2027–50. doi: 10.1177/0042098009359957.

Storper, M. 2013. *Keys to the City: How Economics, Institutions, Social Interaction, and Politics Shape Development*. Princeton: Princeton University Press.

Storper, M., and A. Venables. 2004. Buzz: Face-to-Face Contact and the Urban Economy. *Journal of Economic Geography* 4 (4):351–70. doi: 10.1093/jeg/4.4.351.

Thirsk, J. 1973. The fantastical folly of fashion: The English stocking knitting industry, 1500-1700. In *Textile History and Economic History: Essays in Honour of Miss Julia De Lacy Mann*, ed. N. B. Harte and K. G. Ponting, 50–73. Manchester: Manchester University Press.

Trinder, B. 2000. Industrialising Towns, 1700-1840. In *The Cambridge Urban History of Britain, Vol II: 1540-1840*, ed. P. Clark, 805–30. Cambridge: Cabridge University Press.

van Zanden, J. L. 2009. *The long road to the industrial revolution: The European economy in a global perspective, 1000-1800*. Leiden: Brill.

von Tunzelmann, G. N. 1981. Technical progress during the Industrial Revolution. In *The Economic History of Britain since 1700, Volume 1: 1700-1860*, ed. R. Floud and D. McCloskey, 143–63. Cambridge: Cambridge University Press.

von Tunzelmann, G. N. 1993. Technological and Organizational Change in Industry during the Early Industrial Revolution. In *The Industrial Revolution and British Society: Essays in Honour of Max Hartwell*, ed. P. K. O'Brien and R. Quinault, 254–82. Cambridge: Cambridge University Press.

Wahl, F. 2016. Does medieval trade still matter? Historical trade centers, agglomeration and contemporary economic development. *Regional Science and Urban Economics* 60: 50–60. doi: 10.1016/j.regsciurbeco.2016.06.011.

Wallerstein, I. 1989. *The Second Era of Great Expansion of the Capitalist World-Economy, 1730's-1840's. The Modern World-System*, Vol. III. Berkeley: University of California Press.

Weil, D. N. 2008. *Economic Growth*. New York: Addison Wesley.

Wrigley, E. A. 1967. A simple model of London's importance in changing English society and economy 1650-1750. *Past and Present* 37:44–70. doi: 10.1093/past/37.1.44.

Wrigley, E. A. 1978. Parasite or Stimulus: The Town in a Pre-Industrial Economy. In *Towns in Societies: Essays in Economic History and Historical Dociology*, ed. P. Abrams and E. A. Wrigley, 295–310. Cambridge: Cambridge University Press.

Wrigley, E. A. 1985. Urban Growth and Agricultural Change: England and the Continent in the Early Modern Period. *Journal of Interdisciplinary History* 15 (4): 683–728. doi: 10.2307/204276.

Wrigley, E. A. 1987. *People, Cities and Wealth*. Oxford: Basil Blackwell.

Wrigley, E. A. 1990. Brake or Accelerator? Urban Growth and Population Growth before the Industrial Revolution. In *Urbanization in History: A Process of Dynamic Interactions*, ed. A. van der Woude, A. Hayami, and J. de Vries, 101–12. Oxford: Clarendon Press.

Wrigley, E. A. 2004. *Poverty, Progress, and Population*. Cambridge: Cambridge University Press: Cambridge.

Wrigley, E. A. 2006. The transition to an advanced organic economy: Half a millennium of English agriculture. *The Economic History Review* 59 (3):435–80. doi: 10.1111/j.1468-0289.2006.00350.x.

Wrigley, E. A. 2010. *Energy and the English Industrial Revolution*. Cambridge: Cambridge University Press.

Wrigley, E. A. 2014. Urban Growth in Early Modern England: Food, Fuel and Transport. *Past & Present* 225: 79–112. doi: 10.1093/pastj/gtu032.

Zeileis, A. 2004. Econometric Computing with HC and HAC Covariance Matrix Estimators. *Journal of Statistical Software* 11 (10):1–17. doi: 10.18637/jss.v011.i10.